Applied Problems

Part One

1.1 Consider the following data: 65% of all radar contacts are surface ships, 23% are aircraft, and 18% are both surface ships and aircraft. Let S be the event that a radar contact is a surface ship and let A be the event that the contact is an aircraft.

(a) Compute the probability $P(A \cup S)$ that the contact is either a surface ship or an aircraft.
(b) Compute $P(A \cap S')$ and explain your answer in words.
(c) Write down (like the preceding parts) the symbolic version of: "The probability that the contact is neither an aircraft nor a surface ship." Compute this probability.
(d) Compute the probability that the contact is either an aircraft or a surface ship, but not both.

1.2 A fleet of eight fast attack submarines, #1 through #8, is to be assigned to patrol three regions A, B, and C, with 2 subs going to A, 5 to B, and 1 to C.

(a) In how many different ways can this be done?
(b) In how many ways can this be done if sub #1 must go to region A?

1.3 There are 6 pilots and 4 NFOs being screened to command 3 separate squadrons. The ten aviators are to be interviewed by the Wing CO.

(a) How many ways can the ten aviators be lined up awaiting their interviews?
(b) How many ways can the 3 COs be chosen?
(c) What is the probability that all 3 COs are chosen from the NFOs?
(d) What is the probability that all 3 COs are chosen from the pilots?

1.4 From a crew of 12, consisting of 3 women and 9 men, a team will be picked at random. Assume in this problem that all team members have equivalent work duties and thus the order of choice is irrelevant.

(a) How many ways can a team of 4 be chosen?
(b) How many ways can a team of 4 be chosen with no women?
(c) How many ways can a team of 4 be chosen with one woman?
(d) How many ways can a team of 4 be chosen with at least one woman?

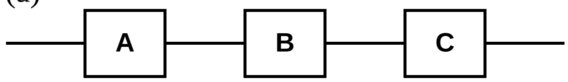
Now assume the 12 crewmen will be broken up into three teams of 3, 4, and 5 members each.

(e) How many ways can this be done?
(f) How many ways can this be done with the team of 3 consisting of all women?
(g) What if the 12 crewmen are to be broken up into three indistinguishable teams of 4 members each? How many ways can this be done?

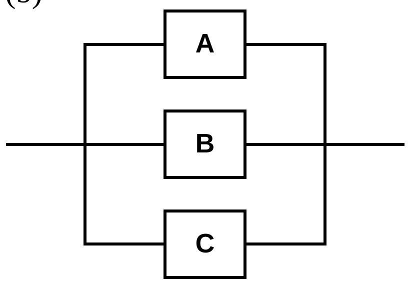
1.5 In each electrical system below, the components work with reliability (probability of working) given by the table to the right. Assume the failures of the components are independent. Compute the probability that the entire electrical system works.

component	reliability
A	0.7
B	0.8
C	0.9

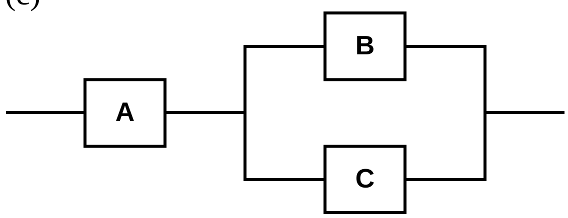
(a)



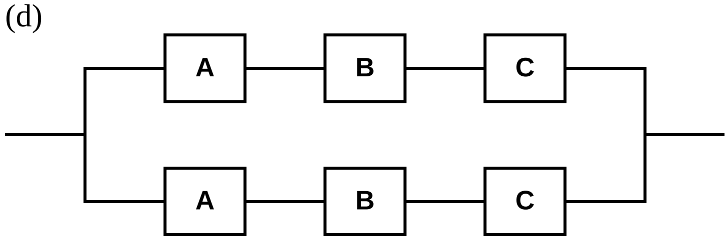
(b)



(c)



(d)



1.6 The following questions are based on the electrical systems in the preceding problem.

(a) In part (c) of the preceding problem, compute the probability that component **C** does not work given that the entire system works.

(b) In part (d) of the preceding problem, the configuration of (a) has been stacked $n = 2$ times to cause the entire system to be more reliable due to this *redundancy*. How tall ($n \geq ?$) must the redundancy "stack" be in order for the entire system have reliability at least 0.98?

1.7 Assume 80 % of all military personnel brought to trial are guilty (G) of the crime for which they are charged. Of those who are innocent (I), the jury acquits (A) 95 % . Of those who are guilty, the jury convicts (C) 90 % .

(a) What percent of cases brought to trial involve individuals who are both innocent and acquitted?

(b) What percent of cases brought to trial involve individuals who are both guilty and convicted?

(c) What percent of cases brought to trial end up with the individual being acquitted?

(d) What percent of acquitted individuals are in fact guilty?

(e) If an individual is convicted, then what is the probability that he/she is in fact innocent?

1.8 You are steaming in the Gulf of Sidra on routine freedom of navigation exercises. While performing these exercises you are attacked by a Flogger that fires three ASMs at your position. You have the following information available to you. Your probability of successfully destroying a missile with your SAM system is 0.4 for each inbound. Your CIWS has a probability of 0.8 of destroying each inbound when they close. Finally, each missile has a 0.95 probability of successfully completing its attack if it gets inside CIWS range. Compute the probability that your ship is hit by at least one of the ASMs. State your assumptions and explain your solution.

1.9 There is an air strike and a Tomahawk strike planned against an airfield with revetted aircraft positions. If *P*(successful air strike) = 0.75 and *P*(successful Tomahawk strike) = 0.60 and the strikes are independent, then what are the following probabilities:

(a) *P*(both strikes are successful)
(b) *P*(only the air strike is successful)
(c) *P*(at least one strike is successful)

Now assume that the strikes are dependent and that P(air strike succeeds | Tomahawk strike succeeds) = 0.9.

(d) What is the probability that the air strike succeeds and the Tomahawk strike succeeds?
(e) What is the probability that the air strike fails and the Tomahawk strike succeeds?
(f) What is the conditional probability that the Tomahawk strike succeeds given the air strike fails?

1.10 Rocket motors for your SM1MR missiles come from one of three possible manufacturers: Estes Industries (20%), Revell Corporation (50%) and Monogram Heavy Industries (30%). Based on historical data, Estes has a 5% failure rate, Revell has an 8% failure rate, and Monogram has a 2% failure rate. You do not know which manufacturer made the motors for your missiles. However, you do know that all the motors came from the same manufacturer.

(a) When you attempt to fire the first missile, it is a dud. What is the revised probability that your motors came from Estes? From Revell? From Monogram?
(b) Now suppose that the second missile you attempt to fire is a successful launch. What are the revised probabilities based on these two outcomes?

1.11 You must transit into restricted waters by one of two passages so that you can make repairs to your inoperative SAM system. You know that each inlet is guarded by one Nanuchka armed with ASMs. Intelligence reports that one of the two small craft is out of missiles and the other has only one missile left. The missile's Pk (probability of kill) against you is 0.7. Both patrol boats are also armed with a 76MM gun which has a Pk against you of 0.2. If the patrol boat fires a missile at you then you can engage it at long range and have a 0.8 probability of killing the patrol boat before it can engage you with guns. Assume that the passages are narrow enough so that you must pass within gun range of each patrol boat (if it is not sunk), and that if it does not fire a missile at you the two of you will engage simultaneously with guns as soon as you are in gun range. What is the probability that you successfully transit into the restricted waters? State your assumptions and explain your solution.

Part Two

2.1 The probability that a single radar set, when working properly, will detect an enemy plane when scanning a certain region is 0.7. We have 5 working radar sets set up to cover that region, each operating independently of the others. Compute the probability that:

(a) exactly four sets will detect the plane;
(b) at least one set will detect the plane.
(c) Assume that fighters will be scrambled if at least two radars show a detection. Compute the probability of a scramble if an enemy plane enters that region.

2.2 Independent firings of artillery rounds have a fixed probability p of hitting a practice target for any one round.

(a) If $p = 0.8$, then what are the chances you will observe 42 hits or more when you fire 50 rounds?
(b) If $p = 0.7$, then what are the chances you will observe 42 hits or more when you fire 50 rounds?
(c) Assume there were 42 hits among 50 rounds. Explain why you can safely discard the assertion that $p \le 0.7$.

2.3 An unmanned aerial vehicle program sends a single UAV on the same difficult reconnaissance mission each day. Assume that each day's mission is successfully completed with probability $p = 0.7$. Assume a binomial distribution is applicable in order to solve the following problems.

(a) Compute the probability that in 14 days the mission will be successfully completed 10 times.
(b) Compute the probability that in 14 days the mission will be successfully completed 10 or fewer times.
(c) Compute the probability that in 14 days the mission will be successfully completed at least 12 times.
(d) What would the daily mission success probability p have to be so that in a 14 day period the mission would be successfully completed at least 12 times with probability 0.75 ? Give your answer correct to at least 2 decimal digits.
(e) Write down briefly why a binomial distribution might *not* be applicable to this scenario.

2.4 Your platform carries a certain type of missile. The manufacturer guarantees each missile to hit its target with probability 0.8; assume this actually is true and that each missile operates independently of the others. Assume you fire 5 missiles.

(a) What is the probability that all 5 missiles hit the target?
(b) What is the probability that no missile hits the target?
(c) What is the probability that exactly 3 missiles hit the target?
(d) What is the probability that at least 3 missiles hit the target?

Assume you fire 20 missiles.

(e) What is the probability that at least 15 missiles hit the target?
(f) At least *n* of the 20 missiles will hit the target with probability 0.95 or more. Compute the largest possible integer value for *n*.

Assume your mission is to knock out a target and that intelligence says that this will require 25 missile hits.

(g) What is the minimum number of missiles you need to fire in order to be at least 95% certain of knocking out the target?

2.5 Six of 13 radar sets do not work properly. Assume 5 are selected at random from the 13 to be tested. Compute the probability that among these 5:

(a) all work properly
(b) none work properly
(c) at least two work properly.

2.6 SUNA Inc. makes small turbines and 3.5% are of them are defective. The Navy needs 40 defect-free turbines.

(a) If the Navy buys 42 SUNA turbines, then what is the probability it gets at most 2 defectives?
(b) If the Navy buys 50 SUNA turbines, then what is the probability it will get less than 40 that are defect-free?
(c) What is the minimum number of turbines that you recommend the Navy purchase so as to be at least 99.9% sure of having 40 that are defect-free?

2.7 You ordered truck batteries to ship overseas. Your battery supplier assures you that in your shipment of 1000 batteries, no more than 20 will fail to meet your specifications. Being a savvy statistician you decide to test this claim. You randomly choose 50 of these batteries to test and find 3 among them that do not meet specs. Assume the manufacturer's failure rate is in fact 2%. Compute the probability of you finding 3 or more defectives among your sample of size 50.

2.8 There is a bag of 20 fresh batteries for "calculator emergencies" stored on the company wardroom table. Unfortunately, some plebe comes along and mixes in 4 dead batteries, thinking the bag is for recycling. On your way to your SM230 exam you notice your TI has gone dead. You know you're behind schedule so plan on swapping out your 4 old batteries when you get to class. You grab 6 batteries from the bag of 24 as you rush to the exam.

(a) Compute the probability that you have 6 fresh batteries.
(b) Compute the probability that you have no fresh batteries.
(c) Compute the probability that you have at least 4 fresh batteries.

2.9 In a lot of 30 missiles, 4 are defective. A random sample of 8 is selected from the lot and tested. Compute the probability that:

(a) exactly one of the sampled missiles tests defective;
(b) none of the sampled missiles tests defective;
(c) at least 2 of the sampled missiles test defective.

2.10 Artillery has a probability 0.2 of hitting a practice target on any one round. The target is considered destroyed when it has been hit by a total of three rounds.

(a) Compute the probability that the target is destroyed when the 3^{rd} round is fired (i.e., three hits in a row).

(b) Compute the probability that the target is destroyed when the 4^{th} round is fired.
(c) Compute the probability that the target is destroyed after the 10^{th} round. (Hint: this is equivalent to the event that there are 2 or fewer hits among the first 10 rounds.)

2.11 You are part of a six ship battle group transiting a mined section of the Straights of Hormuz. The probability that any one ship detonates a mine is 0.6. Assume independence. If a ship detonates a mine, it is sunk. Also, assume that there are sufficient mines laid that the lethality of the minefield does not diminish with attacks.
(a) What is the probability that no more than 2 ships detonate a mine?
(b) What is the expected number of ships out of your battle group to successfully transit the straights?
(c) What is the probability that all the ships of the battle group successfully transit the straights?

2.12 You are attempting to determine the proper number of missiles to launch to kill a target. You need to have at least two harpoon hits to destroy the target. You know the following probabilities:

P(target detecting a harpoon) = 0.7
P(target successfully engaging a harpoon | harpoon detected) = 0.6
P(harpoon hits | target does not detect and successfully engage the harpoon) = 0.8

(a) What is the probability that any one launched harpoon hits the target?
(b) How many harpoons should you fire to have an expected number of hits at least two?

2.13 Suppose numerous exercise firings of the Hatchet missile have indicated that there is a quality control problem at the factory. On the average, 7 of every 100 were defective in that they failed to fire on command. Test firing destroys the booster section so missiles that are test fired cannot be shipped (very expensive). The CEO decides to minimize the number of defectives shipped by sampling 5 from each lot of 100 and rejecting the entire lot (send back for rework) if one or more fail to fire on command.
(a) What is the expected number of defectives in the sample of size 5?
(b) What is the standard deviation of the number of defectives in the sample of size 5?
(c) Compute the hypergeometric probability that a lot will be rejected.

2.14 You determine that inbound missiles are arriving at your position at a rate averaging 5 missiles per hour. Assume a Poisson process.
(a) What is the probability of eight missiles arriving in a given two hour period?
(b) What is the probability of two or more missile arrivals in a given two hour period?

2.15 Manufacturer Alpha produces a continuous band of videotape that is used to fill your unit’s supply of videocassettes. Assume the number of flaws in the videotape follows a Poisson distribution and that there are on average 4 flaws per 100 linear feet.
(a) What is the probability that in a 50-foot length of videotape there will no flaws?
(b) What is the probability that in a 50-foot length of videotape there will be at least 3 flaws?
(c) What is the probability that in a 500-foot length of videotape there will be at least 30 flaws?

2.16 You are defending a small bridge against attack. The enemy has a squad of 10 troops: 6 riflemen and 4 machine gunners. Your single round probability of killing a rifleman is 0.5 and your single round probability of killing a machine gunner is 0.4. You have 10 rounds left. You decide to fire one round at each enemy as they appear on the hill, one at a time, and then your squad will defend with fixed bayonets. From experience, you know that you can hold the position if you begin hand-to-hand combat with 7 or less of the enemy. What is your probability of success? State your assumptions and explain your solution.

2.17 Your aircraft tracking facility is under review. At 0800, your morning shift staff of 8 technicians begins work. As each aircraft comes into your region, one staff member begins an initial surveillance that lasts for at least 30 minutes. Because of the level of surveillance, each staff member can only track one aircraft at any one time. Assume that aircraft come into your region following a Poisson process at the rate of 20 per hour.

(a) What is the probability that in the first 15 minutes (from 0800 to 0815) your staff will not be able to track all aircraft that come into your region?
(b) What is the probability that in the first 30 minutes your staff will not be able to track all aircraft that come into your region?
(c) Over what time interval $0 \le t \le T$ are you 99% certain of being able to track all aircraft that come into your region?
(d) What is the maximum aircraft arrival rate for which you could be 99% certain of tracking all aircraft coming into your region during the first 30-minute interval?
(e) Assume that 99% of the time you must be able to track all aircraft coming into your region over the first 30-minute interval. Assume the aircraft arrival rate is 20 aircraft per hour. What is the minimum number of staff that you must have available in order to accomplish your task?

Part Three

3.1 Assume that the number of years that a Naval Academy graduate stays in active duty is normally distributed and has a mean of 7 years and a standard deviation of 3 years.

(a) What percentage of Naval Academy graduates will stay in for 10 years or longer?
(b) 75% of the Naval Academy graduates will stay longer than y years; compute y.

3.2 The seal on a gas mask has a mean lifetime of 10 years and a standard deviation of 1 year. Assume a normal distribution.

(a) What is the probability that any given seal will last between 10 and 12 years?
(b) What is the probability that any given seal will last less than 9 years?
(c) If the manufacturer warrantees the gas masks for y years, then what value should y be so that only 3% of the gas masks fail during the warrantee period?

3.3 A certain type of jet can use afterburners at full fuel capacity (with no external tanks) for a mean of 45 seconds before it runs out of fuel. Assume this time is normally distributed with a standard deviation of 5 seconds. What is the probability that one of these jets with full fuel and no external tanks can burn for 50 seconds to get to friendly territory and avoid interception? Assume the jet needs no more fuel once in friendly territory; it will ditch.

3.4 The A/OA-10 Thunderbolt has the AN/GAU-8 20mm Avenger seven-barrel gattling gun. Assume that the barrel diameter of the gun has to be 30 ± 0.01 mm for it to properly fire a certain type of depleted uranium round. A factory produces barrels with mean diameter 30 mm and standard deviation 0.005 mm.

(a) Assuming a normal distribution, what percentage of these barrels will have to be scrapped?
(b) Re-compute the percentage in the preceding part for the case when the factory mean diameter is 29.992 mm (and the standard deviation remains the same).

3.5 When a certain ship class comes into production, any number of that class is known to have a mean life span of 10.2 years before it needs an overhaul. Assume this life span is normally distributed with a standard deviation of 0.6 years.

(a) Compute the probability that any given ship of the class will last more than 10.5 years before needing an overhaul.
(b) Compute the probability that among 20 ships of this class that are chosen at random, 15 or fewer will need an overhaul before 10.5 years.

3.6 Assume that the maximum distance a Trident C3 missile travels is normally distributed with a mean of 1,000 nm and a standard deviation of 75 nm.

(a) What is the probability of a C3 missile traveling more than 1,100 nm?
(b) What is the probability that in four independent firings of C3 missiles, none will travel further than 1,100 nm?
(c) What is the probability that in four independent firings of C3 missiles, the average distance traveled will be less than 1,100 nm?

3.7 Assume the height of a midshipman is normal with population mean 70.7 inches and population standard deviation 2.56 inches.

(a) Compute the probability that one mid will be shorter than 68 inches.
(b) Compute the probability that one mid will be taller than 6 feet.
(c) If two mids are picked at random, then what is the probability they will both be taller than 6 feet?
(d) Compute the 3rd quartile q_3 for a mid's height. (This is the height below which 75% of midshipmen heights fall.)
(e) Compute the 90th percentile for a mid's height. (This is the height below which 90% of midshipmen heights fall.)
(f) If 10 mids are picked at random, then what is the probability that at least 3 will be shorter than 68 inches?
(g) If 10 mids are picked at random, then what is the probability that their average height will be greater than 72 inches?

3.8 Amplifiers for your satcom transceiver exhibit an exponential failure rate, with the average time between failures equal to 6000 hours.

(a) What is the probability that an amplifier selected at random fails in the first 8000 hours of use?
(b) What is the probability that an amplifier selected at random fails after 4500 hours of use?

(c) What is the probability that an amplifier selected at random fails after 2000 hours, given that it has already lasted for 4000 hours?

3.9 You have just climbed into your life raft after having ejected from your F/A-18 after what had been a perfect mission until you ran out of JP5. You determine that ships appear to be passing you in a random fashion. No two ships ever pass by you at the same time. Ship arrivals are independent, and the rate appears constant at 3 per hour.

(a) What is the probability that a ship will pass in the next ten minutes?
(b) That ship does not see you. What is the expected time until the arrival of the next ship?

Part Four

4.1 A random sample of 30 oxygen tank hoses had an average useful lifetime of 8.4 years and standard deviation 0.9 years. Compute a 95% confidence interval for the mean useful lifetime of these hoses. Assume lifetime is normally distributed.

4.2 Assume that the life expectancy of the aluminum wing support beam of an F/A-18 hornet is normally distributed. A random sample of seven of these beams showed an average lifetime of 16.7 years and a standard deviation of 1.3 years. Compute a 90% confidence interval for the support beam life expectancy.

4.3 The seal on a gas mask has a mean lifetime of 10 years and a standard deviation of 1 year. Assume a normal distribution.

(a) What is the probability that a random sample of 5 of these seals will have an average lifetime between 9.5 and 10.5 years?
(b) How large should the sample size be in order to be at least 95% confident that the sample mean will lie within 0.5 years of the population mean?

4.4 The acceptable diameter of a barrel of an Abrams tank gun is important to maintain because the rounds will wear away at the interior with use. Each of ten tank guns independently fired a fixed number of rounds and the resulting diameter measurements (in mm) are given in the following table.

17.69	20.80	21.36	19.31	18.70	17.52	20.44	18.31	20.76	18.79

(a) Generate a boxplot for this data and report the sample median.
(b) Compute a 90% confidence interval for the mean diameter of the barrel after firing this number of rounds. Assume barrel diameter is normally distributed.

4.5 Assume that the shelf life of one MRE is normally distributed.

(a) A random sample of 20 of supplier Alpha's MREs had a mean shelf life of 4.25 years and a standard deviation of 0.65 years. Compute a 95% confidence interval for the mean shelf life of this supplier's MREs.
(b) A random sample of 25 of Supplier Bravo's MREs had a mean shelf life of 5.4 years and a standard deviation of 0.8 years. Compute a 95% confidence interval for the difference in the mean shelf life. Does there seem to be a statistically significant difference in the mean shelf life for the two suppliers? Explain.

4.6 A CO2 scrubber on a submarine has a limited life due to particle impurities in the air. Two manufacturers, Alpha and Bravo, are interested in supplying scrubbers. When checking out random samples from each manufacturer, the following table of lifetimes (in months) is obtained.

Alpha	6.6	6.6	6.2	6.0	7.5	6.7	5.8		
Bravo	2.1	8.2	6.0	7.5	8.0	9.1	7.4	6.6	8.0

Use this data and assume that the lifetime of a CO scrubber is normally distributed.

(a) Generate a double boxplot that compares the two sets of data side-by-side.
(b) Construct a 90% confidence interval for the mean lifetime of Alpha's scrubber.
(c) Construct a 90% confidence interval for the mean lifetime of Beta's scrubber.
(d) Construct a 90% confidence interval for the difference in mean lifetimes of the two manufacturers' scrubbers. Is there statistical evidence that one scrubber lasts longer than the other? Explain.
(e) Supplier Bravo claims that the first data entry is in error. Repeat all of part (d) with the 2.1 removed from the data.
(f) Explain why removing this data point without justification is unethical ("lying with statistics").

4.7 Assume that each 5.56mm round of ammo has a factory rated probability p of misfiring and that firing performances of individual rounds are independent of one another. LCPL Wilcox fires a burst of 500 rounds resulting in 80 misfires. You are instructed by your CO to estimate from this data the value of p. To fulfill your task, construct a 95% confidence interval for p and write a short "report" stating your conclusion so that your CO will understand it.

4.8 The A/OA-10 Thunderbolt has the AN/GAU-8 20mm Avenger seven-barrel gattling gun. Assume that the barrel diameter of the gun has to be 30 ± 0.01 mm for it to properly fire a certain type of depleted uranium round. Fifty barrels were randomly selected from a factory's production and 8 did not meet specifications. Compute a 95% confidence interval for the proportion p of this factory's barrels that do not meet specs. (Compare this interval with the one in the preceding problem.)