SM362 Syllabus / Homework **Prof. Baker / Spring 05**

Text: Contemporary Abstract Algebra by Joseph Gallian, 5th edition, Chapters 1-11.

Lesson	Topics	Homework
1	Well Ordering, Division Algorithm, Divisors	p23 1,4,8,12,16,17
2	GCD, Primes, Fund. Thm. of Arithmetic	
3	Induction, Equivalence Relations	p23 21,46,49
4	Modular Arithmetic	p23 3,6,11,13,28,39,42
5	Mappings	p23 50
6	Symmetries of the Square	p37 1,2,3
7	D_4 and S_n	p37 6,7,8,12,13
8	Definition of a Group , Examples	p53 3,4,5,7,9,14
9	Elementary Group Properties	p53 25,26,29,32,34,35
10	Finite Groups and Subgroups	p67 1,2,4,6,11,12,14,17,23
11	Important Subgroups: <a>, C(a) , Z(G)	p67 24,29,38,41,50,51
12	Review	
13	**Hour Exam 1** **Mon. Feb. 7**	
14	Cyclic Groups	p82 1,2,4,5,8,14
15	Cyclic Groups	p82 17,19,25,40,43
16	Subgroups of Cyclic Groups	p82 49,50,51,52,53,59,62
17	Permutation Groups: S_n	p111 1,2,3
18	Cycles in S_n	
19	Decomposition	p111 4,6,7,9,11
20	Even and Odd Permutations	p111 12,13,14,24,28,30
21	Group Isomorphisms	p129 1,4,6,7
22	Cayley's Theorem	p129 38,39
23	Automorphisms	p129 2,3,15,17,30
24	Review	
25	**Hour Exam 2** **Wed. Mar. 9**	
26	Cosets	p145 1,2,3,4,8,9
27	Lagrange's Theorem	p145 10,12,14,18,20
28	Lagrange's Theorem	p145 20,25,26,31,33,40
29	External Direct Product	p162 4,5,8,10,11,14,23
30	Z_n and U_n as External Direct Products	p162 28,32,39,41,46,47
31	Normal Subgroups	p186 1,3,4,8
32	Factor Groups	p186 9,10,16,19,34,40,41
33	Factor Groups , Cauchy's Theorem	p186 46,47,50,53
34	Internal Direct Products	p186 69
35	Group Homomorphisms	p205 1,2,3,4,8
36	Group Homomorphisms	p205 12,20,21,25
37	Fund. Thm. of Group Homomorphisms	p205 30,37,41,46
38	Review	
39	**Hour Exam 3** **Mon. Apr. 18**	
40	Fund. Thm. of Finite Abelian Groups	p219 2,5,8,15
41	Fund. Thm. of Finite Abelian Groups	p219 17,19,22,34
42,43,44	Review	

Supplementary Problems for Extra Credit

S1. Prove that $n^3 - n$ is always divisible by 3 , by induction on n.

S2. If p is an odd prime, show that p is of the form 4n+1 or 4n+3 for some n.

S3. If p>3 is an odd prime, show that p is of the form 6n+1 or 6n+5 for some n.

S4. Show than no integer u = 4n+3 can be written as $u = a^2 + b^2$, where a and b are integers.

S5. Prove by induction that for a set having n elements, the number of 1-1 mappings of the set onto itself is n! .

S6. Let $G = D_n$, the dihedral group of order 2n. Prove that:
(a) If n is odd and a ¿ G satisfies ab = ba for all b ¿ G, then a = e.
(b) If n is even show that there is an a ¿ G, a ¿ e , such that ab = ba for all b ¿ G .

S7. If G is a group in which $(ab)^i = a^i b^i$ for three consecutive integers i , then G is abelian. (Note: This is false if it's only true for 2 consecutive integers.)

S8. If A,B are subgroups of an abelian group G, let AB ={ab | a ¿ A, b ¿ B} . Prove that AB is a subgroup of G .

S9. p111, #52.

S10. p131, #30

S11. p162, #60

S12. p186, #44

S13. p186, #60

S14. p189, #56

S15. If G is a non-abelian group of order 6, prove that G ¿ S_3 .

S16. Prove that a group of order p^2 , p a prime, has a normal subgroup of order p.

S17. Prove that a group of order p^2 is abelian.

S18. Find all isomorphism classes of groups of order 9.

S19. Find all isomorphism classes of groups of order 8.