Mathematics Department

Professor Mark D. Meyerson
Chair

Mathematics provides a logical framework and a language indispensable to understanding the technical world in which we live. The following description summarizes the many contributions to this field of study made during the past academic year by the faculty and midshipmen majors of the Mathematics Department of the U.S. Naval Academy (USNA). The results cited reveal the great scope, diversity, and applicability of mathematics and offer glimpses of its intellectual beauty and appeal.

Several midshipmen conducted research projects either as Honors Mathematics Majors or in specially created projects under the guidance of faculty members. Several faculty spent many hours serving as mentors and readers for capstone projects required of all midshipmen majors. Listed below are our honors mathematics majors, their projects, and their faculty mentors:

Shaun P. Hayes: "Planning the Clearance of Open Ocean Minefields"
(Professor W. Charles Mylander)

Daniel Post: "Toeplitz Matrices Over Finite Fields"
(Professor Geoffrey L. Price)

Once again, the USNA Mathematics Department produced a wide range of scholarly work that appeared as technical reports or as publications in refereed journals throughout the world. Dozens of articles appeared as applications of mathematics or as pure mathematical research. Topics that Mathematics Department faculty researched include: mathematics pedagogy, chaos and dynamical systems, mathematical physics and cosmology, algorithms for computers, wavelets, cryptology, ocean acoustics, navy ship degaussing, image recognition, quantum computing, and basic mathematical research in areas such as algebra, analysis, combinatorics, differential equations, matrices, number theory, operator theory, and statistics.

In addition to many independent research projects, several research projects were sponsored in whole or in part by a variety of sources, such as:

Department of Health and Human Services
Honeywell
Office of Naval Research
Naval Research Laboratory
Naval Air Warfare Center
Naval Surface Warfare Center
Naval Academy Research Council
National Science Foundation
National Security Agency

During the past year, members of the USNA Mathematics Department presented the results of their scholarly activities on over sixty occasions at professional mathematical meetings and colloquia throughout the United States and abroad. This activity, along with publication, enhances the academic stature of the Naval Academy and promotes the professional growth and reputation of those individuals involved. Through research activity, the faculty expand their intellectual horizons and stay vital in their disciplines. They contribute to the discovery of new mathematics. And they develop new materials and ideas that they can share with midshipmen students in their mathematics courses and research projects.

Sponsored Research

Applications and Theory of Controlling Symbol Dynamics: Communicating with Chaos

Researcher: Associate Professor Erik M. Bollt
Sponsor: National Science Foundation (NSF)

This project addresses problems in controlling chaos and controlling symbol dynamics with small perturbations. Recent work has demonstrated the utilization of symbolic dynamics representation of controlled chaotic orbits for communications with chaotic signal generators. The evolution of trajectories of a chaotic dynamical system is equivalent to symbolic dynamics in an appropriate symbol space. The recent advent of controlling chaos using the OGY (Ott, Grebogi, and Yorke) technique and a variety of targeting algorithms, with physical applications, has demonstrated that chaos can be mastered, and inherent instabilities can be used as an advantage in allowing small deliberate perturbations to cause large signal variations. Coupling control of chaos through small perturbations with learning the grammar of the corresponding symbol dynamics means that the control perturbations are actually a coding scheme on the original dynamics. Controlling symbol dynamics using a map based description of the dynamics requires resolution of the following issues: learning the response of map iterates to variations in the control parameters, learning the semi-conjugacy, or coding function, between the dynamics of the map on the attractor and the grammar of the corresponding symbol dynamics, and finding the minimal grammar which is dependent on the appropriate choice of the partition in phase space. In particular, extensions include on-going work on communicating in higher dimensional dynamical systems. Investigations include issues of practical noisy environment control versus bandwidth trade-off and issues of ergodicity, learning system response to controls, and symbol dynamics of a chaotic dynamical system which is known only by time-series embedding of experimental data, and the further development of practical grammar learning algorithms.

RUI: Combinatorial Control of Chaos, Symbolic Dynamics, Optimal Control and Inverse Frobenius-Perron Problem

Researcher: Associate Professor Erik M. Bollt
Sponsor: National Science Foundation (NSF)

This project addresses global control strategies of chaotic dynamical systems. In this research program, control strategies based on transfer operator representations of the action of a chaotic dynamical system, on its invariant subspace are being developed. The objective in this research is to develop these new techniques, with the primary goal being that they might naturally be used by experimentalists. To ensure such practicality, the researcher has been fostering on-going relationships, and developing new relationships, with experimental groups.

By definition, a chaotic dynamical system is highly sensitive to small variations of states or parameters. During the last decade, a major emphasis of dynamical systems theory research has been focused on the realization that sensitive dependence provides that control of a chaotic dynamical system can be flexible and efficient. However, the majority of the research has relied on local analysis of variations, which has limited application for global control objectives.

Poroelastic Media

Researcher: Professor James L. Buchanan
Sponsor: National Science Foundation (NSF)

An important practical problem in ocean acoustics is the recognition of the shape and/or composition of an object in the water or lying on the seabed. The investigator and his collaborators made progress on the problem of recognizing the location and shape of an object floating in the ocean under the assumption that the seabed is perfectly reflecting. The method employed was the Intersecting Canonical Boundary Approximation (ICBA) which attempts to match the acoustic field locally to the field of a cylinder. This method works well unless the source/receiver is located at a near null of the acoustic field. Hence for this method to be successful the frequency of the source must be varied to avoid such nulls.

Human bone is a poroelastic medium consisting of an outer cortical layer surrounding a high porosity cancellous interior with an interstitial lipid fluid. An important question is can the porosity of the cancellous bone be ascertained by acoustic interrogation, thereby determining whether the bone is osteoporotic. This year the investigator worked on developing a finite elements code for calculating the acoustic field arising from cancellous bone in order to determine such parameters as its porosity, permeability, pore size and moduli of elasticity, knowledge of which would permit characterization of the health of the bone.

The Effects of Sound on the Marine Environment (ESME)

Researcher: Professor James M. D'Archangelo
Sponsor: Naval Research Laboratory (NRL),
Advanced Information Technology Branch

The majority of time was spent learning about the parameters of the problem and assisting in the development of a proposal of research. The goal of the research is to develop a set of software tools for evaluating the impact of diverse sound sources on organisms in the underwater environment. The impact of sonar on cetaceans is of particular concern. The input to these tools will include the parameters of the acoustic sources: local variables, environmental variables, and relevant spatial and temporal scales of environmental change, particularly with respect to climatology and related marine mammal habits. The output from the tools will be predictions of the ESME effects of proposed operations, and guidelines for modifying the acoustic aspects of the proposed operations as required.

There are three components of the ESME program: (a) Modeling and Simulation Architecture, (b) Sound Sources and Propagation Loss Modeling, and (c) Biological Models. Our work at NRL will primarily be concerned with part (a) the Modeling and Simulation Architecture. Our primary task will be to integrate the products of the other two ESME components into simulation tools mentioned above. There are three future deliverables: a Monte-Carlo based Discrete Event Simulation model, an Entity-Based Simulation Model, and a Tactical Decision Aid.

Closed-loop Degaussing Using Both Naval Vessels and Scale Physical Models

Researcher: Associate Professor Gary O. Fowler
Sponsor: Naval Surface Warfare Center (NSWC)

Naval vessels containing ferrous material emit detectable magnetic signals. NSWC administers a project that studies and measures this phenomenon. Data is collected from naval vessels under a variety of conditions and from scale models under similar conditions. A goal of the project is to connect these data in a manner that will allow measurements from the scale model to predict measurements on the naval vessel. This analysis is both statistical and mathematical in nature. It includes both the design of the experiments and the analysis of the data

Discriminating and Co-discriminating Groups

Researcher: Professor Anthony M. Gaglione
Sponsor: Naval Academy Research Council (NARC)

Discriminating and co-discriminating groups were first introduced by Baumslag, Myasnikov and Remeslennikov with an eye toward applications to the universal theory of various groups. This project is concerned with proving that a number of groups satisfy certain separation conditions which are the defining conditions for discrimination and co-discrimination. In other words, we are investigating whether certain groups are discriminating and others are non-discriminating. In order to explain what these conditions are, we need to fix some terminology. Let G and H be groups. We say that G *separates* H provided that for every nontrivial element h of H there exists a homomorphism $\phi_h : H \to G$ such that $\phi_h(h) \neq 1$. We say that G *discriminates* H provided that for every finite nonempty subset S of H of nontrivial elements there is a homomorphism $\phi_S : H \to G$ such that $\phi_S(h) \neq 1$ for all h in S. We term G *discriminating* provided that it discriminates every group which it separates. The theory of algebraic geometry over groups is a very useful tool for these investigations. We have been investigating several problems posed by A. Myasnikov some of which are:

Problem 1: Describe in terms of Ulm or Szmielew invariants Abelian discriminating groups.

Problem 2: Is it true that the only one-relator groups which are not domains (in the sense of algebraic geometry over groups) are those which contain Abelian normal subgroups?

Problem 3 : Describe one-relator groups which contain Abelian normal subgroups.

Problem 4: Is the restricted wreath product ZwrZ discriminating?

Problem 5: Are the free metabelian groups discriminating?

Problem 6: Are the non-Abelian finitely generated (f.g.) nilpotent groups of class 2 (nil-2) discriminating? In particular, are the free f.g. nil-2 groups discriminating?

We have answered Problems 4 and 5 in the negative and we have partial results concerning Problem 6 and Problems 1-3, so this project is on-going. The results mentioned here are contained in the papers "Discriminating Groups," (accepted for publication by the *Journal of Groups Theory*) and "Groups Whose Universal Theory is Axiomatizable by Quasi-identities" (in preparation).

On Toric Varieties and Codes

Researchers: Professor W. David Joyner and Assistant Professor William N. Traves
Sponsor: National Security Agency (NSA)

This project investigates the construction and properties of Goppa codes — methods to encode data for fast and accurate transmission across a noisy or unstable channel. These codes are linear error-correcting codes that quite often have fast decoding algorithms. Goppa codes associated to algebraic curves are well-studied objects. One way to define the code is, roughly speaking, to evaluate a linear subspace of functions on the curve at a list of n points on the curve. This vector space of n-tuples gives rise to a Goppa code of length n. In particular, the data transmission properties of these codes are related to the algebraic and geometric properties of the curves. This project aims to extend these constructions to algebraic and projective surfaces, in particular, to toric surfaces.

Nature of Spacetimes with Mild Singularities and Cauchy Horizons

Researcher: Professor Deborah A. Konkowski
Sponsor: National Science Foundation (NSF)

In this project, spacetimes with mild singularities and Cauchy horizons are being studied. Mild singularities include quasiregular and nonscalar curvature singularities. In the case of quasiregular singularities, particle paths end suddenly with no warning from infinitely-increasing tidal forces, while in the case of nonscalar curvature singularities some, but not all, particles moving near the singularity feel infinite tidal forces. A Cauchy horizon is the boundary of the causal development of spacetime. Cauchy horizons and singularities are intimately connected, including through the cosmic censorship conjecture, which continues to be debated.

In particular, this researcher is studying (1) A quantum-mechanically singular spacetimes, (2) two previously-developed stability conjectures, and (3) the global structure of mild and/or unusual singularities. This work is done in collaboration with T.M. Helliwell (Harvey Mudd College).

In the first case, following Horowitz and Marolf and work begun by T.M. Helliwell and D.A. Konkowski, classical test particles are replaced with quantum test wave packets to test the singularity structure of spacetimes which classically possess mild singularities. Preliminary results were presented at conferences in Rome, London, Texas, Michigan and Washington, D.C. An essay on the subject won an Honorable Mention in the Year 2000 Gravity Research Foundation Essay Contest and will appear as a paper in the June volume of *General Relativity and Gravitation*.

In the second case, conjectures proposed by T. M. Helliwell and D. A. Konkowski to predict whether various mild singularities and Cauchy horizons are stable are being investigated further. Thus far, the singularity conjecture always holds true but the Cauchy horizon conjecture misses purely Weyl singularities which occur in the plane wave and exact mass-inflation spacetimes. A summary of this work was written up for the upcoming GR16 conference in Durban, South Africa and D. A. Konkowski has been invited to present a talk at the conference in the session on "Mathematical Studies of the Field Equations."

In the third case, spacetimes with nonscalar and quasiregular singularities are under investigation. This includes preliminary work on: (1) spacetimes with nonscalar singularities due to Siklos, (2) quasiregular singularities in the context of dislocations and disclinations and in the context of Columbeau's extended theory of distributions, and (3) the structure of some spacetimes with directional singularities.

The Mathematics of Entanglement

Researcher: Associate Professor Robert B. Lockhart
Sponsor: Naval Research Laboratory (NRL)

This work, which is a continuation of previous summers' work done with Dr. M. Steiner at the Naval Research Laboratory, has as its goal the establishment of various mathematical properties for the sets of separable and entangled states of finite dimensional quantum systems. The purpose is to aid in understanding these systems, which are at the heart of quantum computing and quantum information.

The method used is linear algebra and Hilbert space theory along with geometric ideas. For instance, in examining the size of the set of low rank separable states, the researcher showed that that set could be embedded into the appropriate Grassmann and Stiefel manifolds. An application of Sard's theorem then showed that the low rank separable states form a set of measure zero within the set of low rank states and in fact have a complement which is generic. A publication summarizing this result is being prepared.

The main result of this period's work is the Sandwich Theorem for separable states and the construction of convex neighborhoods of entangled states for the maximally entangled states and others like them. The Sandwich Theorem shows that all separable states lie between two easily determined hyperplanes. As the number of particles increases, the distance between these planes goes to zero. This result has been submitted and is on the Los Alamos National Laboratory e-print website.

Vector Implementation of Godunov's Method for Euler's Equations

Researcher: Associate Professor Thomas J. Mahar
Sponsor: Naval Surface Warfare Center, Indian Head (NSWC-IH)

Godunov's method for the Euler equations for fluid flow was implemented in MatLab as vector code. The fluid was allowed to consist of two fluids, each with its own equation of state and thermodynamic properties. The interface movement was treated as a pure advection phenomenon for this first coding. Shock fronts were captured quite well this way, while material interfaces became smeared. The very large pressure and density changes at the material interface cannot be modeled well as a pure advection process. Work on improving the performance of the code at interfaces will continue with the implementation of some version of the ghost fluid method.

Adaptive Analytic Methods for a Cauchy Problem

Researcher: Professor Peter A. McCoy
Sponsor: Naval Research Laboratory (NRL)
Directed Energy Effects Branch, Code 6330

The technical objective is to develop and adapt analytic solutions for the mixed boundary value problem (Cauchy Problem) with certain geometries that are to be used in validating the Naval Research Laboratory High Accuracy Scattering and Propagation Code (HASP). The application of the code is to model a cavity with an aperture radiated by two or more electromagnetic beams. The problem concerns radiation effects on interior components of the NATO GENEC missile assembly.

The problem evolved in two directions. The first was numerical and concerned absorbing boundary conditions for the Enquist-Majda one-way wave equation in the context of 2-D and 3-D electromagnetic scattering. The investigator developed a homogeneous semi-isotropic FDTD scheme for the EM-equation that lead to an improvement over the standard Mur finite-differencing scheme in the 2-D and 3-D in terms of the simplicity of the stability and dispersion calculations. However, the scheme did not yield the anticipated simplification of the Yee differenced Maxwell equations. The second direction concerned adapting wavelet methods to the HASP code. The method pursued involved a combination of wavelet and transfinite interpolation.

Combinatorial Problems

Researcher: Associate Professor T. S. Michael
Sponsor: Naval Academy Research Council (NARC)

Several strong mathematics majors have completed impressive honors theses under this researcher's direction in recent years. The purpose of this research project was to bring some of these theses up to the standard of publishable work in scholarly journals. Two joint papers are now in progress and will be submitted to good journals. These two papers include the general theorems on tilings obtained by Midshipman Richard Bower (U.S. Naval Academy, Class of 1999).

The Object-Image Equations of N Points

Researcher: Professor George C. Nakos
Sponsor: Naval Air Warfare Center (NAWC)

We assume that a set of three-dimensional points produces a two-dimensional image. Our goal is to describe relations between the coordinates of the object points and the image points without using the parameters of the transformation that has produced this image. These are known as *object-image equations*. Both the object points and the image points are determined up to a projective colineation. We base our approach on earlier work by Quan. Stiller, Asmuth, and Wan have produced a different set of object-image equations.

This is the last stage of a long project suggested by the sponsor that required several summers' work. The resulting paper, co-authored with the sponsor, is currently prepared for publication.

The Impact of Calculus Reform on Long-term Student Performance

Researcher: Professor Howard L. Penn
Sponsor: National Science Foundation (NSF)

The purpose of this research project is to determine the long-term effect of the use of technology and other reformed calculus ideas on the performance of students in their upper division courses. The U.S. Naval Academy is one of several institutions participating in this study. The goals include:

- comparing the performance of reform and traditional calculus students in courses beyond calculus.
- examining students prior to graduation from college to determine these students' fundamental notions of calculus.
- determining the extent to which potential employers value the ideals supported by calculus reform efforts.

This researcher is serving as the on-site evaluator for the Naval Academy. His duties include:

- identifying student subjects for interviews.
- compiling background data on student subjects.
- compiling project data.
- analyzing the data.
- developing human subjects review protocol.

This project is in the initial phase. The data will be collected during the 2001-2002 academic year and it will be analyzed during the summer of 2002. Currently, the procedures for human subject protocol are being developed.

Shifts on Operator Algebras

Researcher: Professor Geoffrey L. Price
Sponsor: National Science Foundation (NSF)

One of the key problems in the theory of von Neumann algebras is to study and to classify the position of subfactors of a prescribed index in the hyperfinite II_1 factor R. In some ways this problem resembles the Galois theory of automorphisms of finite groups: in fact, the group-theoretic notions of index, normality, and conjugacy all have analogues in the theory of subfactors. This researcher has studied a family of subfactors in R on which one can define a sort of non-commutative version of the Bernoulli shift of index 2. These shifts are called Powers shifts. For each Powers shift there is a corresponding bitstream of 0's and 1's which defines the shift. The structure of the shift is reflected in certain properties possessed by the bitstream, the most conspicuous of which is whether or not the bitstream is eventually periodic. In fact, the bitstream is eventually periodic if and only if there is a finite positive integer k such that the kth power s^k of the shift s has range $s^k(R)$ with nontrivial relative commutant in the hyperfinite factor R. It has been shown that all Powers shifts with relative commutant index 2 are cocycle conjugate (a natural sort of equivalence for automorphisms and endomorphisms on von Neumann algebras) and has made some progress on the higher relative commutant case. Recently, an unexpected connection between the conjugacy classes of Powers shifts of finite commutant index and certain classes of polynomials over the finite field of two elements has been discovered, which has led to a complete classification of these shifts up to conjugacy.

RAT-STATS Verification and Validation

Researcher: Professor Thomas J. Sanders
Sponsor: Department of Health and Human Services,
Office of Inspector General, Office of Audit Services

During the period 1 June 2000 to 15 August 2000, modules of the statistical software *RAT-STATS* were examined for purposes of verification and validation. *RAT-STATS* is a software package developed by the Office of Audit Services, Office of Inspector General, Department of Health and Human Services. The current version of *RAT-STATS* is DOS based and a MS Windows version is currently being developed. The purpose of this project is the verification and validation of the initial release of that software. Specific results are available on request.

The code was carefully examined to ensure that the formulas given in Reference (a) were accurately calculated. Part of this involved calling objects written in "parallel code" to verify that the results of calculations were as expected for the data inputted. Exact agreement was required in the calculation of integer values, but a tolerance of $10^{-06} = .000001$ was allowed in the calculation of floating point numbers. This was to account for round-off error differences due to different coding techniques. This was considered more than adequate since results are typically reported to 2 decimals (nearest cent) and never reported to more than 3 decimals (tolerance of .0005). Special attention was paid to technical calculations contained in each module, e.g., pseudo-random number generation, estimates of proportions and populations, estimates of varinces, and confidence limits.

Data sets used to examine the module calculations included ones provided by Dr. Al Kvanli and ones newly created. An attemp was made to anticipate possible user input errors to see how the program would respond. Outputs in all formats allowed were examined to ensure that they agreed with the results of the calculations.

Some of the modules examined contain errors significant enough to require correction before certification can be done. Several of the modules were either not inspected or their inspection was incomplete. Specific details on the module inspections are available on request.

Helicopter Obstacle Detection

Researchers: Professor Thomas J. Sanders, Professor W. Charles Mylander,
and LtCol Scott H. Poindexter, USMC
Sponsor: Honeywell DAS

Honeywell DAS is currently developing an Infra-Red LIDAR system to do Air Data measurement and obstacle detection. They have been successful so far with one flight demonstration and are scheduled to conduct another later this year.

Hopefully this will lead to their developing a product rather rapidly as they take this technology and turn it into a manufacturable product.

Mr. Dick Krulas, of Honeywell DAS, approached us for help on finding the optimal scan pattern for a helicopter in its various flight regimes. There appeared to be a lot of data for radar searches using RF signals but very little research done on laser pulse search and detections.

We were able to help Mr. Krulas decide on an appropriate measure of performance and did initial evaluations of several possible scan patterns. These were passed on to the development team at Honeywell DAS and the replies that came back were favorable. The next step in this investigation is to try to get an appreciation of the statistical effectiveness of these patterns.

Generic Initial Ideals in the Weyl Algebra

Researcher: Assistant Professor William N. Traves
Sponsor: Naval Academy Research Council (NARC)

The general linear group acts in a natural way on the Weyl algebra of differential operators on affine space. Given a generic element of the general linear group, the initial ideal of its action on a fixed ideal is well defined. As in the commutative case, these generic initial ideals have a combinatorially rigid structure. This project attempts to classify all generic initial ideals in the Weyl algebra. One expects that the structure should depend on the type of term order chosen. This project is ongoing.

Closed Loop Degaussing

Researcher: Professor John C. Turner
Sponsor: Naval Surface Warfare Center (NSWC)

The purpose of this ongoing classified project is to develop algorithms for Closed Loop Degaussing. A considerable databank has been developed on the current test platform. As we enter the third phase of research development, we consolidated the database and investigated analyzing current data in advance of the next sea trial. This analysis revealed that the data that had been collected at the three sites was inconsistent for our purposes. This inconsistency had not been observed in the earlier phases, which considered the data in less detail. To the level of detail needed for the third phase, the three sources of data are not sufficiently consistent to be used.

This observation led to a review of the processing of the data. This revealed an error in processing that had not previously been noticed. The error is currently being fixed and the data re-processed.

A sea trial that had been scheduled for summer 2001 was canceled. It has been tentatively rescheduled for May 2002.

Image Recognition Problems

Researcher: Professor Peter R. Turner
Sponsor: Naval Air Warfare Center-Air Defense (NAWCAD), Patuxent River

Growing out of earlier work for the Naval Air Warfare Center, the problems of image recognition and moving object location in computer vision were studied using a linear algebra and vector-geometric approach. This yielded a simple approach to the solution of the image recognition problem and for the application of this same algorithm to the problem of locating (in three-dimensional space) an object from two-dimensional images. This was all the subject of a long term research collaboration sponsored again by NAWCAD, Patuxent River this year.

Independent Research

Flow of Weights and the AT Property for Product States of the Gauge Invariant and Rotation Invariant CAR Algebras

Researcher: Professor B. Mitchell Baker

In this work, type III factors obtained by restricting product states of the CAR algebra A to the gauge or U(1)-invariant algebra A^T have been studied. This work is a natural extension of earlier work, and uses the many tools that have been developed to refine the study of purely infinite factors and ITPFI (infinite tensor product of type I) factors in general. The principal result from this investigation shows the flow of weights for the group-invariant factors is approximately transitive whenever the corresponding eigenvalue list does not converge to zero; moreover for lists slowly converging to zero, there are non-AT flows. This demonstrates that most of these group invariant factors are in fact ITPFI, but there are surprising non-ITPFI examples. In addition the research shows all ITPFI factors are contained in the group invariant M^T factors. Finally, we contain computable characterization of the S and T sets (Connes' isomorphism invariants) for our M^T and M^G factors.

Integrating Image Processing Research into the Discrete Mathematics Curriculum for Majors

Researcher: Professor Carol G. Crawford

This ongoing research extends prior work by the author in the area of state-of-the-art applications of graph theory to real world problems in image processing and computer vision. These applications include Robot Vision System Design and Automated Fingerprint Identification. These topics are based on the author's funded research from The Federal Bureau of Investigation, The Office of Naval Research, The National Institute for Standards and Technology and David Taylor Naval Research Lab.

A major goal of this project is to develop real world examples for use in the Discrete Mathematics Courses for the Mathematics Major. These applications include graph representations for robot vision systems design and graph representations of fingerprint minutiae maps for neural networks matching and classification of fingerprint images. These examples have provided a foundation for team projects in these courses. Also, the author has created a yearly presentation based on these materials for the Math Major Open House.

Most recently, the author presented results of this work at the International Conference of The Society for Industrial and Applied Mathematics, July 11, 2000 in Puerto Rico. The paper was entitled "Graph Representations for Image Processing."

Varietal Amalgams and Groups which Satisfy a Freiheitssatz

Researcher: Professor Anthony M. Gaglione

The classical *Freiheitssatz* or *independence theorem* of Magnus says that in a one-relator group $G=\langle x_1,\ldots,x_n;R\rangle$, where the relator R is cyclically reduced and involves all the generators, the subgroup generated by $x_1,\ldots,x_{n-1}$ is free on these generators. The object of this project is to investigate the more general situation where X and Y are disjoint sets of generators, the group A has the presentation $A=\langle X;Rel(X))\rangle$ and the group G has the presentation $G=\langle X;YRel(X),Rel(X,Y)\rangle$. Then we say that G satifies a *Freiheitssatz*, abbreviated FHS, relative to A if $\langle X\rangle_G=A$. In other words, the subgroup of G generated by X is isomorphic to A. The group A is then called a *FHS factor* of G. In this language, Magnus' original FHS can be phrased as : A one-relator group satisfies a FHS relative to the free group on any proper subset of the generators. FHS factors arise naturally in the class of group amalgams, i.e., free products with amalgamation and HNN groups. In these cases the normal form theorems (see *Combinatorial Group Theory* by Lyndon & Schupp) imply that the amalgam factors inject into the amalgam under the identity mapping, i.e., the amalgam factors are FHS factors. In particular, Magnus' original proof of the Freiheitssatz and subsequent proofs of this result all utilize this fact, that the amalgam factors are FHS factors. Fine and Rosenberger have given

a complete description and overview of the FHS problem which we investigate in this project. (See B. Fine and G. Rosenberger, "The Freiheitssatz and its Extensions,'' *Cont. Math*. 169 (1994), 213-252 - both Fine and Rosenberger are working with me on this project). This project was also concerned with the FHS problem in the context of group varieties. If V is a variety of groups given by the laws W and G is a group then G *relativized* to V is the group $G_V = G/W(G)$ where W(G) is the verbal subgroup of G relative to the laws W, in other words, we impose the laws of W on G. This G_V is the largest possible quotient of G which is in the variety V. Hence a *one-relator V-group* is a one-relator group relativized to the variety V. A *varietal amalgam* is a group amalgam (fee product with amalgamation or HNN group) with factors from V and then relativized to the variety V. We have proven two main results in this direction. First, we have given some sufficient conditions for the amalgam factors in a relativized free product with amalgamation to be FHS factors. We showed that this is always true in relativized abelian groups. We also have constructed examples to show that this is not true in general. Second we have proven a FHS for certain one-relator nilpotent groups. This has special conditions on the relator and we have given examples to show that it is not true in general. We have also constructed examples to show that several basic results in one-relator group theory do not carry over to the relativized case, however, many do when the variety is the variety of abelian groups.

Rapid Binary Gage Function to Extract a Pulsed Signal Buried in Noise

Researchers: Associate Professor Sonia M. Garcia,
Professor William J. Bagaria (Aerospace Engineering Department),
Associate Professor Colin P. Ratcliffe (Mechanical Engineering Department),
and Visiting Professor Richard P. Fahey (Aerospace Engineering Department)

The motivation for this research was the need to extract pulsed signals that are buried in noise. The signals of interest have a short pulse duration compared to the period of the signal, and they are weak. A consideration when analyzing these signals is that the length of the recorded signal be as short as possible. In this way, factors, which might change the period of the signal over the short term, such as a variable Doppler shift, can be detected. If the signal is recorded, and analyzed over a long period of time, the short time variations would be averaged out. This particular example has a half sine wave pulse shape.

Classic techniques for recovering information about noisy signals are based on the Fourier transform method. If the signal is digitized then the FFT (discrete Fast Fourier transform) is the method of choice. The FFT is so computationally efficient, that if the autocorrelation or cross-correlation function is desired, they are obtained by first finding the Fourier transform then taking the inverse transform. When dealing with digitized data, certain precautions must be taken when the data is analyzed. The signal must be digitized based on the Nyquest sampling rate, which in turn is based on the highest desired frequency component of the signal. This is to avoid the appearance of false aliases in the discrete Fourier transform. To avoid this in practice, the data is passed through a low pass filter before being analyzed. The digitizing rate of the signal is also dependent on the equipment being used to capture, digitize, and record the signal. There are three primary considerations, which are equipment dependent. The first is the digitizing rate. The second is the storage rate. The third is the amount of storage space required.

Ultralogics and Probability Models

Researcher: Professor Robert A. Herrmann

Using the Nonstandard Physical World model, and nonstandard analysis this research has established the following: For an appropriate language *L* and any real number *p* such that 0 d•*p* d•1 there exists an ultrachoice function **C* and a ultralogic P_p with the following properties. (1) When P_p is applied to an encoded source {**G**}, a hyperfinite set of "events" $\{a_1, ¼, a_n, ¼, a_n\}$ is obtained such that for any "n" trials, $\{a_1, ¼, a_n\}$ is a finite identified "event" sequence, where each a_i determines the labeled event **E** or labeled non-event **E**'. (2) The labeled events in (1) are sequentially determined by **C*, where *C* determines a sequence g_{ap} of relative frequencies that converges to *p*. (3) The sequence of relative frequencies g_{ap} determine by **C* cannot be differentiated from a sequence produced by theory dependent random chance.

The new notion of the product consequence operator is introduced. It is shown that for any discrete distribution that exhibits a probability mass function (frequency function), there is a hyperfinite ultralogic that generates, in the same manner as does P_p, the predicted results for the distribution. A paper formalizing these results has been accepted for journal publication.

Standard and Hyperfinite Unifications for Physical Theories

Researcher: Professor Robert A. Herrmann

Using the Nonstandard Physical Model and nonstandard analysis, this research has established the following: Let a set of physical theories is represented by a nonempty subset $\{S^{V}_{N(j)} \mid j \hat{I} N\}$ of the lattice of consequence operators defined on a language L. It is established that there exists a unifying injection S defined on the nonempty set of significant representations for natural-systems M Ì L. If WÎ M, then S_W is a hyperfinite ultralogic and È$\{\mathbf{S}^{\mathbf{V}}_{\mathbf{N}(j)}(\mathbf{W}) \mid j \hat{I} N\} = S_W(*\mathbf{W})$ Ç **L**. A product hyperfinite ultralogic P is defined on internal subsets of $*\mathbf{L}^m$ and shown to represent the application of S to $\{W_1, ¼, W_m\}$ Ì M. There also exists a standard unifying injection S_W such that $S_W(*\mathbf{W})$ Ì $*\mathbf{S}_{\mathbf{W}}(\mathbf{W})$. A paper formalizing these results has been accepted for journal publication.

Algebraic Structure of the Set of Multiple Harmonic Sums

Researcher: Professor Michael E. Hoffman

Purpose: Multiple harmonic sums have appeared in the mathematical physics literature in recent years. While there is now a considerable literature on the algebraic structure of the multiple zeta values, there has been relatively little attention paid to the multiple harmonic sums themselves. There are in fact many algebraic relations among the multiple harmonic sums that are a consequence of their being homomorphic images of the quasi-symmetric functions.

Results: Many algebraic identities among the S_I can be derived from known facts about QSym. In addition, there is an automorphism È of QSym that corresponds, under the mapping to sequences of harmonic sums, to a certain automorphism of series (which is the identity after reduction modulo a prime p.) This gives a "mod *p* duality" result for multiple harmonic sums.

Status: The author's results on the algebra of finite harmonic sums and their mod *p* duality was presented in a Special Session talk at the 2001 Winter Meeting of the AMS. A paper including the mod *p* duality result is in preparation. Study of the algebra of multiple harmonic sums continues.

Hopf Algebras of Trees and Quantum Field Theory

Researcher: Professor Michael E. Hoffman

In his study of renormalization in quantum field theory, Dirk Kreimer introduced a commutative Hopf algebra whose generators are rooted trees. Later, in joint work with Alain Connes, Kreimer related his Hopf algebra of trees to some Hopf algebras arising in Connes' program of noncommutative geometry. These algebras present many interesting algebraic and combinatorial questions. The researcher has worked his way through Kreimer's and Connes' original papers on the subject, as well as some related work, with the goal of understanding the algebra involved. Kreimer's Hopf algebra of trees was related in one published paper to an earlier Hopf algebra defined by Grossman and Larsen, however, the published result is incorrect, and understanding the relation between the two constructions will require further work. Also, Kreimer's Hopf algebra is itself an interesting object, and relating it to previously studied algebras could prove fruitful.

Results: Study of some combinatorial aspects of Kreimer's algebra led the researcher to define a notion of "covering space" for a ranked poset. This seems to have significance beyond this immediate context.

Status: The author gave a series of lectures on Kreimer's and Connes' Hopf algebras, which he has written up as a text of about 25 pages; this may be the basis of a future publication. In addition, he has written and submitted a paper based on his definition of the covering space of a ranked poset.

Multiplicity One for SL(2)

Researchers: Professor W. David Joyner and Associate Professor Courtney Moen

We apply the trace formula to the study of cuspidal automorphic representations of *SL(2,A)* as compared with the genuine cuspidal automorphic representations of the 2-fold metaplectic cover *SL(2,A)**. The main result is a multiplicity one theorem for most automorphic representations of SL(2), where A denotes the ring of adeles of a totally imaginary number field.

Discrete Time Control Systems and Algorithms. Digital Signal Processing, Analytic Theory of Continued Fractions, J-fractions, and Stieltjes Continued Fractions

Researcher: Associate Professor John Sigurds Kalme

The analytic theory of continued fractions is of purely mathematical interest, but also has applications to electrical engineering. Continued fractions give alternative ways for determining conditions for polynomials to have roots with negative real parts (different from the Hurwitz criterion), and are important in studying stability of systems, are useful in determining regions in which roots lie, and determining the number of roots in a given region. The expansion of rational functions of a variable z into a J-fraction and Stieltjes continued fractions are useful in certain problems in electrical engineering, for example electrical networks, and digital ladder networks.

Classification of Singularities with SHEEP/CLASSI

Researcher: Professor Deborah A. Konkowski

Ways to apply computer algebra programs (e.g., SHEEP/CLASSI) to the problem of singularity classification are being investigated. This computer algebra research was begun during a recent sabbatical in London at Queen Mary and Westfield College with Professor Malcolm MacCallum's group and has been continued during periodic visits to London. Although it is impossible to classify topological singularities such as quasiregular singularities in this manner, scalar curvature and non-scalar curvature singularities should yield to analysis. Ways to study scalar and nonscalar curvature singularities using a complete listing of C^0- curvature invariants and frames related to parallel propagated orthonormal ones are currently under consideration.

Explicit Construction of Complete Kähler Metrics of Saper Type by Desingularization

Researcher: Associate Professor Caroline G. Melles

This work is a continuation of earlier research on the geometry of singular complex analytic spaces. A description of the construction of complete Kähler metrics on the smooth part of any singular subvariety of a compact Kähler manifold is provided. The metrics have a particularly simple local formula, involving essentially a product of distances to the centers of blow-ups used to resolve the singularities of the subvariety. A paper for the *Pacific Journal of Mathematics* has been prepared, and it includes detailed descriptions of the behavior of sheaves of ideals under blow-up maps.

Revisions of the planned publication include a more detailed description of direct images of ideals under blow-up maps, a description of a C-infinity patching argument in terms of individual blow-ups of smooth centers, and more material on how such metrics might contribute to the proof of a conjecture by Cheeger, Goresky, and MacPherson relating L2-cohomology and intersection cohomology.

Hadamard Embeddings

Researcher: Associate Professor T. S. Michael

This research is almost complete. Given positive integers m and n, what is the smallest order F(m,n) of a Hadamard matrix that contains an m-by-n submatrix of all 1's? The researcher obtains upper and lower bounds for F(m,n) that are better than the bounds in the current literature.

Crossing Numbers of Famous Graphs

Researcher: Associate Professor T. S. Michael

The crossing number of a graph G is the minimum number of edge intersections among all drawings of G in the plane. This parameter is important in electrical engineering, for instance, where one seeks to minimize the number of wire crossings (requiring a bridge) in the layout of a circuit. Visual proofs that determine the crossing numbers of the Petersen graph, the Heawood graph, and the Clebsch graph, among others, are provided. This research is in progress.

Games on Oriented Graphs

Researcher: Associate Professor T. S. Michael

This in-progress research investigates a zero-sum two-person game defined by an oriented graph. Researchers have recently shown that if the oriented graph is a tournament, then the optimal strategy is unique. In this research, some necessary and sufficient conditions for uniqueness of the optimal strategy for more general classes of oriented graphs are obtained.

Representation Theory of U(2,1)

Researcher: Associate Professor Courtney Moen

This project deals with the representation theory of U(2,1) over a p-adic field. In a previous paper, the researcher used the theory of reductive dual pairs to construct a series of representations of this group. These are particularly interesting representations due the connection with theta series and number theory. Several other authors have constructed representations of p-adic reductive groups using the theory of Hecke algebras. If one specializes their techniques to U(2,1), one can ask for the relationship between the two methods of construction. In particular, it is not obvious that the set of representations constructed by means of Hecke algebras includes the set constructed by means of dual pairs.

Functions in Generalized Moulis Classes

Researcher: Associate Professor E. J. Moulis

Let N be the set of all (complex-valued) functions analytic in the unit disk |z| < 1, having the form f(z) = z + sum[a(n)*z^n, n=2..infinity]. This research continues the study of the class U(k,c,b,L) of generalized Pinchuk functions consisting of those functions in N which, with J(f) = 1 – 1/c +(z/c)*(f'/f). z = r*e^(it), r in [0,1), t in [0,2*Pi), c a non-zero complex number and L in (-Pi/2,Pi/2), b in [0,1) and k greater than or equal to 2, we have, using MAPLE syntax,

Int [|Re{e^(it)*J(f) – b*cos(L)|,t,t=0..2*Pi] < k*(Pi)*(1-b)cos(L).

When z*f '(z) is in U(k,c,b,L), f is said to belong to a generalized Moulis class V(k,c,b,L). This researcher introduced the class V(k,0,0,L) in his Ph. D. thesis. We try to find sharp bounds in terms of the real parameters k, b and L and the complex number c, for the geometric mapping properties of functions in these classes, including distortion bounds on |f(z)| and |f '(z)| , rotation bounds on |arg(f ' (z)| as well as bounds on the moduli of the series coefficients a(n).

The Brown-Peterson Cohomology of Two Lens Spaces

Researcher: Professor George C. Nakos

The author computes the Brown-Peterson (BP) cohomology of a product of two lens spaces. This is a new result that vastly extents the author's corresponding paper on projective spaces that appeared in the *Michigan Mathematical Journal*.

The BP-homology of a product of two lens spaces consists of a Tor module and a tensor module. Although the tensor module was somewhat understood by experts, the Tor module was totally unknown. In this work the author gives a complete description of the Tor module and sheds new light in the tensor module. A by-product is the calculation of the ideal of the v_n-torsion of the tensor module. The Tor product has a surprisingly nice but *complicated* interpretation: it is a direct sum of the Brown-Peterson cohomology of single lens spaces of various sizes.

The author has dualized these results to the case of BP-homology. Although he has a complete description, a very lengthy detailed calculation is still ahead.

Mathematical Foundation for Empirical Mode Decomposition

Researcher: Associate Professor John F. Pierce

Utilizing the Hilbert transform, N. Huang developed a remarkably successful and robust empirical methodology to extract multi-resolution information from time series data in a nonlinear (signal-dependent) manner (*Proc. R. Soc. Lond. A.* (1998) 454, 903-995; *Annu. Rev. Fluid Mech.* 1999, 31:417-57). By an empirical "sifting" process, the method resolves the signal into a finite sum of intrinsic modes, each of which isolate features of the signal at a particular time scale. In contrast to most methods of representing signals, the intrinsic modes are not a basis set of functions imposed externally on the signal, and whose features are known prior to considering the signal. Rather, they emerge from the signal, are peculiar to it, and their features are not known prior to considering the signal.

As successful as the technique has been in its empirical applications, there are issues about the rational basis for its success, and concern for delineating its scope of validity. For example, are there time series sufficiently complex that sifting will cycle toward, but not converge to a specific intrinsic mode? Are there attributes delineating an intrinsic mode that independent of the context, by which someone "untutored" in the particular signal would infer, correctly, the sifting process delivers an intrinsic mode? Are there formal results that affirm that iterative sifting of a signal will produce a finite set of intrinsic modes and a single, residual trend?

Wavelet Compression with Redundancy for Wireless Communication

Researcher: Associate Professor John F. Pierce

Wireless communication places constraints upon applications that include dynamically varying bandwidth availability, noisy and temporary loss of channels, multi-path reflections, and true channel throughput issues that render current encoder/decoder technologies inapplicable.

The inherent scalability and the progressive features of wavelet-based compression and encoding make it attractive to exploit the full potential of delivering information within the parameters of wireless communication. Wavelet-based applications revolutionized encoder decoder technology in the tethered environment of high-performance Ethernet and modem networks. Here one could handle encoded data within the optimal conditions of large bandwidth and few interference issues. In the presence of large bandwidth, data error handling could be treated straightforwardly. Wavelet-based encoding activity could be directed towards quality and performance, with little activity directed toward issues arising in the transport of data.

The opposite is the case with wireless communication. Because of the severely limited bandwidth, and because of the noisy channels for communication, data error handling becomes an intrinsic part of compression, encoding, and decoding. Compression that over-represents a signal (redundant information) provides a means by which information can be conveyed, though data is lost in transmission. We examine the issues of how wavelet-based frames can provide redundant compression at the applications layer that is consonant with the data error handling protocols operating at the physical layers of the ISOP.

Combinatorics of Clique Sequences

Researchers: Associate Professor T. S. Michael
and Assistant Professor William N. Traves

An n-clique in a graph G is a complete subgraph of G with n vertices. We are interested in the structure of the graded poset of cliques in a graph. This interest takes many forms and avenues, including the study of the rank generating polynomial (the clique polynomial) and the Hopf-algebra structure of these posets. This project is ongoing.

Commutative Algebra and Graph Theory

Researcher: Assistant Professor William N. Traves

There are many commutative ideals that are naturally associated to graphs. Perhaps the best-studied is the ideal associated to a simplicial complex. Another interesting ideal is the "edge ideal" of a graph. This is the topic of a recent book by R. Villareal. Our aim in this project is to extend some of these new ideas and methods to hypergraphs, producing a new series of results about simplicial complexes. This project, just beginning, complements previous work by S. Faridi.

Computational Algorithms in Algebraic Number Theory

Researchers: Associate Professor JoAnn S. Turisco and Professor W. David Joyner

This project is a continuation of previous work that involves the use of Maple and Mathematica for computations, and to generate examples with the goal of making progress towards the solution of some open and difficult problems in algebraic number theory.

One part of this work is the explicit computation of forms associated to smooth maps between Euclidean spaces. These computations, which involve the determination of eigenvalues of large matrices whose elements are multivariable polynomials, have produced generalized hypergeometric functions.

The problem of the existence of normed bilinear maps from an algebraic viewpoint, using Jordan algebras and triple systems is also being investigated. Examples are generated with Maple and Mathematica by associating to each map a family of rectangular matrices which satisfy certain idempotent and anti-commutative properties.

In addition, work on Maple and Mathematica programs which determine prime numbers and class numbers of certain algebraic number fields is continuing.

Problem sets, primarily in elementary number theory, are being written and will be included in Applied Abstract Algebra, co-authored by D. Joyner, R. Kreminski and J. Turisco.

Rapid Arithmetic Coding

Researcher: Professor Wm. Douglas Withers

The data-compression technique of *arithmetic coding* is known to provide optimum coding performance in compressing a sequence of symbols, each of which takes particular values with known probabilities. As a practical matter, two particular aspects of the method are recognized as leaving room for improvement: the estimation of probabilities for a sequence of symbols derived from a real data set rather than generated by a specified random process, and the speed of coding. Improvements in both have been sought since the original discovery of arithmetic coding more than twenty years ago. The researcher, in an ongoing study of the problem, has developed the second of two algorithms combining rapid arithmetic coding and probability estimation, which provides compression performance superior to any previously known algorithms.

Research Course Projects

Major League Baseball: Analyzing and Predicting Changes in Salary

Researcher: Midshipman 1/C Rob Blackwell, USN
Adviser: Professor Peter P. Andre

This study tries to predict average professional baseball salaries from a list of economic variables. It also shows that there is a shift in determination of baseball salaries after the change in the free agent policy of the baseball major leagues. Using time series techniques this study describes a model in which the change in baseball salaries is treated as a function of national economic variables including the previous increase in salaries. The economic variables are only weakly correlated with the changes in baseball salaries. The data does indicate that a shift did occur with the change in the free agent policy. The model predicts the changes in salaries well before the free agent change but poorly after the change. This indicates that there are now other factors which are critical for salary determination.

Bridging the Gap Between Average Salaries in Professional Sports

Researcher: Midshipman 1/C Michael Cunningham, USN
Adviser: Professor Peter P. Andre

This study examines the relationship between the average salaries of the three major professional sports: basketball, football and baseball. The study uses time series techniques to analyze a model which predicts changes in the average professional basketball salaries from earlier changes in the professional basketball, baseball and football salaries. The model shows a strong dependence of changes in basketball on football salaries and changes in baseball salaries over the previous year. It shows a negative dependence over the variables from two years prior indicating some serious trending of the data.

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Effects of the Olympics on the Host Country's Growth

Researcher: Midshipman 1/C Erin Emery, USN
Adviser: Professor Peter P. Andre

This research studies the possible effect that hosting the Olympics has on the host country's gross domestic product. Time series techniques were used to measure the dependence of the GDP of Austria, Canada and Korea on several economic input variables one of which measured the time of development of an infrastructure for the Olympic Games. The effect of the Games was found to be small and the direction of the effects varied across hosts.

Planning the Clearance of Open Ocean Minefields

Researcher: Midshipman 1/C Shaun P. Hayes, USN
Adviser: Professor W. Charles Mylander

The problem of planning the clearance of an open ocean minefield was addressed in this project. Two independent sensors sweep the minefield. It is assumed that probabilities that each sensor will detect and classify mines and non-mines which are minelike objects are know from test data. Then using a prior distribution of mines and minelike objects for the area to be cleared Bayes Theorem is used in the development of a system to score each minelike object in the area swept. The scores are the probabilities each detected minelike object is a mine. An algorithm is provided to clear a channel of maximum width through the in a fixed about of time. Each minelike object's location, score, and depth are taken into in planning the channel clearing operation. The paper also points out that the planning the clearness of the entire minefield to a pre-specified probability is equivalent to the traveling salesman problem.

Cooperative Organic Mine Defense, Human/Computer Interface

Researcher: Midshipman 1/C Shaun P. Hayes, USN
Adviser: Professor Thomas J. Sanders
Sponsor: The Johns Hopkins University Applied Physics Laboratory

This project addressed the Human/Computer Interface (HCI) of the Cooperative Organic Mine Defense (COMID) with respect to information requirements, symbology and display of such requirements, and necessary functions of the HCI.

COMID is the near real-time sharing and data fusion of information provided by surface, subsurface, and airborne assets (containing advanced systems capable of detecting and classifying mines) attached to the Carrier Battle Group (CVBG) and/or Amphibious Ready Group (ARG). COMID will be available to all forces (as necessary) and will supplement current forward-deployed in-theater Mine Countermeasure (MCM) assets, or will provide MCM capability where no such forward-deployed forces are. Data fused by COMID will be displayed on the Common Tactical Picture (CTP).

Comparison of Two Simulation Models of AQS-14 Sonar System

Researcher: Midshipman 1/C Nikolaus Keapproth, USN
Adviser: Professor Thomas J. Sanders
Sponsor: Naval Research Laboratory (NRL),
Advanced Information Technology Branch

The project was a comparison between two simulation models of the AN/AQS-14 Side Scanning Minehunting Sonar in the Joint Semi-Automated Forces (JSAF) simulation system. The existing "baseline" AQS-14 model uses a lookup table using a few parameters from the simulation, whereas the newer model uses a special server to model the interactions of the sound waves as they propagate through the water. The server for the second model, called PC-SWAT (Shallow Water Acoustic Tool), requires detailed information from the simulation about the oceanographic environment in which the sonar and target are operating. Parameters passed to this server are a sound velocity profile (SVP), range, bearing, salinity, frequency, source level, depth, bottom type, along with many others.

Subjectively, the two different models exhibit very different behaviors under identical circumstances. The baseline model will detect nearly every contact (mine) out to a seemingly arbitrary maximum range, and none beyond this point. The PC-SWAT model will detect fewer contacts – roughly about 10% as many as the baseline – and only those located very close to the sonar. Thus the more sophisticated model indicates a much shorter effective range for the AQS-14 than the baseline model reflects, and is more consistent with actual behavior.

Since the baseline model accepts significantly fewer environmental parameters from the simulation, it is less dependent on environmental factors. Therefore, the baseline model behaves nearly identically at a given range in any sea state, depth, or temperature profile. This is simplistic and inaccurate, and detrimental to an effective simulation experience. By allowing the user to take advantage of environmental factors that would be present in a real life exercise or situation, the simulation becomes more realistic and can be a more potent training tool.

PC-SWAT can be a viable and ultimately very useful training tool as part of the Joint Semi-Automated Forces simulation system. Before this happens, however, it must be enhanced and implemented further.

Predicting the NASDAQ

Researcher: Midshipman 2/C Erik Kenny, USN
Adviser: Professor Peter P. Andre

This researcher tried to predict variations in the level of NASDAQ market from economic variables including the variations in the level of the New York Stock Exchange. Using time series techniques, the research modeled changes in the NASDAQ as a linear function of economic variables. The model did not predict the NASDAQ very well. It did show a change in behavior at the advent of the internet which spawned a collection of fast growing volatile dot.com companies. The model did show that the federal interest rate did not have a significant effect on the performance of the NASDAQ.

The Impact of Political Party Representation in Congress on the Gross Domestic Product

Researcher: Midshipman 1/C Nicholas Lockwood, USN
Adviser: Professor Peter P. Andre

This paper studied the dependence of the Gross Domestic Product on the political party strengths in the United States Congress. Using time series methods, this paper modeled the GDP as a function of the percent of Republicans in the Senate and the House of Representatives in addition to some national economic variables. The dependence on political party seems to be weak and showed opposite sign for the two houses.

Short and Long Run Price Elasticities of Petroleum

Researcher: Midshipman 1/C Brad McCreedy, USN
Adviser: Professor Peter P. Andre

This research studied the nature of the demand for petroleum. The model described the consumption of petroleum as a function of price and gross domestic product. By using logarithms for all of the variables, the investigator found the elasticities as the regression coefficients.

It was determined that the consumption of petroleum was surprisingly inelastic to changes in price in both the short and long runs.

Toeplitz Matrices Over Finite Fields

Researcher: Midshipman 1/C Daniel Post, USN
Adviser: Professor Geoffrey L. Price
Sponsor: National Science Foundation (NSF)
Research in Undergraduate Institutions (RUI) Grant

Through his work on shifts on operator algebras Price has been led to the study of Toeplitz matrices over finite fields. The connection between the two subjects stems from the fact that the structure of the algebras on which the Powers shifts act is determined by the nullities of a sequence of Toeplitz matrices associated to these algebras. A *Toeplitz matrix* over a finite field is a square matrix (i.e., it has the same number of rows as columns) that is defined by the following properties: (i) the matrix has 0's along its main diagonal (ii) it is constant along the other diagonals, and (iii) it is skew-symmetric. Such a matrix is easily seen to be determined by its entries along the top row. One may use a bitstream of 0's and 1's to define a sequence of Toeplitz matrices $\{T_n: n=1,2,\ldots\}$ by letting T_n be the nxn matrix whose first row consists of the first n elements of the bitstream. In earlier work with R. T. Powers and with other undergraduate coauthors Price has shown that the sequence $\{n_n\}$ of the nullities of the matrices $\{T_n\}$ (the nullity of an nxn matrix is the difference of its size n and its rank) has the property that it is the concatenation of a string of numbers of the form 1,2,…,m-1,m,m-1,…,1,0. In his honors work Midshipman Post studied such sequences with the use of a number of computer programs written by Price. The programs indicated (and Post eventually proved) that there is a connection between the nullity sequences and a family of polynomials over finite fields with certain symmetry properties. More precisely, if $n_{n+1} = 1$ then by the definition of nullity there is a vector **v** which spans the (right) kernel of the matrix T_{n+1}. This vector is a palindrome (i.e., its entries are the same left to right as right to left). Conversely, any such vector is the single element generating the kernel for exactly $2^{n/2}$-1 nonzero Toeplitz matrices. Post has shown that if one associates the polynomial p(x) of degree n whose coefficients are the same as the entries of **v**, then if p(x) is either irreducible or the product of two irreducible polynomials that are "flips" of one another, then each predecessor matrix T_n to T_{n+1} is invertible, i.e., $n_n = 0$. Conversely, if $n_n = 0$ for each predecessor T_n to T_{n+1} then p(x) must have one of the forms above. Post has more general results which should be worked out in detail over the next few months.

Defining Nations: An In-Depth Look into GDP Determinants

Researcher: Midshipman 1/C Douglas Santillo, USN
Adviser: Professor Peter P. Andre

This paper studied the growth of the GDP some of the chief trading partners of the United States. Using time series techniques this paper modeled the gross domestic products of Mexico, Canada and the United Kingdom as functions of economic variables including some trade variables. The paper tried to find support for the theory that the GDP of poor countries respond more quickly to changes in the level of capital per worker than do rich countries. The data from these three countries did not support the above theory of convergence.

Economic and Political Factors that Influence the United States Defense Budget

Researcher: Midshipman 1/C Scott Shadforth, USN
Adviser: Professor Peter P. Andre

This research examined the defense budget of the United States from economic variables, including some political variables such as the party affiliation of the President and the party affiliation of the chairman of the Senate's Armed Services Committee. Using time series techniques, this paper models the defense budget as a linear of function of its economic and political input variables. This model did not predict the defense budget very well indicating that there were more important variables to be considered. The political variables seemed to have little influence and indicated an increase in defense spending under Democratic control which was inconsistent with the author's original estimates.

Publications

Journals (Refereed) Manuscripts

BOLLT, E., Associate Professor, STANFORD, T., Assistant Professor, Lai, Y-C, and Zyczkowski, K., "What Symbolic Dynamics Do We Get With A Misplaced Partition? On the Validity of Threshold Crossings Analysis of Chaotic Time-Series," *Physica D*. 154, 3-4, pp. 259-286 (2001).

An increasingly popular method of encoding chaotic time-series from a physical experiments is the so-called threshold crossings technique, where one simply replaces the real valued data with symbolic data of relative positions to an arbitrary partition at discrete times. The implication has been that this symbolic encoding describes the original dynamical system. On the other hand, the literature on generating partitions of nonhyperbolic dynamical systems has shown that a good partition is nontrivial to find. It is believed that the generating partition of non-uniformly hyperbolic dynamical system connects "primary tagencies", which are generally not simple lines as used by a threshold crossings. Therefore, we investigated consequences of using itineraries generated by a non-generating partition. We did most of our rigorous analysis using the tent map as a benchmark example, but showed numerically that our results likely generalize. In summary, we found the misrepresentation of the dynamical system by "sample-path" symbolic dynamics of an arbitrary partition can be severe, including (sometimes extremely) diminished topological entropy, and a high degree of nonuniqueness. Interestingly, we found topological entropy as a function of misplacement to be devil's staircase-like, but surprisingly nonmonotone.

Klebanoff, A. and BOLLT, E., Associate Professor, "Convergence of the DL-algorithm for Finding Periodic Orbits," *Chaos, Solitons, and Fractals*, 12, pp.1305-1322 (2001).

We rigorously studied a recent algorithm due to Davidchack and Lai(DL) for efficiently locating complete sets of hyperbolic periodic orbits for chaotic maps. We give theorems concerning sufficient conditions on convergence and also describing variable sized basins of attraction of initial seeds, thus pointing out a particularly attractive feature of the DL-algorithm. We also point out the true role of involutary matrices which is different from that implied by Schmelcher and Diakonos and propagated by Davidchack and Lai.

Billings, L. and BOLLT, E., Associate Professor, "Probability Density Functions of Some Skew Tent Maps," *Chaos, Solitons and Fractals*, 12, pp. 365-376 (2001).

We considered a family of chaotic skew tent maps. The skew tent map is a two-parameter, piecewise-linear, weakly-unimodal, map of the interval *F_{a,b}*. We show that *F_{a,b}* is Markov for a dense set of parameters in the chaotic region, and we exactly find the probability density function (pdf), for any of these maps. It is well known, (BG), that when a sequence of transformations has a uniform limit *F*, and the corresponding sequence of invariant pdf's has a weak limit, then that invariant pdf must be *F* invariant. However, we show in the case of a family of skew tent maps that not only does a suitable sequence of convergent sequence exist, but they can be constructed entirely within the family of skew tent maps. Furthermore, such a sequence can be found amongst the set of Markov transformations, for which pdf's are easily and exactly calculated. We then apply these results to exactly integrate Lyapunov exponents.

BOLLT, E., Associate Professor, STANFORD, T., Assistant Professor, Lai, Y-C, and Zyczkowski, K., "Validity of Threshold-crossing Analysis of Symbolic Dynamics from Chaotic Time Series," *Physical Review Letters*, 85(16), pp. 3524-3527.

A practical and popular technique to extract the symbolic dynamics from experimentally measured chaotic time series is the threshold-crossing method, by which an arbitrary partition is utilized for determining the symbols. We address to what extent the symbolic dynamics so obtained can faithfully represent the phase-space dynamics. Our principal result is that such practice leads to a severe misrepresentation of the dynamical system. The measured topological entropy is a Devil's staircase-like, but surprisingly nonmonotone, function of a parameter characterizing the amount of misplacement of the partition.

BOLLT, E., Associate Professor, "Model Selection, Confidence, and Scaling in Predicting Chaotic Time-Series," *International Journal of Bifurcation and Chaos*, Vol. 10, No. 6, pp.1407-1422. (2000).

Assuming a good embedding and additive noise, the traditional approach to time-series embedding prediction has been to predict pointwise by (usually linear) regression of the k-nearest neighbors; no good mathematics has been previously developed to appropriately select the model (where to truncate Taylor's series) to balance the conflict between noise fluctuations of a small k, and large k data needs of fitting many parameters of a high ordered model. We present a systematic approach to: 1) select the statistically significant neighborhood for a fixed (usually linear) model, 2) give an unbiased estimate of predicted mean response together with a statement of quality of the prediction in terms of confidence bands.

Xie, Qingxian, Chen, Guanrong and BOLLT, E., Associate Professor, "Hybrid Chaos Syncronization and Its Application in Information Processing," *Math Comput Modeling*, accepted for publication.

In this paper, through numerical studies, we explored a new methodology for chaos synchronization via a hybrid (generalized plus identical) synchronization. A wide range of signals, generated by an unknown dynamical system, can be synchronized by the hybrid chaotic system. The signal can then be stored for future application such as password and message identification. Each finite-length signal can in principle be labeled and stored by a unique number, provided that the key hybrid system parameter used for the purpose is suitably chosen within a one-to-one mapping range. The new methodology enables us to encode an arbitrary signal accurately and efficiently. Sufficient numerical simulations are shown to verify the proposed design. Potential applications of the developed hybrid chaos synchronization system include information storage, message identification, and certain types of secure signal and image communication.

BOLLT, E., Associate Professor and Klebanoff, A., "A New and Simple Chaos Toy," *International Journal of Bifurcation and Chaos*, accepted for publication.

We present two new, and perhaps the simplest yet, mechanical chaos demonstrations. They are both designed based on a recipe of competing nonlinear oscillations. One of these devices is simple enough that using the provided description, it can be built using a bicycle wheel, a piece of wood routed with an elliptical track, and a ball bearing. We provide a thorough Lagrangian mechanics based derivation of equations of motion, and a proof of chaos based on showing existence of an embedded Smale horeshoe using Melnikov's method. We conclude with discussion of a future application.

BOLLT, E., Associate Professor, "Combinatorial Control of Global Dynamics in a Chaotic Differential Equation," *International Journal of Bifurcation and Chaos*, accepted for publication.

Controlling chaos has been an extremely active area of research in applied dynamical systems, following the introduction of the Ott, Grebogi, Yorke (OGY) technique in 1990 [Ott *et al*, 1990], but most of this research based on parametric feedback control uses local techniques. Associated with a dynamical system which pushes forward initial conditions in time, transfer operators, including the Frobenius-Perron operator, are associated dynamical systems which push forward ensemble distributions of initial conditions. In [Bollt, 2000a; Bollt, 2000b; Bollt & Kostelich, 1998], we have shown that such global representations of a discrete dynamical system are useful in controlling certain aspects of a chaotic dynamical system which could only be accessible through such a global representation. Such aspects include invariant measure targeting, as well as orbit targeting. In this paper, we develop techniques to show that our previously discrete time techniques are accessible also to a differential equation. We focus on the Duffing oscillator as an example. We also show that a recent extension of our techniques by Gora and Boyarsky [GÌra & Boyarsky, 1999] can be further simplified and represented in a convenient and compact way by using a tensor product.

BUCHANAN, J.L., Professor, Gilbert R.P., and Wirgin, A., "Implementation of the ICBA Method for Solids of Revolution," *Problemi Attuali Dell'Analisi e Della Fisica Mathematica*, pp. 41-57, 2000.

The Intersecting Canonical Body Approximation ICBA is a method for determining the shape of an unknown object from measured values of the acoustic field generated by scattering of an incident wave by the object. In this paper we consider objects that are solids of revolution of the form $r = R + \delta f(z)$ where z is the depth variable and δ is small. We use the method of variation of boundaries to develop recurrence relations for the coefficients in the representation of the scattered field. This permits use to avoid the "inverse crime" by affording a means of calculating the scattered field that is different from the technique used by ICBA to invert the field.

BUCHANAN, J.L., Professor, Gilbert R.P., Wirgin A., and Xu, Y., "Identification, by the Intersecting Canonical Domain Method, of the Size, Shape and Depth of a Soft Body of Revolution Located Within an Acoustic Waveguide," *Inverse Problems*, 16, pp.1709-26, 2000.

The Rayleigh hypothesis is employed to solve the forward problem of scattering of a known, arbitrary acoustic wave form a body of revolution immersed in a shallow body of water. The acoustic wavefields obtained in this way are then employed as simulated data for the inverse problem of the determination of the (unknown) size, shape and depth of the unknown body, which is known to be acoustically soft, axi-symmetric in shape and located on the vertical axis of a Cartesian reference system within the shallow water waveguide. This fully 3D inverse problem is solved by the intersecting canonical domain method, using multifrequency scattered field data, for two types of bodies: a non-convex (indented) spindle and a conical seamount.

GAGLIONE, Anthony M., Professor, (co-author), "Discriminating Groups,'' *Journal of Group Theory*, accepted for publication.

A group G is termed *discriminating* if every group separated by G is discriminated by G. In this paper, we answer several questions concerning discrimination which arose out of G. Baumslag, A. Myasnikov and V. Remeslennikov, "Discriminating and Co-Discriminating Groups,'' *J. Group Theory* 3 (2000) 467-479. We prove that a finitely generated equationally Noetherian group G is discriminating if and only if the quasivariety generated G is the minimal universal class containing G. Among other results, we show that the non-abelian free nilpotent groups are non-discriminating. Finally we list some open problems concerning discriminating groups.

JOYNER, W. David, Professor, "Invariant distributions on the *n*-fold metaplectic covers of *GL(r,F), F* p-adic," *J. Fourier Anal. and Appl.*, 7(2001) pp.343-358.

We describe the unitary and tempered dual of the n-fold metaplectic covers of SL(2,F), where F is a p-adic field with p not dividing 2n. We show that any tempered distribution on the n-fold metaplectic covers of SL(2,F) or of GL(r,F) (satisfying certain assumptions) may be expressed as a distributional integral over the tempered dual. We also show that any invariant distribution on the n-fold metaplectic covers of SL(2,F) or of GL(r,F) is supported on the tempered dual.

JOYNER, W. David, Professor, "On Finite Dimensional Representations of Non-connected Reductive Groups," *J. of Lie Theory*, 10 (2000) pp. 269-284.

We extend the classification of irreducible finite dimensional representations of almost simple algebraic groups over an algebraically closed field of characteristic zero to certain non-connected groups G where the component group is cyclic. We also extend some of Steinberg's results on the adjoint quotient to these non-connected groups. These results are used to describe the geometry of twisted conjugacy classes of the connected component of G. As an application, we show that there is a ''functorial'' correspondence between virtual (finite dimensional) characters of twisted invariant representations of G and virtual characters of an endoscopic group H of G.

KONKOWSKI, Deborah A., Professor (co-author), "Quantum Singularity of Quasiregular Spacetimes," *Gen. Relativ. Grav.*, 33, pp.1131-1136 (2001)

Some of the mildest singularities in classical general relativity are shown to be singular quantum mechanically as well. A class of the mild, topological singularities known as quasiregular singularities remains singular when probed by quantum wave packets. These static spacetimes possessing dislocations and disclinations are quantum-mechanically singular since the spatial portion of the wave operator is not essentially self-adjoint and thus the evolution of a test quantum wave packet is not uniquely determined by the initial wave function.

This paper won an Honorable Mention in the Year 2000 Gravity Research Foundation Essay Contest.

KIDWELL, Mark E., Professor, and STANFORD, Theodore, Assistant Professor, "On the z-degree of the Kauffman Polynomial of a Tangle Decomposition," *AMS/IP Studies in Advanced Mathematics*, Vol. 21, (2001).

A link diagram is divided into k tangles $\{T_i\}$. Tangle T_i has N_i crossings and a longest bridge of length B_i. Then the Dubrovnik polynomial of the link (a variant of the Kauffman polynomial) has z-degree less than or equal to $k - 1 + S(N_i - B_i)$.

LOCKHART, Robert B., Associate Professor, "Optimal Ensemble Length of Mixed Separable States," *Journal of Mathematical Physics*, Vol 41, issue 10, pp.6766-6771.

The ensemble length of a mixed separable state is the minimal number of pure separable states needed in convex combination to produce it. The spectral theorem shows that one needs at least the rank. A result of Uhlmann shows

that the square of the rank will suffice. In this paper it is shown that generically, with the exception of bipartite states in which one particle is a qubit, that one needs more than the rank. In fact, for systems of qubits, the ensemble length is fairly close to the square of the rank. These results are important for using separability testing algorithms. The method used to prove the theorems is Sard's theorem.

MARUSZEWSKI, Richard F., Professor, "Asymptotes and Spreadsheets," *AMATYC Review*, Volume 21, #1, pp 27-31.

This paper discusses the spreadsheet implementation of an algorithm for finding the approximate solution of a differential equation near an asymptote. The material is accessible to students in a standard differential equations course and is intended to enhance their understanding of numerical algorithms.

McCOY, Peter A., Professor, "On the Singularities of Rational Legendre Series," C*omplex Variables: An International Journal*, pp.369-374. (2000).

A classical theorem of Z. Nehari connects the singularities of Legendre Series in one complex variable with those of an associated analytic function. The author uses integral transform methods to solve an analogous problem for series whose terms are the quotients of products of Legendre Series.

McCOY, Peter A., Professor, "On Radiating Solutions to the Helmholtz Equation and Inverse Scattering," *Applicable Analysis*, accepted for publication.

This paper concerns the structure of radiating solutions of the Helmholtz equation for acoustic problems in 3-D. The purpose is to locate sources for radiating solutions by analyzing their far-field patterns. This information is inferred from the angular distribution of values for the far-field pattern.

NAKOS, George C., Professor, and Williams, Robert M., "A Fast Calculation of the Characteristic Polynomial," *Mathematica for Education and Research*, Vol. 9, No. 1.

We introduce J. Abdeljaoued's work on the computation of the characteristic polynomial over any commutative ring. The work is based on an improved sequential version of the Berkowitz algorithm, which in turn is based on Samuelson's Formula. This approach yields the fastest and most efficient algorithm of its kind. The algorithm is completely free of division and it applies to any commutative ring with unit. The Leverrier-Fadeev algorithm is also fast but requires characteristic zero, or characteristic at least greater than the size of the matrix.

PRICE, Geoffrey L., Professor, and Arveson, William B., "Tensor Products of Completely Positive Semigroups," *Journal of Evolution Equations*, 1, accepted for publication.

Since virtually the beginning of the study of quantum mechanics, physicists have used operator algebras as models to understand the behavior of quantum mechanical systems. In this setting the elements of an operator algebra *A* are viewed as the physical observables, and there is a one-parameter group of automorphisms a = {a_t: t e R} defined on *A* so that for each observable x the operator $a_t(x)$ represents the evolution of x at time t. Physicists say that the system "returns to equilibrium" if there is a normal state w on *A* such that for any other normal state r on *A*, the states roa_t approach w for large t. Such a state is called an absorbing state. Recently R. T. Powers has studied quantum dynamical systems called E_0-semigroups. These are similar to the one-parameter groups of automorphisms above but they may not move backward in time. An important question in the subject is to determine if there are E_0-semigroups that have no absorbing states. Powers showed that for a large class of these semigroups each E_0-semigroup in the class is equivalent to an E_0-semigroup with an absorbing state. In this paper the authors have shown that there are examples of E_0-semigroups that are pure (i.e., there are no nontrivial subalgebras of *A* left globally invariant) but which have no absorbing states. On the other hand by Powers' result such E_0-semigroups are equivalent to pure E_0-semigroups that do have an absorbing state.

TRAVES, William N., Assistant Professor, "Tight Closure and Differential Simplicity," *Journal of Algebra*, Volume 228, pages 457-476, 2000.

The behavior of the Hasse-Schmidt algebra under etale extension is used to show that the Hasse-Schmidt algebra of a smooth algebra of finite type over a field equals the ring of differential operators. These techniques show that the formation of Hasse-Schmidt derivations does not commute well with localization, providing a counterexample to a question of Brown and Kuan; their conjecture is reformulated in terms of the Hasse-Schmidt algebra. These techniques also imply that a smooth domain is differentially simple. Tight closure is used to show that the test ideal is Hasse-Schmidt stable. Indeed, differentially simple rings of prime characteristic are strongly F-regular.

Saito, Mutsumi and TRAVES, William N., Assistant Professor, "Differential Algebras on Toric Varieties," *Contemporary Mathematics*, accepted for publication.

This paper studies algebras of operators associated to a semigroup algebra. The ring of differential operators is shown to be anti-isomorphic to the symmetry algebra and both are described explicitly in terms of the semigroup. As an application, we produce a criterion to determine the equivalence of A-hypergeometric systems. Conditions under which associated algebras are finitely generated are studied. These results are sufficient to establish Becker's conjecture in the semigroup case. As well, an algorithm is provided to compute the composition series of D-modules over semigroup algebras.

TURNER, Peter R., Professor, "Residue Polynomial Systems," *Theoretical Computer Science*, 281 (2001) 21.

In this paper the basic ideas and properties of the Residue Polynomial System are presented. This system is an analog of the Residue Number System for integer representation and arithmetic. The basics of the representation are manifestations of standard results in polynomial interpolation. The biggest problem with residue representations is in performing division. In a polynomial ring however, this turns out to be a tractable problem. An algorithm for polynomial division is presented. This is then extended to other polynomial algorithms which render RPS a potentially practical system for polynomial manipulation within Computer Algebra Systems.

TURNER, Peter R., Professor, "Teaching Scientific Computing through Projects," *Journal of Engineering Education*, 90 (2001) 79 – 83.

Traditional numerical methods courses face several difficulties. There is a performance problem for students who need to combine different skills. Students spend so much time mastering the details that they often miss the point as to why there is a need for scientific computing.

Using projects drawn from practical situations, and a good high-level package such as MATLAB, can overcome much of this. The difficulty associated with learning scientific programming is greatly alleviated. This in turn frees more time for studying more advanced methods.

The need to combine ideas from different areas makes this an ideal candidate for both team-teaching and student teamwork on the projects. In this paper, we discuss the use of this approach in a variety of scientific computing courses at different levels.

WITHERS, Wm. Douglas, Professor, "Augural Image Zooming," *Dr. Dobbs Journal*, 323 (August, 2000), pp. 48-58.

This article presents a fundamentally new method for zooming images, termed the *augural* method. The goal of the augural method is to zoom images with a minimum of computation, while keeping smooth regions smooth and sharp edges sharp. The method described here serves to double the scale of an image; the same principles have been applied to zoom an image to any desired size.

WITHERS, Wm. Douglas, Professor, "A Rapid Probability Estimator and Binary Arithmetic Coder," *IEEE Transactions on Information Theory*, accepted.

We present a new integrated algorithm for binary arithmetic coding and probability estimation, competitive in speed and compression performance with state-of-the-art algorithms such as the QM-coder and Z-coder. The chief innovation is representing the bracketing interval width both directly and by a logarithmic approximation. Performance is evaluated by experiments using bilevel-image data sets. An open-source software version is available.

Conference Proceedings

McCOY, Peter A., Professor, "Electromagnetic Scattering in 3-D," *Proceedings of the IMACS 2000,* D. Schulz, B. Wade, J. Vigo-Aguiar & S.K. Dey (editors), May (Y2K) pp. 334-337.

The author uses integral transform methods in several complex variables to deduce information about the singularities of electromagnetic fields from an analysis of their radiation patterns.

MELLES, Caroline G., Associate Professor, (editor with Ruth I. Michler), "Singularities in Algebraic and Analytic Geometry," *AMS Contemp. Math.,* 266, 2000.

This volume contains the proceedings of an AMS special session held at the 1999 Joint Mathematics Meetings in San Antonio. The participants were an international group of researchers studying singularities from algebraic and analytic viewpoints. The contributed papers contain original results as well as some expository and historical material. Topics include the role of valuation theory in algebraic geometry with recent applications to the structure of morphisms; algorithmic approaches to resolution of equisingular surface singularities and locally toric varieties; weak subintegral closures of ideals and rings; direct-sum decompositions of finitely generated modules; construction and examples of resolution graphs of surface singularities; Jacobians of meromorphic curves; investigation of spectral numbers of curve singularities using Puiseux pairs; Gröbner basis calculations of Hochschild homology for hypersurfaces with isolated singularities; and the theory of characteristic classes of singular spaces – a brief history with conjectures and open problems.

MELLES, Caroline Grant, Associate Professor and Milman, Pierre, "Single-Step Combinatorial Resolution via Coherent Sheaves of Ideals," Singularities in Algebraic and Analytic Geometry, *AMS Contemp. Math.,* 266 (2000) pp.77-88.

We consider complex analytic varieties whose singularities may be resolved by a finite sequence of blow-ups along smooth centers, each of which is an intersection of smooth normal crossings divisors, arising either as strict transforms from the base space or as exceptional divisors from previous blow-ups. We give an algorithm for replacing such a sequence of blow-ups by a single blow-up along a coherent sheaf of ideals which is locally generated by monomials in appropriate coordinates.

MICHAEL, T. S., Associate Professor and Pinciu, Val, "Multiply Guarded Guards in Orthogonal Galleries," *Proceedings of the International Conference of Computing and Information* (ICCI-2000).

Let P be a rectilinear polygon (the art gallery) with n sides. We wish to select a set G of points (the guards) so that (a) every point in the polygon is visible to some guard; and (b) every guard is visible to at least k other guards. What is the minimum number of guards g(n,k) that is guaranteed to suffice for all polygons with n sides? We prove that g(n,k) = (k+1)floor(n/6) + d, where d=1 if n is 2 less than a multiple of 6, and d=0.

TURNER, Peter R., Professor, "The Symmetric Level-Index System with an application to Chaos," *Proceedings of CSCC2000,* Athens, Greece, July 2000. (invited paper)

In this paper we present a brief summary of the Symmetric Level Index, SLI, system of number representation and computer arithmetic with only enough detail to allow the reader to understand the remaining discussion. This discussion is focused on an elementary example of chaos, the logistic map or shift map. This map, which was discussed by Ulam and von Neumann in the context of random number generators, is one of the classical one-dimensional examples of a chaotic map. However, in the conventional realm of floating-point computing, the chaotic behavior is completely lost. To a great extent the correct behavior of this map is recovered if the arithmetic is performed in the SLI system. This statement is valid for all three of the distinguished cases, where the process is seeded with an irrational, a rational, or a rational with a power of 2 denominator.

Technical Reports, Books and Book Chapters

JOYNER, W. David, Professor, Rubik's Cube, Merlin's Machine, and Other Adventures with Groups, to be published by The Johns Hopkins University Press, 2002.

In this book, we develop the basics of group theory and create group-theoretical models of Rubik's cube-like puzzles. On the practical side, we also discuss the solution strategy for the Rubik's cube in some detail. Some solution strategies are briefly discussed for similar puzzles (the '15 puzzle', the 'Rubik tetrahedron' or pyraminx, the 'Rubik dodecahedron' or megaminx, the skewb, the 'hockeypuck', and the 'masterball') as well. The important point to remember, though, is that group theory is a powerful tool with many real-world applications. Solving puzzles happens to be just one of them. Because of our Rubik's cube focus, the approach in this book is different from some other texts.

McCOY, Peter A., Professor, "Zonal Harmonics," *Kluwer Encyclopedia of Mathematics,* Supplemental Vol. III, Professors M. Hazewinkel (Managing Editor) & Rob Hoksbergen, Coordinating Editor, in press.

This encyclopedia entry is a short survey of zonal harmonics and associated applications for a function-theoretic point of view.

MEYERSON, Mark D., Professor, "When a Sum of Functions is Increasing," solution to problem 10768, *American Mathematical Monthly,* 108, January 2001, p. 82.

Showing that there is a continuous function from the reals to the reals such that no differentiable function added to it gives an increasing function. Still true if differentiable is replaced by continuously differentiable and continuous by differentiable. Not true, in this latter case, if continuously differentiable is replaced by real analytic and differentiable by continuously differentiable.

Smith, Karen E., Kahanpää, Lauri, Kekäläinen, Pekka, and TRAVES, William N., Assistant Professor, An Invitation to Algebraic Geometry, University text series, published by Springer-Verlag, New York, (2000).

The aim of this book is to describe the underlying principles of algebraic geometry, some of its important developments in the twentieth century, and some of the problems that occupy its practitioners today. It is intended for the working or the aspiring mathematician who is unfamiliar with algebraic geometry but wishes to gain an appreciation of its foundations and its goals with a minimum of prerequisites. Few algebraic prerequisites are presumed beyond a basic course in linear algebra.

After explaining the basic context of algebraic geometry, we treat classical constructions, smoothness and the resolution of singularities. We end with a treatment of maps to projective space and line bundles. This book is unusual in that it attempts to give a broad overview of some highly technical mathematics without getting mired in details.

TURNER, Peter R., Professor, and Williams, Robert M.,"Small Linear Systems Solutions for Image Identification in Computer Vision," NAWCADPAX TR 2001, 42 pp.

In this paper we present solutions to some of the fundamental problems in computer vision using simple vector-geometric and linear algebraic techniques. The problems discussed are image recognition and the three-dimensional location of a moving object in an otherwise known image.

The methods used rely on simple linear homogeneous systems which result from the natural image-object equations. The dimension of these linear systems is small, 6x6 for the classical six-point image identification problem.

The model used remains valid for points, lines, or a combination of these. The algorithm is applicable to the most general image projection models. Potential difficulties resulting from the colinearity or coplanarity of data points are examined. The numerical stability of the algorithm is also considered.

TURNER, Peter R., Professor, and Williams, R., M., "A Simple Explanation of Projective Invariants for Image Identification with an Application to Point-selection," NAWCADPAX TR 2000, 40 pp.

In this paper we reinterpret the fundamental results on using projective invariants for image identification in simple vector-geometric and linear algebraic terms. The basic problem discussed is image identification using six points. The geometric interpretation also sheds light on the selection of appropriate signature points to improve the numerical stability of the image identification algorithm. This is the "full" version of the journal submission with the same title listed above.

WARDLAW, William P., Professor, "A Generalized General Associative Law," *Mathematics Magazine,* Vol. 74, No. 3, June 2001, pp. 230-233.

This paper first reviews various proofs of the generalized associative law in the literature and points out the shortcomings and fallacies in some of them. Then it is recommended that the general associative law be replaced with a generalization, namely that n-associativity implies $(n + 1)$-associativity when $n > 2$.

Presentations

BAILEY, Craig K., Professor, "The Mathematics of GPS," MD-DC-VA section meeting of the MAA, Lexington, Va., 20-21 April 2001.

BAKER, B. Mitchell, Professor, "Flow of Weights and the AT Property for Product States of the Gauge Invariant and Rotation Invariant CAR Algebras," U. of New Hampshire, Great Plains/Canadian Operator Theory Symposium, May 2001.

BAKER, B. Mitchell, Professor, "Flow of Weights and the AT Property for Product States of the Gauge Invariant and Rotation Invariant CAR Algebras," Southeastern Sectional Meeting of AMS, Birmingham, Alabama, June 2000.

BAKER, B. Mitchell, Professor, "Are GICAR Product State Factors Infinite Tensor Products of Type I Factors?" University of Pennsylvania, Philadelphia, PA, April 2001.

BOLLT, E., Associate Professor, "Synchronism and Occurrence of Low-Dimensional Chaos in High-Dimensional Dynamical Systems," Siam Dynamical Systems Conference'01, Snowbird Utah, May 20-24, 2001.

BOLLT, E., Associate Professor, "Is Symbolic Threshold Crossing Analysis of Chaotic Time-Series by a Nongenerating Partition Valid?" Siam Dynamical Systems Conference'01, Snowbird Utah, May 20-24, 2001.

BOLLT, E., Associate Professor, "Combinatorial Control of Global Dynamics in a Chaotic Differential Equation," Worcester Polytechnical Institute, MA, April 2001. (invited colloquium)

BOLLT, E., Associate Professor, "What Symbolic Dynamics Do We Get With A Misplaced Partition? On the Validity of Threshold Crossings Analysis of Chaotic Time-Series," American Physical Society Meetings, Seattle, WA, March 2001. (invited plenary talk)

BOLLT, E., Associate Professor, "A New and Simple Chaos Toy," Dynamics Days 2001, University of North Carolina, Chapel Hill, NC, January 2001.

BOLLT, E., Associate Professor, "Global Control of Chaos: Choosing Invariant Measure and the Inverse-Frobenius Perron Problem," Los Alamos National Laboratory, Los Alamos, NM, November 2000. (invited colloquium)

BOLLT, E., Associate Professor, "Are Threshold Crossings Reasonable?" Pacific Rim Dynamical Systems Conference (DS00), Maui, Hawaii, August 2000.

BOLLT, E., Associate Professor, "Combinatorial Control of Global Dynamics: Theory and Experiment," Pacific Rim Dynamical Systems Conference (DS00), Maui, Hawaii, August 2000.

BOLLT, E., Associate Professor, "Low Dimensional Chaos in High Dimensional Phase Space. How Does it Occur?" Pacific Rim Dynamical Systems Conference (DS00), Maui, Hawaii, August 2000.

BOLLT, E., Associate Professor, "Controlling Invariant Density: An l^∞ Solution to the Inverse Frobenius-Perron Problem," ISCAS2000-IEEE International Symposium on Circuits and Systems, Geneva Switzerland, June 2000.

CRAWFORD, Carol G., Professor, "Graph Representations for Image Processing," Society for Industrial and Applied Mathematics Annual Meeting, Puerto Rico, July 11, 2000.

GAGLIONE, Anthony M., Professor, "Discriminating Groups," U.S. Naval Academy Mathematics Department Seminar, October 2000.

GAGLIONE, Anthony M., Professor, "Discriminating and Non-discriminating Groups," Albany Group Theory Conference, 14 October 2000.

GAGLIONE, Anthony M., Professor, "Discriminating Groups," sectional meeting of the AMS held at Columbia Univ., November 2000.

GAGLIONE, Anthony M., Professor, "On Discriminating and Squarelike Groups I," Zassenhaus Group Theory Conference, New College of University of South Florida, January 2001.

GAGLIONE, Anthony M., Professor, "Residual Properties of Groups and Logic: an Overview," at Queens College (CUNY) of New York, 14 March 2001.

GAGLIONE, Anthony M., Professor, "Squarelike and Discriminating Groups," NY Group Theory Seminar, CUNY Graduate Center, New York, NY, 16 March 2001.

GAGLIONE, Anthony M., Professor, "What is the Probability that Two Elements of a Group Commute?" Fairfield University, Fairfield CT, 10 April 2001.

GAGLIONE, Anthony M., Professor, "Varietal Amalgams and a Freiheitssatz," sectional meeting of the AMS held at Stevens Institute of Technology, 29 April 2001.

HOFFMAN, Michael, Professor, "The Algebra of Multiple Zeta Values," contributed talk for AMS Conference on Mathematical Challenges of the 21st Century, Los Angeles, Ca., August 7, 2000.

HOFFMAN, Michael, Professor, "Sum Fun with Sequences," U.S. Naval Academy Mathematics Colloquium, September 6, 2000.

HOFFMAN, Michael, Professor, "Polynomials Defined by Derivatives and Their Applications," University of Maine Undergraduate Colloquium, Orono, ME, November 10, 2000.

HOFFMAN, Michael, Professor, "An Overview of Recent Work on Multiple Zeta Values and Multiple Polylogarithmic Constants," University of Maine Colloquium, Orono, ME, November 11, 2000.

HOFFMAN, Michael, Professor, "Multiple Harmonic Sums and Quasi-Symmetric Functions," Special Session on Integrals and Series Throughout Mathematics, AMS Winter Meeting, New Orleans, La., January 11, 2001.

KAPLAN, Harold M., Professor, "C Macros to Check that Subscripts Lie Within Their Array Bounds," Mid-Atlantic Probability and Statistics Day, Saturday, American University, Washington, D. C., April 21, 2001.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Quasiregular Spacetimes," Contributed Talk at The Nineth Marcel Grossmann Meeting, University of Rome "La Sapienza", Rome, Italy, July 7, 2000.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Quasiregular Spacetimes," Contributed Talk at XIII International Congress on Mathematical Physics, Imperial College, London, U.K., July 18, 2000.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Quasiregular Spacetimes," Talk at 10th Midwest Gravity Meeting, Rochester, Michigan, October 28, 2000.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Quasiregular Spacetimes," Poster Presentation at 20th Texas Symposium on Relativistic Astrophysics, Austin, Texas, December 10-15, 2000.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Quasiregular Spacetimes," Contributed Talk at Spring 2001 Meeting of the American Physical Society, Washington, D.C., May 1, 2001.

MAHAR, T. J., Associate Professor, "A Vector Version of Godunov's Method for the Euler Equations of Fluid Flow," Naval Science Warfare Center, Indian Head, MD, 19 October 2000.

MARUSZEWSKI, Richard F., Professor, "One Way Functions," Cryptology Seminar, U.S. Naval Academy Mathematics Department.

McCOY, Peter A., Professor, "On Electromagnetic Scattering in 3-D," SIAM Annual Meetings, Mini-Symposium on Special Functions and Orthogonal Polynomials, San Juan, Puerto Rico, July 13, 2000.

McCOY, Peter A., Professor, "The Field Singularity Problem and Source Location," Applied Computational Electromagnetics Society Annual Meeting, Naval Postgraduate School, Monterey, CA, Special Session on "H-Infinity for Antennas," March 21, 2001.

MELLES, Caroline G., Associate Professor, "A Generalization of Chow's Theorem for Coherent Sheaves of Ideals," AMS Meeting Special Session, San Francisco, California, October 21, 2000.

MELLES, Caroline G., Associate Professor, "Some Recent Progress in Resolution of Singularities," U.S. Naval Academy Mathematics Department Colloquium, Annapolis, Maryland, March 21, 2001.

MICHAEL, T. S., Associate Professor, "Galleries, Guards, and How a Marine Solved a Problem of Erdos," The George Washington University, Combinatorics Seminar, March 2001.

MICHAEL, T. S., Associate Professor, "Tiles, Guards, and Galleries," University of Nevada Reno, Mathematics Department Colloquium, November 2000. (invited colloquium)

MICHAEL, T. S., Associate Professor, "Valiant Guards and How to Pack the Marine Corps Way," United States Naval Academy, Mathematics Department Colloquium, October 2000.

MICHAEL, T. S., Associate Professor, "Two New Art Gallery Theorems: Guarding the Guards," Ninth International Conference on Graph Theory, Combinatorics, Algorithms, and Applications, Kalamazoo, Michigan, June 2000.

PENN, Howard L., Professor, "Home Run Hitting and Halley," MD-DC-VA Sectional Meeting of the Mathematical Association of America, Fall Meeting, November 2000, Washington, DC.

SANDERS, Thomas J., Professor, "Mathematics and Science Programs, Facilities, and Resources at U.S. Naval Academy," DMSO Workshop on Mathematics and Science at Service Academies, 26-27 June 2000 Defense Modeling & Simulation Office (DMSO).

TRAVES, William N., Assistant Professor, "Secret Sharing," U.S. Naval Academy Summer Seminar, June 2000.

TRAVES, William N., Assistant Professor, "Differential Operators on Toric Varieties," AMS-IMS-SIAM Research Conference, Mount Holyoke College, June 2000.

TRAVES, William N., Assistant Professor, "Incidence Hopf Algebras," three lectures, George Washington University, September and October 2000.

TRAVES, William N., Assistant Professor, "Differential Operators on Toric Varieties," Special Session on Differential Algebra, AMS Section Meeting, New York, November 2000.

TRAVES, William N., Assistant Professor, "Fair Cake Cutting After Su," MD/DC/VA MAA Meeting, Bowie State University, November 2000.

TRAVES, William N., Assistant Professor, "Commutative Algebra and Internet Reliability," Mid-Atlantic Algebra Conference, Virginia Tech., March 2001.

TRAVES, William N., Assistant Professor, "Commutative Algebra and Network Reliability," Towson State University, April 2001.

TRAVES, William N., Assistant Professor, "Commutative Algebra and Network Reliability," George Washington University, April 2001.

TRAVES, William N., Assistant Professor, "Keeping Secrets," U.S. Naval Academy Summer Seminar, June 2001.

TURNER, John C., Professor, "Spreadsheet for Fisher's Exact Test," Western Conference on Teaching Statistics, March 2000.

TURNER, John C., Professor, "Confidence Intervals Using Probability," University of Colorado at Denver, March 2000.

TURNER, John C., Professor, "Confidence Intervals Using Probability," Joint Statistical Meetings, Indianapolis, IN, August 2000.

TURNER, Peter R., Professor, "The Symmetric Level-Index System with an Application to Chaos," Proceedings of CSCC2000, Athens, Greece, July 2000. (invited paper)

WARDLAW, William P., Professor, “Matrix Representations of Finite Fields,” American University, Washington, DC, 19 September 2000. (invited talk)

WARDLAW, William P., Professor, U. S. Naval Academy Mathematics Department Colloquium Talk, “What is the Rank of a Matrix over a Commutative Ring?” U. S. Naval Academy, Annapolis, MD, 15 November 2000.

WARDLAW, William P., Professor, “What is the Rank of a Matrix over a Commutative Ring?” Fall meeting of the MD-VA-DC Section of the MAA, American University, Washington, DC, 18 November 2000.

WARDLAW, William P., Professor, “What is the Rank of a Matrix over a Commutative Ring?” annual meeting of the AMS and MAA, New Orleans, LA, 11 January 2001.

WARDLAW, William P., Professor, “A Problem of Kaplansky,” Spring meeting of the MD-VA-DC Section of the MAA, Virginia Military Institute in Lexington, VA, 21 April 2001.

WITHERS, Wm. Douglas, Professor, “Augural Image Zooming,” Department of Mathematics, Georgia Institute of Technology, 25 January 2001.