Mathematics Department

Professor Mark D. Meyerson
Chair

Mathematics provides a logical framework and a language indispensable to understanding the technical world in which we live. The following description summarizes the many contributions to this field of study made during the past academic year by the faculty and midshipmen majors of the Mathematics Department of the U.S. Naval Academy. The results cited reveal the great scope, diversity, and applicability of mathematics and offer glimpses of its intellectual beauty and appeal.

Several midshipmen conducted research projects as Honors Mathematics Majors, or in specially created projects under the guidance of faculty members. Faculty spent numerous hours serving as mentors and readers for capstone projects required of all midshipmen majors. Listed below are our three honors mathematics majors in the Class of 2004, their project titles, and their faculty mentors:

Midshipman 1/C Aron T. Foster, USN, "Probabilistic Coding Theory Algorithms Implemented in GAP" (Professor W. David Joyner)

Midshipman 1/C Charles R. Platt, USN, "A study of the Traveling Salesman through a variety of search algorithms" (Professor W. Charles Mylander)

Midshipman 1/C William R. Sheridan, III, USN, "A Linear Programming Approach to Modeling the Effect of Consolidating US Navy Fuel Types," (Commander David R. Spoerl, USN)

Regularly meeting throughout the year are four colloquia series, in pure mathematics, applied mathematics, operations research, and teaching methods. These bring in outside speakers and they also provide a forum for department members and mathematics majors to present their work. In addition, there are weekly seminars in topics of special interest, such as Riemann Surfaces, Homological Algebra, and modeling the Chesapeake Bay.

Once again, the Naval Academy's Mathematics Department produced a wide range of scholarly work that appeared as technical reports or as publications in refereed journals throughout the world. Dozens of articles appeared as applications of mathematics or as pure mathematical research. Topics that Mathematics Department faculty researched include:

mathematics pedagogy, chaos and dynamical systems, mathematical physics and cosmology, algorithms for computers, wavelets, cryptology, ocean acoustics, navy ship degaussing, image recognition, quantum computing, optimization theory, and basic mathematical research in areas such as algebra, analysis, combinatorics, computational geometry, differential equations, differential geometry, matrices, number theory, operator theory, and statistics.

In addition to many independent research projects, several research projects were sponsored in whole or part by a variety of sources, such as:

Department of Defense, High Performance Computing Initiative
Department of Health and Human Services
National Aeronautics and Space Administration
National Science Foundation
National Security Agency
Naval Research Laboratory
Naval Surface Warfare Center
Office of Naval Research

During the past year, members of the Mathematics Department presented the results of their scholarly activities on over fifty occasions at professional mathematical meetings and colloquia throughout the United States and abroad. This activity, along with publication, enhances the academic stature of the Naval Academy and promotes the professional growth

and reputation of those individuals involved. Through research activity, the faculty expanded their intellectual horizons and stayed vibrant in their disciplines. They contribute to the discovery of new mathematics, and they develop new materials and ideas that they can share with midshipmen students in their mathematics courses and research projects. Of special note this year, Professor Michael E. Hoffman was awarded the Naval Academy Alumni Association Civilian Faculty Award for Excellence in Research.

Sponsored Research

Cocycles for One-parameter Flows of B(H)

Researchers: Assistant Professor Alexis Alevras,
Dr. Robert Powers (University of Pennsylvania) and Professor Geoffrey L. Price
Sponsor: Naval Academy Research Counsel (NARC to AA)

E_0-semigoups of B(H), *i.e.,* one-parameter semigroups of endomorphisms on the algebra of bounded operators on a Hilbert space H, were studied. E_0-semigoups model evolution of quantum systems that is irreversible in time. The objective of this study was to characterize these objects up to a natural equivalence, called cocycle conjugacy.

A natural cocycle conjugacy invariant for an E_0-semigroup a is the set of local cocycles, i.e. of strongly continuous families {Vt} of operators that commute with a and satisfy the cocycle relations Vt+s=Vtat (Vs). This set has a multiplicative structure as well as an order structure on its positive elements and contains a lot of information about a; in particular the unitary local cocycles form a group that may be appropriately viewed as the automorphism group of the E_0-semigroup.

While local cocycles for the standard, type I examples of E_0-semigoups were computed early on by W. Arveson (unitary cocycles) and R. Bhat (positive contractive cocycles) nothing was known about other classes of E_0-semigoups. In this work, we computed (for the first time) local cocycles for more exotic, type II examples. More precisely, we provided a complete description of the local cocycles of a family of E_0-semigoups of type II0, including information on the multiplicative and order structures; in particular we identify the "automorphism" group of unitary local cocycles. The description is at the level of the so called boundary weight associated with the E_0-semigroups.

Graph Representation for Computer Vision: Aspect Graphs and Sphere-of-Influence Graphs for Images

Researcher: Professor Carol G. Crawford
Sponsors: The Federal Bureau of Investigation (FBI) and Office of Naval Research (ONR)

The researcher applies prior results in graph representations for the pre- and post-processing of images to the development of pedagogical tools for undergraduate courses in Graph Theory and Combinatorics. Earlier publications presented Aspect Graphs for Robotic Vision and Sphere-of Influence Graphs for Automatic Fingerprint Identification. This project is devoted to the creation and design of classroom implementations incorporating the results of this research.

Quantitative Population Models for Cell-Based Biosensor Arrays

Researcher: Professor James M. D'Archangelo
Sponsor: Naval Research Laboratory (NRL)

The rapid detection and identification of potentially toxic substances has assumed new urgency given the increasing threat of weapons of mass destruction. The firing patterns of neurons can be potentially used to characterize the presence and effect of both toxins and benign stimulants. The objectives of this project are:

(1) To design, implement, and test realistic analytical mathematical models of individual cultured living neurons, and networks of cultured neurons.

(2) To refine and improve the neuron models based upon laboratory measurements and modeling of neuron geometry, electrical membrane characteristics (ionic channels), and synaptic conductivity (i.e. network behavior) of cultured neurons.

(3) To validate these models by predicting the behavior of living cultured neurons in response to diverse neurotoxins (*e.g.,* channel blockers, synaptic blockers).

Thus far, the researchers have learned to specify and execute neuron models using both the neural simulation systems NEURON, which is maintained at Yale University, and GENESIS (General Neural Simulation System), which was developed at the California Institute of Technology. Simple multi-compartmental models of spinal motoneurons have been developed using MATLAB and NEURON. These models represent the different components of the physical neuron (e.g. axons, dendrites, cell body, axon hillock). The excitable axonic membrane is modeled with an extended Hodgkin-Huxley model which has five ionic channels. Simple network models (i.e. two-neuron loops) have been implemented in both MATLAB and NEURON, including realistic models for synaptic transmission.

Data Assimilation Applied to Chesapeake Bay Trajectory Modeling

Researcher: Associate Professor Gary O. Fowler
Sponsor: Defense Modeling and Simulation Office (DMSO)

Deterministic models for calculating trajectories of fluid flow can be improved by including stochastic elements. There are several competing and complementary methods for including the stochastic elements. This on-going project is producing a literature review of the various methods, is developing software in MATLAB or MATHEMATICA to implement and is evaluating the various methods.

Data assimilation is the process of using empirical information to improve a forecast from a deterministic model. The best linear method of data assimilation is the Kalman filter. This filter requires estimates of variability of the empirical information and of the variability of the deterministic forecast. Methods of estimating these variances using techniques like simulation and statistical bootstrapping are being investigated and evaluated.

Model aggregation is the process of incorporating several good models to produce a better model. In the last decade there has been great interest and progress in model aggregation, especially in biological settings. These methods are being investigated and evaluated in the context of fluid trajectories.

On the Development of an Analysis of the Comparison of MODIS Data with HARLIE and MPLNET Data

Researchers: Associate Professor Sonia M. F. Garcia
and Mr. Geary Schwemmer (National Aeronautics and Space Administration)
Sponsor: National Aeronautics and Space Administration (NASA) Code 912 - GSFC

This research has two components: (1) continuation of the study of calibration techniques for data from the Holographic Airborne Rotating Lidar Instrument Experiment (HARLIE) as an extension of a collaboration over for the last three summers between the researchers; and, (2) a study for comparison and analysis of the results of the Moderate Resolution Imaging Spectroradiometer (MODIS) satellite view of aerosols and data from ground-based measurements from HARLIE. The second project is a new collaboration with the MODIS Group, Climate and Radiation Branch, Code 913, and involves the use of MODIS and the Micro-pulse Lidar Network - MPLNET a network of lidar instruments.

Conformal Anomaly and Determinants of Curves

Researcher: Assistant Professor Amy E. Ksir
Sponsor: Naval Academy Research Council (NARC)

This project is part of a larger project being undertaken by researchers in the mathematics and physics departments at the University of Maryland and at Howard University. The focus of this group is foundational aspects of string theory.

One object of interest in this study is the conformal anomaly, a physical quantity calculated in the process of solving conformal field theories. While its definition comes from physics, it is a geometric quantity: for a given curve of

genus g, one can compute the determinant of the period matrix. It seems *a priori* that this determinant will depend heavily on the choice of a Lagrangian subspace of homology cycles for the period integrals. However there is evidence both from conformal field theory and from small genus examples that there is a canonical choice of subspace which will allow this determinant to be well-defined.

One objective of this project is to investigate the conformal anomaly and try to define it mathematically. It is hoped that a proof can be put forth that is mathematically well-defined and that it is additive with respect to a connected sum (which is also expected from the physics).

The methods used in this study include results in algebraic geometry, complex analysis, and physics. So far the investigation has been carried out in a few small genus cases, with positive results. Much of the work during thus far has consisted of studying the existing literature and learning background material.

Partially Ordered Sets and Hopf Algebras

Researcher: Professor Michael E. Hoffman
Sponsors: Naval Academy Research Council (NARC), Max-Planck-Institut für Mathematik (MPI), German Academic Exchange Service (DAAD)

In earlier work involving the Hopf algebra of rooted trees introduced by Dirk Kreimer to provide an algebraic approach to renormalization in quantum field theory, the investigator introduced a particular combinatorial structure on the partially ordered set (poset) of rooted trees. This type of structure can be given to many other posets of long-standing interest in combinatorics, including integer partitions, integer compositions, and planar rooted trees. The investigator formulated a precise definition of this structure as an "updown category," and developed a theory for such objects.

During his stay at the Max-Planck-Institut für Mathematik (MPI), the investigator wrote a draft paper "Updown posets," which has been distributed in the MPI preprint series. Subsequently he gave a series of lectures on the subject while visiting the Ruhr-Universität Bochum (supported by the DAAD, the German Academic Exchange Service), and in discussions with Ralf Holtkamp and Lothar Gerritzen has developed some of the ideas further. The investigator plans to submit a revised version of the paper for publication sometime during the coming academic year.

Relations of Mod *p* Multiple Harmonic Sums

Researcher: Professor Michael E. Hoffman
Sponsor: Max-Planck-Institut für Mathematik (MPI)

Multiple zeta values (MZVs), also known as multiple harmonic series, have been a rapidly expanding area of research in the past 15 years. Besides their intrinsic interest as number-theoretic objects, they have appeared in several computational results in perturbative quantum field theory. In fact, quantum field theory calculations also make use of the finite multiple harmonic sums, and some of the algebraic techniques developed for MZVs can be used to study the finite sums as well. If we reduce take sums up to p-1, where p is a prime, and reduce mod p, many other relations are introduced. It seems that it might well be possible to develop a general theory to account for all relations in this case.

During his stay at the Max-Planck-Institut für Mathematik (MPI), the investigator collected various algebraic results about the mod p multiple harmonic sums, and used them to calculate a generating set for all the mod p sums in each degree up to 9. The results of this calculation were presented in "Quasi-symmetric functions and mod p multiple harmonic sums," distributed as an MPI preprint. At the close of his time at MPI, the investigator had a very enlightening discussion with the Institut's director Don Zagier, who found that the computation in degree 9 was apparently incomplete (*i.e.,* the generating set is not smallest possible). The investigator plans to complete the calculation and submit a revision of the preprint for publication.

Toric Codes

Researcher: Professor W. David Joyner
Sponsor: National Security Agency (NSA)

This project investigates the construction and properties of error-correcting codes associated to toric varieties. One evaluates a linear subspace of functions on the variety at a list of n points on the variety. This vector space of n-tuples gives rise to a toric code of length n. In particular, the data transmission properties of these codes are related to the algebraic and geometric properties of the toric variety. Many examples are given. For small q, many of these codes have parameters beating the Gilbert-Varshamov bound. In fact, using toric codes, we construct a [n,k,d]=[49,11,28] code over GF(8), which is better than any other known code listed in Brouwer's tables for that n and k. This work has been accepted for publication in *Applicable Algebra in Engineering, Communication and Computing* (AAECC).

Representations of Finite Groups on Riemann-Roch Spaces

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir
Sponsors: National Security Agency (NSA to WDJ) and Naval Academy Research Council (NARC to AEK)

If G is a finite subgroup of the automorphism group of a projective algebraic curve X, and D is a divisor on X which is invariant under G, then there is a natural action of G on the Riemann-Roch space L(D). The purpose of this investigation was to compute the trace of this representation. The main objective was to obtain a formula for the trace which depends on only the genus of X/G, the types of ramification of the cover X → X/G, and the traces of irreducible representations of G, and could be easily computed. These computations were carried out using theorems from algebraic geometry, number theory, and representation theory. Part of the formula was obtained using a result from a previous paper of Ksir's, for a special class of groups and divisors. We were able to generalize these results to a much larger class of groups and divisors, and to significantly simplify formulas existing in the literature. This project has been completed, and the work has been submitted for publication.

Representations of Finite Groups on Riemann-Roch Spaces, III

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir
Sponsor: National Security Agency (NSA)

We continue the work started in "part II", concentrating on the case where the curve is defined over a field of "bad" characteristic (*i.e.,* the characteristic of the ground field divides the order of the automorphism group of the curve). For example, we have made progress on the case of the curve y^2 = x^p – x over a field of characteristic p.

On the Variety of Borels in Relative Position

Researchers: Professor W. David Joyner and Adjunct Professor Pablo Lejarraga
Sponsor: National Security Agency (NSA)

First analysis: Let G be a connected semi-simple group defined over an algebraically closed field k, T a fixed Cartan, B a fixed Borel containing T, S a set of simple reflections associated to the simple positive roots corresponding to (T,B), and let X = G/B denote the Borel variety. We show that the variety of n-tuples of Borels in relative position w are smooth and indicate why this result is, in one sense, best possible. Such varieties were first introduced by Deligne and Lusztig as a technical tool in representation theory. Lately they have appeared in other contexts, such as error-correcting codes. Our main results assume that k has characteristic 0 and uses Grothendieck's theory of schemes.

Second analysis: Using completely different methods, we prove that the canonical projections between two such varieties P^1-bundles. This implies that these varieties are smooth.

Fundamental Lemma for the 2-fold Metaplectic Cover of SL(2) over a p-adic field F

Researcher: Professor W. David Joyner
Sponsor: National Security Agency (NSA)

A well-known conjecture states that if f, f ” are elements of the respective Hecke algebras (f a function on SL(2,F) and f “‘ on the 2-fold cover) which correspond under the Satake isomorphism then the orbital integrals on the “elliptic set” are equal, up to a “transfer factor.” We prove this (and more) in the case of the unit elements.

RUI: Classical Singularities, Quantum Singularities and Cauchy Horizons

Researcher: Professor Deborah A. Konkowski
Sponsor: National Science Foundation (NSF)

This project involved a study of spacetimes with mild singularities and Cauchy horizons. Mild singularities include quasiregular and nonscalar curvature singularities. In the case of quasiregular singularities, particle paths end suddenly with no warning from infinitely-increasing tidal forces, while in the case of nonscalar curvature singularities some, but not all, particles moving near the singularity feel infinite tidal forces. A Cauchy horizon is the boundary of the causal development of spacetime. Cauchy horizons and singularities are intimately connected, including through the cosmic censorship conjecture, which continues to be debated.

In particular, the project included a study of: (1) a quantum-mechanically singular spacetimes, (2) two previously-developed stability conjectures, and (3) the global structure of various singular spacetimes. This work was done in collaboration with T. M. Helliwell (Harvey Mudd College) and students at both Harvey Mudd College and at the U.S. Naval Academy.

Further work includes: (1) Following Horowitz and Marolf and work begun by T. M. Helliwell and D. A. Konkowski, classical test particles were replaced with quantum test wave packets to test the singularity structure of spacetimes which classically possess mild singularities. The results were presented at conferences in Milan (Italy), Rio de Janeiro (Brazil) and Denver (Colorado). A talk was also given in the Naval Academy’s Pure Mathematics Seminar. (2) Conjectures proposed by T. M. Helliwell and D. A. Konkowski to predict whether various mild singularities and Cauchy horizons are stable are being investigated further. (3) Numerical studies of singular spacetimes are underway with B. Berger (NSF) and spacetimes with nonscalar and quasiregular singularities are under analytical investigation.

Algebraic Surfaces and Seiberg-Witten Curves

Researcher: Assistant Professor Amy E. Ksir
Sponsor: Naval Academy Research Council (NARC)

A configuration of NS5 five branes and stacked D4 branes in type IIA string theory lifts to a smooth M5 brane in M theory. This M5 brane gives the Seiberg-Witten curves for the induced N=2 supersymmetric Yang-Mills theory on the stacked D4 branes. When D6 branes and orientifold sixplanes are added to the IIA picture, this affects on the one hand the gauge group and matter content of the induced N=2 theory, and on the other hand the geometry of an algebraic surface in which the Seiberg-Witten curves sit in M theory.

The overall objective is to give a complete description of the algebraic surfaces and curves associated to each gauge theory. One piece of this is to reconcile two different descriptions of the geometry of each surface and curve. The plan is to use these results to determine subleading terms in the instanton expansion of the holomorphic prepotential for each theory. Techniques from algebraic geometry, string theory, gauge theory, and complex analysis will be used. Results thus far have involved tests of previous conjectures for the surface in the case of two orientifold planes of opposite charges (O6+ O6-). The initial conjecture was not quite correct, and efforts are underway to modify it. The case of two negatively charged orientifolds (O6- O6-) will be considered next. When the global mass is nonzero, the surface is no longer algebraic.

Twin Screw Extrusion of Energetic Materials

Researchers: Assistant Professor Anastasios Liakos and Dr. H. Bruck (University of Maryland)
Sponsors: Naval Surface Warfare Center, Indian Head Division (NSWC-IH)
and Naval Academy Research Council (NARC to AL)

This project is still in progress. A second-generation three dimensional program has been written in C to try to predict the values of the flow variables (velocity, pressure, stress). In addition, research was done to mesh the complex geometry of the flow domain of the extruder.

The Geometry of Entanglement

Researcher: Professor Robert B. Lockhart
Sponsor: Naval Research Laboratory (NRL) Radar Division

We started a study of the effects of relativity on entanglement. This is currently a very active area in quantum information theory, with many new papers appearing. The first part of the project was spent in a literature review, to learn about relativistic quantum field theory and the representation theory of the Lorentz group. This literature effort is on-going since the field is extensive. Our goal is to be able to analyze what happens to entanglement of two particles when one of them is in a moving frame. Applications of this abound; for instance, communicating with a satellite by using entangled photons.

The Analytic Ghost Fluid Method

Researcher: Associate Professor Thomas J. Mahar
Sponsor: Naval Surface Warfare Center, Indian Head Division (NSWC-IH)

The analytic ghost fluid method was developed to analyze the ghost fluid method as a numerical method for multi-material compressible flows. The analytic method provides a tool to check whether the ghost fluid method generates correct solutions to compressible flows in which a material interface interacts with a shock wave. Results derived with the analytical method indicate that the ghost fluid method mishandles this interaction and gives incorrect predictions about the resulting flow. Indeed, the ghost fluid method misplaced the shock wave inside of a compression wave in one run.

Modeling of the Atmospheric Propagation of a Laser Beam

Researcher: Professor Peter A. McCoy
Sponsor: High Energy Laser Program-Joint Tactical Office

The object of this project is to develop a solution, mathematical analysis supported by computation of the propagation of a laser beam through a homogeneous isotropic medium. An interesting and novel solution of the Paraxial Wave Equation (PWE) has been developed using integral transform methods. This model not only gives a complete solution of the PWE but introduces a class of radiating solutions similar to evanescent radiation in classical electromagnetic theory. A lower bound on the energy radiated by these solutions is given.

Horn's Eigenvalue Inequalities in Combinatorics

Researcher: Associate Professor T. S. Michael
Sponsor: Naval Academy Research Council (NARC)

Horn's inequalities in matrix analysis relate the eigenvalues of a sum of Hermitian matrices to the eigenvalues of the summands. The underlying notion is one of convexity. We apply similar notions of convexity to obtain inequalities among the components of the normalized Perron vector of a tournament matrix. Our general inequalities give information about the relative rankings of players in a round-robin tournament. This work is being prepared for publication in a journal covering combinatorial matrix theory.

Globe Triangularizations for an Ozone Movement Model

Researchers: Professor George C. Nakos and Dr. Doug Allen (Naval Research Laboratory)
Sponsor: Naval Research Laboratory (NRL), Remote Sensing Physics Lab

The quantity, concentration, and movement of the ozone in the stratosphere are of great importance to atmospheric and other scientists. A factor in that interest is the ozone depletion noted in the last few decades because of the chlorine gases that escape, due to human activity. The atmospheric ozone layer protects life from the sun's harmful radiation. A large ozone vortex has created a hole in the stratosphere above the Antarctic region. In the mid 1990s, French scientists discovered a smaller ozone vortex with a hole above the *Arctic region*. This surprised the atmospheric community. The success of this discovery was due to the better geometry used for the solutions of the partial differential equations of motion. The traditional use of spherical harmonics was abandoned and triangularizations of the *regular icosahedron* were used. The disadvantage of this method was it produced irregular triangles that left 30% of the earth uncovered. In this work the geometry was changed so that the initial polyhedron used is a pentakis dodecahedron (60 triangular faces). The resulting triangles are superior in that they are off only up to 22% from being equilateral. In addition, the entire surface of the earth is covered to any desired accuracy. The algorithms generated are very fast and efficient. Previous methods required supercomputing. The current method produces better results even on a 0.5-gigahertz laptop computer.

The Impact of Calculus Reform on Student Performance in Subsequent Courses at the United States Naval Academy

Researcher: Professor Howard L. Penn
Sponsor: National Science Foundation (NSF)

The Naval Academy was one of six institutions of higher learning participating in a study of the effects of calculus reform. There were three portions of the study: (1) a study of grades in calculus and follow on courses comparing the class of 2002 with the class of 1996. While there were changes in some follow-on courses, these were attributed to changes in those courses rather than changes in calculus I. (2) An attitude survey of graduating Midshipmen. On the whole, the Naval Academy graduates were positive in their reaction to the various questions. and (3) Interviews and a test given to 12 temporary duty assignment ensigns. They did very well on most of the questions on this exam. The Naval Academy portion of the report has been completed. The combined report is still in preparation.

Characterizing Surface Current Features in the Chesapeake Bay Using Lagrangean Flow Methods

Researchers: Professor Reza Malek-Madani, Associate Professor John F. Pierce, and Associate Professor Gary O. Fowler
Sponsor: Defense Modeling and Simulation Office (DMSO)

Over the last decade, methods of dynamical systems have been used to examine fluid flow from a Lagrangean perspective to extract information about the transport of materials by surface currents. The models developed have been applied successfully to study the dynamics of transport in the Monterey Bay, producing results that were unexpected, highly provocative, quite promising and imminently verifiable. We seek to identify the factors that support or inhibit the application of these methods to the Chesapeake Bay. The factors are both analytical and empirical. Analytically, there are many issues involving the specifics of the initial and boundary value PDE problem upon which the dynamical system is based for the Chesapeake Bay that distinguish it from the model for the Monterey Bay. Empirically, an adequate sample of surface current data is essential to the success of the dynamical method. We will examine how feasible it is to fuse the surface current measurements from tow data and the interpolations from it that National Oceanic and Atmospheric Administration (NOAA) can provide for the Chesapeake Bay with the high frequency radar data available at the mouth of the bay to provide a sample adequate to drive the dynamic model.

Spatio-Chromatic Transforms for Compression of Color Images

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers
Sponsor: Naval Academy Research Council (NARC to IP)

This project seeks to improve compression of color images by designing a new non-linear predictive coding technique that exploits the interdependence in spatially proximate pixels between the color components of images. A raw image file is specified as three components: red, green and blue for each image location (pixel); such an image can be considered as a three-dimensional matrix of size *width* × *height* × 3. Currently predominant image-compression techniques, such as JPEG and JPEG2000, use two transforms as part of coding color images: a color-component transform which interrelates the three color components independently on each pixel and a spatial transform, (such as a discrete wavelet or cosine transform) which interrelates proximate pixel values independently on each color component. However, even after these two transforms obvious redundancies exist among the three image components. To exploit these redundancies, the researchers have developed the *eidochromatic transform*, an additional integrated transform step following the color-component and wavelet transforms. In experiments this transform has been found to reduce the entropy of the C_b and C_r components by up to 40% for a given level of quality (as measured by mean-square error). When JPEG 2000 is modified to incorporate the eidochromatic transform, the overall file-size required for a color image has been found to decrease by up to 27%.

Multi-Resolution Contour-Based Image Coding

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers
Sponsor: Naval Academy Research Council (NARC to IP)

This project concerns compression of either color or black-and white images, using an approach radically different from the popular JPEG and JPEG 2000 standards. Our objective is to exploit the sharp-edge structures in an image to represent it more compactly, while maintaining an acceptable visual quality for the decompressed image. The correlation between spatial locations of pixels has been exploited by almost all existing algorithms, but the driving principle of these algorithms is elimination of data in a simple (say linear or cubic) dependency from proximate pixels. We propose a recurrent coding technique that also exploits the discontinuities in data near the contours of an image.

The popular wavelet approach to image compression describes an image in terms of a linear basis of elements which are self-similar and which also reflect the smooth nature of many image regions. In this basis, however, a sharp, straight edge must be described as a superposition of many wavelet elements at all scales. The first step in this project is development of a linear basis which shares the wavelet property of self-similarity but is reflective of sharp, straight edge structures rather than smoothness. Such a basis has potential applications in pattern analysis and image enhancement as well as image coding. Following the development of this basis, the remainder of the coding problem can be approached in a fashion similar to standard wavelet image coders.

Shifts of Minimal Index on the Hyperfinite II_1 Factor

Researcher: Professor Geoffrey L. Price
Sponsor: National Science Foundation (NSF)

In previous publications Price has studied the C*-algebras generated by sequences $u_1, u_2, \ldots$ of self-adjoint operators that either commute or anti-commute and that have translation-invariant commutation relations, i.e., there is a sequence $a_0, a_1, \ldots$ of 0's and 1's such that $u_i u_{i+j} = (-1)^{a_j} u_{i+j} u_i$ for all i and j. Price has shown that the C*-algebra A is simple if and only if the sequence $\{\ldots, a_2, a_1, a_0, a_1, a_2 \ldots\}$, obtained by reflecting the sequence of commutation relations, is periodic. In such a situation the algebra A is isomorphic to the CAR algebra. Given this type of construction of the CAR algebra there is a unital *-endomorphism α on A which is uniquely determined by the relations $\alpha(u_i) = u_{i+1}$ for all i. In previous work Price has classified those α's above that have commutant index 2, *i.e.,* the relative commutant algebra $\alpha^2(A) \cap A$ is nontrivial. This is accomplished by showing that there is a one to one correspondence between conjugacy classes of these shifts and the family of polynomials p(x) over GF(2) whose only self-reciprocal factors are of degree 1 or less. In the current work Price has shown that there exist unital *-endomorphisms β on the CAR algebra of commutant index 2 that are not conjugate to the shifts constructed as above. Price has also computed the Connes-Størmer entropy of these shifts.

RAT-STATS II
Researcher: Professor Thomas J. Sanders
Sponsor: Department of Health and Human Services (DHHS), OIG, Office of Audit Services

RAT-STATS is a suite of modules that was initially developed by the Regional Advanced Techniques Staff (RATS) in San Francisco to aid in auditing procedures. This project involved the transfer of statistical-auditing modules into the MS Windows environment. These modules had never been converted into the MS-DOS version of RAT-STATS. The modules include ratio and regression estimation for unrestricted and stratified samples, two-stage variable ratio and regression estimation, attribute discovery sample size, sample size for controlling alpha and beta risks, one-step acceptance sampling, two-step acceptance sampling, and two-step attribute appraisals.

Differential Operators on Toric Varieties

Researchers: Associate Professor William N. Traves and Mutsumi Saito (Hokkaido University)
Sponsor: Naval Academy Research Council (NARC to WNT)

This project investigated the algebraic and combinatorial structure of rings of differential operators on toric varieties and semigroup algebras. It was a long-term project; this was its third year. This year we proved that the ring of differential operators on semigroup algebras is finitely generated precisely when the semigroup is scored, a combinatorial condition related to the Cohen-Macaulayness of the coordinate ring. We also proved that the ring of differential operators is Noetherian under certain conditions. The paper describing our results was accepted by the *Journal of Algebra*, "Finite Generation of Rings of Differential Operators of Semigroup Algebras."

Independent Research

A Dynamical Systems Approach to Cardiac Arrhythmias

Researchers: Professor B. Mitchell Baker and Assistant Professor Irina Popovici

This project concerns the properties of an iterated system proposed by B. M. Baker and R. Kline to model the behavior of the heart, based on the connection between membrane current kinetics, action potential duration and cardiac rhythm.

The starting point is a dynamical system incorporating one or two kinetic parameters which control repolarization, and hence action potential duration. A simply formulated family of even one-dimensional dynamical systems is mathematically capable of producing restitution curves, bifurcation schemes, rhythmic response and chaotic behavior mimicked by cardiac preparations. The family of one- and two-dimensional maps in which the kinetic parameters are modeled follow three basic assumptions: 1) exponential increase of the parameter until a threshold for repolarization is reached, 2) exponential decay of this parameter immediately after threshold is reached, 3) a periodic constraint.

The orbits can be fully recovered from the Poincare return map, which for the one-dimensional, no refractory-period system is a piecewise-linear, unimodal tent map (with one unstable and one stable slope). Due to the work of B.M. Baker, R. Kline, D. Mariusewki (and others) this dynamics is well understood.

The parameter sets studied in this project consist of points where the dynamical system transitions from nearly-periodic behavior to chaos for the models that include a refractory period and for the two-dimensional (two loader) model. This work is in progress. Theorems describing the existence and uniqueness of periodic orbits, and a characterization of the stability of escalator-type orbits of the two-loader cardiac model have been developed.

Poroelastic Media

Researcher: Professor James L. Buchanan

The investigator and his collaborators have been studying the effects of the poroelastic parameters of a seabed on sound transmission in the ocean. The investigator wrote a chapter for a book which summarizes this research.

Human bone is a poroelastic medium consisting of an outer cortical layer surrounding a high porosity cancellous interior with an interstitial lipid fluid. An important question is can the porosity of the cancellous bone be ascertained by acoustic interrogation, thereby determining whether the bone is osteoporotic. This year the investigator worked on developing a way of using numerical contour integration to calculate the acoustic field arising from cancellous bone in the frequency range 100-500 kHz. This solution to the forward problem in turn permits an attack on the inverse problem of determining the poroelastic parameters of cancellous bone by acoustic interrogation. The investigator developed an algorithm which uses parallel processing to accomplish this. The algorithm is not yet fully tested, but the preliminary results look promising.

Density Estimation of Streaming Data Using a Wavelet Kernel Estimator

Researcher: LCDR Kyle Caudle, USNR

Data streams and how to construct a density estimate were studied. The focus was on kernel estimators with special emphasis on wavelets. Additional study was on mixture distributions, because the end result was a mixture of various wavelets that have been translated and dilated to fit the data.

Notions of Discrimination

Researcher: Professor Anthony M. Gaglione

As an outgrowth of the study of algebraic geometry over groups, "discriminating groups" were introduced. Higman proved the existence of a finitely presented group which embeds every finitely presented group which is what we mean by "Higman's universal finitely presented groups." We need to fix some terminology. Let G be a group. We say that G is "discriminating" provided that for every finite nonempty subset S of nontrivial elements of the Cartesian square, $G\times G$, there is a homomorphism $\phi_S : G\times G \to G$ such that $\phi_S(s) \neq 1$ for all s in S.

It is clear that if $G\times G$ embeds in G then G is discriminating. If this is the case, we term G "trivially discriminating" or simply TD. The objectives of this project were continue the investigation of discriminating and TD-groups initiated by this investigator and his colleagues. In order to better capture the axiomatic properties of discriminating groups "squarelike groups" were introduced. A group G is "squarelike" just in case it satisfies exactly the same first-order universal sentences as its direct square, $G\times G$. It is not hard to show that every discriminating group is squarelike.

In this project, we reinterpreted the concept of squarelike groups in terms of discrimination of quasivarieties. This showed that there was a definite relation between our current notion of discrimination and an older version of discrimination of Baumslag, Neumann, Neumann and Neumann – we call this (B+3N) discrimination.

The Axiomatics of Discriminating and Squarelike Groups

Researcher: Professor Anthony M. Gaglione

A group G is "discriminating" provided that for every finite nonempty subset S of nontrivial elements of the Cartesian square, $G\times G$, there is a homomorphism $\phi_S : G\times G \to G$ such that $\phi_S(s) \neq 1$ for all s in S, i.e., if it discriminates its direct square. This research project falls at the intersection of infinite group theory and mathematical logic. In a paper by A. M. Gaglione (et al) entitled "Discriminating Groups", in the *Journal of Group Theory*, 2001, 463-474, it was shown that a discriminating group has the same universal theory (with respect to a first-order language appropriate for group theory) as its direct square. In an effort to further capture this property squarelike groups were introduced. These are groups universally equivalent to their direct squares. The class of squarelike groups is axiomatic and is closure under elementary equivalence of the class of discriminating groups. In this project, we showed that the theory of the squarelike groups is neither decidable nor finitely axiomatizable. Further, with a contribution from Verena Huber-Dyson, we also produced an explicit axiom schema for the class of squarelike groups.

Thomas Fuller and his Calculation Ability

Researchers: Assistant Professor Amy E. Ksir and Sarah J. Greenwald (Appalachian State University)

Thomas Fuller was an African slave brought to America in 1724. His amazing talent with mental arithmetic made him famous and eventually helped to bring about the end of slavery. His story is inspiring to students and can be used as an introduction to computational efficiency and complexity. Research has shown that incorporating the achievements of women and minorities into the classroom is beneficial to all students. The purpose of this project was to investigate Fuller's life and mathematical techniques. Our objective was to develop a set of classroom activities based on his life and work, to teach students about his story, as well as the mathematical ideas behind his computational methods and more modern computational complexity theory. Our methods included library and internet research. This project has been completed, and the resulting article was published.

Representations of Finite Groups on Riemann-Roch Spaces and Goppa Codes

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir

If G is a finite subgroup of the automorphism group of a projective algebraic curve X, and D is a divisor on X which is invariant under G, then there is a natural action of G on the Riemann-Roch space L(D). This investigation was a continuation of our previous work and has two objectives. The first was to extend our results to the case of "bad characteristic," where the characteristic of the field of definition of the curve divides the order of the group G. The second objective was to apply these results to the study of Goppa codes. We began this work by studying some examples, and we are currently testing a conjecture relating automorphisms of the Goppa code to automorphisms of the Riemann-Roch space.

Representations of Finite Groups on Riemann-Roch Spaces

Researchers: Professor W. David Joyner and Associate Professor William N. Traves

Let X be a curve defined over a finite field k and let G be a finite group of automorphisms of X. Suppose that D is a divisor on X that is stable under the action of G. Then G acts on the Riemann-Roch space L(D). We investigated the structure of L(D) as a G-module and discussed an application to coding theory. We are currently investigating a conjecture that relates to permutation decoding and might produce codes with excellent parameters and coding-decoding properties.

Two Types of Amphichiral Links

Researcher: Professor Mark E. Kidwell

Two-component links were classified as component preserving amphichiral (CPA) or component switching amphichiral (CSA). The two-variable Alexander polynomial was used to prove that CSA links with linking number a multiple of four do not exist, answering a question of Livingston. This work provides the first examples of CPA links with linking number 12, 20, or any number of the form four times an odd number.

Classification of Singularities with SHEEP/CLASSI

Researcher: Professor Deborah A. Konkowski

This project involves an investigation of ways to apply computer algebra programs (*e.g.,* SHEEP/CLASSI) to the problem of singularity classification. This computer algebra research was begun during a recent sabbatical in London at Queen Mary and Westfield College with Professor Malcolm MacCallum's group and the work has continued during periodic visits to London. Although it is impossible to classify topological singularities such as quasiregular singularities in this manner, scalar curvature and non-scalar curvature singularities should yield to analysis. Ways to study scalar and nonscalar curvature singularities using a complete listing of C^0- curvature invariants and frames related to parallel propagated orthonormal ones are currently under consideration.

The Mathematics of *Fox Trot*

Researcher: Assistant Professor Amy E. Ksir

Fox Trot is a syndicated comic strip appearing in newspapers across the country and around the globe. Its creator, Bill Amend, was a physics major in college and he often includes mathematical references in the strips. The purpose of this investigation was to analyze some of these references, learn more about Amend and the history of the strip, and present the results as a magazine article for undergraduate math majors. The methods used in this research included library and internet research, and a personal interview (via email) with Amend. This project has been completed, and an article is being prepared for submission.

The Skolem Problem for 2 x 2 Matrices, Arctangents and Recursive Solvability

Researcher: Associate Professor Jody M. Lockhart

Decision problems for groups have been studied very extensively starting in the 1950's. Many problems, including the word problem and the isomorphism problem for finitely presented groups, have been shown to be recursively unsolvable. While it might at first glance seem that all problems about finite sets of matrices over the integers are solvable, this is not the case. Indeed, Patterson showed in 1970 that the mortality problem is unsolvable; a set of n x n matrices is said to be *mortal* if some finite product of elements in the set is the zero matrix and the mortality problem is the problem of deciding if finite sets of n x n matrices with integer entries are mortal.

The problem investigated in this project was the Skolem problem. The Skolem problem is the problem of deciding for a given square matrix with integer entries whether there is some positive power of the matrix that has zero as its entry in the upper right corner. In solving the Skolem problem for 2 x 2 matrices, one sees that it is equivalent to the following problem about arctangents.

Arctangent problem: Given positive integers m and n, is arctan(sqrt(m)/n) a rational multiple of pi?

Results of J. H. Conway, C. Radin, and L. Sadun were used to show that the arctangent problem is solvable and that, therefore, the Skolem problem for 2 x 2 matrices is also solvable.

Determinants of Matrices over the Integers Modulo m

Researchers: Associate Professor Jody M. Lockhart and Professor William P. Wardlaw

The number of matrices over the integers modulo m with determinant k was computed. The problem was first reduced to the case when m and k are both prime powers. This was done by showing that the number of such matrices is multiplicative in m and that the number with determinant a constant times a power of a prime p was equal to the number with determinant the power of p if the constant is not divisible by p. Next, a recursive formula giving the number of matrices with determinant 0 over the integers modulo m where m is a prime power was found. The proof of the formula involved a very careful counting argument. Finally, the number of matrices over the integers modulo m with determinant k where m and k are both prime powers was expressed in terms of the number of matrices with determinant 0.

Explicit Construction of Complete Kahler Metrics of Saper Type by Desingularization

Researchers: Associate Professor Caroline G. Melles and Dr. Pierre Milman (University of Toronto)

In previous work, the researchers constructed Kahler metrics on the smooth part of a singular compact Kahler variety embedded in a larger smooth compact Kahler variety. Their goal was to simplify the construction and show how such metrics can be useful in studying the structure of singular Kahler varieties. The current work describes the use of a coherent ideal sheaf whose blow-up desingularizes the singular variety. Local collections of generating sections of such a sheaf can be used in a natural way to construct local Kahler metrics for the singular variety.

The researchers revised a proof that global Kahler metrics can be constructed on the smooth part of the singular variety by patching with C-infinity partition of unity functions from the larger smooth variety. The revised proof shows that the construction works even when the fibers of the desingularization map are singular. The researchers are currently trying to finish other revisions of the paper, including addition of a section on an application to L2-cohomology.

Analysis of Non-Differentiable Continuous Functions

Researcher: Professor Mark D. Meyerson

This is an investigation of properties of classical non-differentiable continuous functions. First proposed by Weierstrass with a well known version by van der Waerden, these can be easily graphed with current software and the graphs analyzed with Iterated Function Systems.

Maple Animation of Space Filling Curves

Researcher: Professor Mark D. Meyerson

Maple programs are being written to help with the visualization of classical Space Filling Curves. By varying the usual definition, one can produce families of curves with fractional (between 1 and 2) dimensions (fractals).

Games on Oriented Graphs

Researchers: Associate Professor T. S. Michael and Thomas Quint (University of Nevada at Las Vegas)

This research investigates a zero-sum, two-person game defined by an oriented graph. Researchers have recently shown that if the oriented graph is a tournament, then the optimal strategy is unique. Some sufficient conditions for non-uniqueness of the optimal strategies for more general classes of oriented graphs have been obtained.

Functions in Generalized Moulis Classes

Researcher: Associate Professor E. John Moulis, Jr.

Let **N** be the set of all (complex-valued) functions analytic in the unit disk $|z| < 1$, having the form $f(z) = z + \text{sum}[a(n)*z^n, n=2 \ldots \text{infinity}]$. We continue the study of the class U(k,c,b,L), called generalized Pinchuk functions, of those functions in **N** which, with $J(f) = 1 - 1/c + (z/c)*(f'/f)$. $z = r*e^{(it)}$, r in [0,1), t in [0,2*Pi), c a non-zero complex number and L in (-Pi/2,Pi/2), b in [0,1) and k greater than or equal to 2, we have, using MAPLE syntax

$$\text{Int}\ [\ |\text{Re}\{e^{(it)}*J(f) - b*\cos(L)|, t, t=0..2*\text{Pi}] < k*(\text{Pi})*(1-b)\cos(L).$$

When $z*f'(z)$ is in U(k,c,b,L), f is said to belong to a generalized Moulis class V(k,c,b,L). This researcher introduced the class V(k,0,0,L) in his Ph. D. thesis. We try to find sharp bounds in terms of the real parameters k, b and L and the complex number c, for the geometric mapping properties of functions in these classes, including distortion bounds on $|f(z)|$ and $|f'(z)|$, rotation bounds on $|\arg(f'(z)|$ as well as bounds on the moduli of the series coefficients a(n).

Construction of an Ethical Combat Utility Function

Researcher: Associate Professor E. John Moulis, Jr.

An investigation into the construction of a combat utility function (as given in the text *Naval Operations Analysis*, 3rd edition, Wagner, Mylander, Sanders, eds. 1999), continues. The work was started by the midshipmen of SA302, Class of 2003. A way to take into account the normative dimensions of risk exposure allowing for a constant value of life to be associated with survivability in carrying out risky combat missions has been suggested. This provides an alternative to the "lottery indifference" concept of the decision maker, on which the text's utility function is based.

Stochastic Generalized Assignment Problem

Researcher: CDR David R. Spoerl, USN

Solving a stochastic version of the Elastic Generalized Assignment Problem (EGAP), which incorporates independent, identically distributed resource-consumption coefficients. The Stochastic Elastic GAP (SEGAP) is a two-stage stochastic integer program with simple recourse. A deterministic equivalent and solve using the branch-and-price algorithm has been constructed. The model and the math program to solve the SEGAP are being completed.

Compositions of Forms

Researcher: Associate Professor JoAnn S. Turisco

This research involves many open problems in composition of higher degree forms over general fields. At present, the researcher is concentrating on the composition of diagonal forms and the underlying algebras involved.

High-order Non-reflecting Boundary Conditions

Researchers: CDR Vincent J. van Joolen, USN and Dr. Beny Neta (Naval Postgraduate School)

The researchers are continuing their work to further develop numerical techniques for generating high-order Higdon non-reflecting boundary conditions. The technique was successfully developed resulting in some improvement over previous methods, and these results have been submitted for publication.

Midshipman Research Course Projects

A Study of Algorithms to Solve the Traveling Salesman Problem

Researcher: Midshipman 1/C Charles R. Platt, USN
Adviser: Professor W. Charles Mylander

This project examined the Traveling Salesman Problem (TSP) initially studied by Irish mathematician Sir William Rowan Hamilton on or about 1850. Following its initial conceptualization the problem remained almost entirely dormant for the next century until the advent of technological aids in solving TSPs, which are necessary due to the computational complexity of the problems. In 1954 Dantzig solved the first large scale TSP involving the capital cities of each of the 48 contiguous states using a piece of string and his cutting plane method.

This research examined the TSP and three algorithms used to solve these problems. The algorithms studied spanned the variety of approaches taken to solving TSPs and included the Cutting Planes method, based on linear programming methods; the Dynamic Programming method, employing a recursive algorithm; and the Genetic Algorithm, which combines mathematical analysis with random search. To conclude this study, a problem of size 52 was solved by the Genetic Algorithm approach. This problem is larger than Dantzig's original problem with 7.76×10^{65} more possible tours to search than Dantzig's 48 city problem.

Classical and Quantum Singularities in Levi-Civita Spacetimes with Cosmological Constant

Researcher: Midshipman 1/C Cassidi A. Reese, USN
Adviser: Professor Deborah A. Konkowski
Sponsor: National Science Foundation (NSF)

A Levi-Civita (LC) spacetime describes a cylindrically symmetric spacetime with an infinitely thin line mass along the axis for certain parameter values. Here we studied LC spacetimes with a positive cosmological constant (LCC) whose interpretation is less clear.

The classical and quantum singularities in LCC spacetimes were studied. A classical singularity exists in a spacetime that is geodesically incomplete and cannot be extended. Classical singularities may be divided into three categories: quasiregular, nonscalar curvature and scalar curvature. Classical singularities are probed with classical point particles and thus one studies the behavior of classical timelike and null geodesics together with the curvature scalars of the spacetime. Quantum singularities are determined by the behavior of test quantum wave packets. A static spacetime with timelike singularity is quantum mechanically singular if the spatial portion of the scalar (Klein-Gordon) wave operator is not essentially self-adjoint. One thus solves the scalar wave equation and tests the operator for essential self-adjointness to determine if a spacetime is quantum mechanically singular.

The results of this research were presented by the researcher at the National Conference on Undergraduate Research (NCUR) in Indianapolis, IN in April 2004. The work has been accepted to appear in the *Proceedings* of that conference. In addition, these results have been included in a more extensive preprint on LCC spacetimes that will ultimately be submitted to a research journal.

A Polynomial-time Probabilistic Algorithm for the Minimum Distance of an Arbitrary Linear Error-correcting Code

Researcher: Midshipman 1/C Aron T. Foster, USN
Adviser: Professor W. David Joyner

The goal of the project was to examine a polynomial-time probabilistic algorithm to compute the minimum distance of an arbitrary linear error-correcting code. The work was done in the GAP coding-theory package GUAVA. GAP is a computer algebra package whose open source kernel is written in the C programming language. However, most packages (such as GUAVA) and the algorithms described here are written in GAP's own interpreted language.

Global Requirements Calculator

Researcher: Midshipman 1/C Derek M. Jennings, USN
Sponsors: LT William Malpass, OPNAV N81, Department of Defense
and the Bowman Scholar Program (BSP)
Advisers: Professor Thomas J. Sanders and Associate Professor John F. Pierce

The Global Requirements Calculator is an EXCEL program developed by CAPT Bud Bishop, USN and the "Power Projection" Team of N81. The program takes various logistical and operational input variables for our Carrier Battle Groups, and produces a figure which represents the number of Battle Groups necessary to maintain the desired presence "coverage" of the globe.

The Power Projection Team is currently modifying and augmenting the Global Requirements Calculator to reflect the new objectives and resources outlined in "Sea Power 21," the CNO's vision for the future of the Navy. My role in this project consisted of researching manpower figures for a Carrier Battle Group and an Expeditionary Strike Group. These figures were used in a bandwidth calculator, which determined the bandwidth that each battle group required on deployment. The researcher was tasked to develop a calculator to determine the amount of available berthing space at each of the Navy's homeports in the Continental US and Hawaii. In addition, the researcher investigated the docking space (both pier and quay) at each of the Navy's CONUS homeports and Pearl Harbor. An EXCEL document was created which took as inputs the number of each platform in the US fleet, their dimensions, and calculated whether or not they could berth them.

Where the People Are

Researcher: Midshipman 1/C Matthew D. Collinsworth, USN
Sponsor: OPNAV N-81, Department of Defense
Faculty Adviser: Professor Thomas J. Sanders

Concern about the distribution of military personnel versus civilian personnel in the Navy was expressed by the Chief of Naval Operations (CNO), so OPNAV N-81 was tasked with determining the numbers, locations and distributions of people in the Navy. In this project, the researcher determined and described where Navy personnel with Operations

Analysis backgrounds were currently stationed, and what type of billets they held. Data bases were searched and sorted in various ways, until it was clear how personnel with Operations Analysis backgrounds were distributed. Various EXCEL graphs were developed to present the data. As a result of this study, it became clear that Operations Analysis personnel in the Navy could be better utilized in actual Operations Analysis billets.

Simulation and Modeling with Respect to Operational Analysis: Problems and Solutions

Researcher: Midshipman Charles A. Larwood III, USN
Sponsor: LCDR Phil Pournelle, N816- Simulation and Modeling
Adviser: Professor Thomas J. Sanders

Much of the research work in this project dealt with classified material. Within the Operational Analysis Division of the Navy, OPNAV N-81, the N-816 sub-division deals with simulation and modeling. Their job is to supply N-81 with accurate models of possible conflicts of the future using simulation software to analyze upcoming needs of the Navy in terms of manpower, unit locations, equipment, transportation schedules, platforms, etc. This research supported a current, classified effort of the N-816 sub-division.

A Linear Programming Approach to Modeling the Effect of Consolidating U.S. Navy Fuel Types

Researcher: Midshipman 1/C William R. Sheridan, USN
Adviser: CDR David R. Spoerl, USN

The Navy's ability to maintain a presence and show of force around the globe in some of the most adverse conditions and unfriendly places is a key component of U.S. diplomacy. In order for our Navy's ships and aircraft to operate effectively and efficiently they need to be properly supplied. The most basic requirement for a battle group or any combatant to operate for any prolonged amount of time is fuel. In this research, a stochastic model that more accurately reflected the cost of a JP-5 shortage was created. A network representation of the petroleum supply chain from the production of the fuel at refineries to the utilization of the fuel by the combatants was also created.

Professional Publications

Journal (Refereed) Manuscripts

ALEVRAS, Alexis, Assistant Professor, "One Parameter Semigroups of Endomorphisms of Factors of Type II1," *Journal of Operator Theory*, Vol. 51, pp. 161-179. (2004)

Invariants for continuous semigroups of *-endomorphisms of type II1-factors were studied. An index was defined, based on R. Powers' notion of the boundary representation, and computed for all known examples on the hyperfinite II1-factor R as well as for examples on the group von Neumann algebra of the free goup on infinitely many generators. We also introduced the analogue of W. Arveson's continuous tensor product system associated with an E0-semigroup, and showed that it is a complete invariant under cocycle conjugacy.

BUCHANAN, James L., Professor, Gilbert, R. P., Wirgin, A., and Xu, Y.S., "Transient Reflection and Transmission of Ultrasonic Wave in Cancellous Bone," *Appl. Math. Comput*., Vol. 142, No. 2-3, pp. 561-573. (2003)

Cancellous bone is known to be poroelastic in structure. Ultraso0nic wave propagation in cancellous bone can be formulated by using Biot's equations. In this paper we present some results in our ongoing research on the reflection and transmission in ultrasonic waves in cancellous bone. We investigate the relations among reflected waves, transmitted waves and Biot coefficients. We present an algorithm for the determination of the porosity of cancellous bone.

GAGLIONE, Anthony M., Professor, (co-author), "The Axiomatic Closure of Discriminating Groups," *Archiv der Mathematik*, accepted for publication.

We show that the class of squarelike groups is first-order axiomatizable while the class of discriminating groups is not. We then show that the class of squarelike groups is the least axiomatic class containing the class of discriminating groups, i.e. its axiomatic closure. This answers a previously open question of the authors.

GAGLIONE, Anthony M., Professor, (co-author), "Discriminating and Squarelike Groups I: Axiomatics," *Contemporary Mathematics*, accepted for publication.

A group G is "squarelike" provided that the direct square G×G is universally equivalent to G. Every group that is "discriminating" in the sense of Baumslag, Myasnikov and Remeslennikov is squarelike but not necessarily conversely. The class of squarelike groups is axiomatic and is the closure under elementary equivalence of the class of discriminating groups. This paper shows that the theory of squarelike groups is neither decidable nor finitely axiomatizable. Further we exhibit a set of axioms due to V. H. Dyson for the class of squarelike groups.

GAGLIONE, Anthony M., Professor, (co-author), "Discriminating and Squarelike Groups II: Examples," *Houston Journal of Mathematics,* accepted for publication.

At the time of the writing of our first paper, the only examples of discriminating groups known were of two types: abelian groups and groups which embed their direct squares. Here we give a class of finitely generated non-abelian groups were are discriminating but do not embed their direct squares – we term such groups "nontrivially discriminating". In another one of our papers, an example of a squarelike group which was not discriminating was given. This example was not finitely generated and the existence of a finitely generated counterexample was left as an open problem. In this paper, we also produce such a counterexample.

HOFFMAN, Michael E., Professor, "Combinatorics of Rooted Trees and Hopf Algebras," *Transactions of the American Mathematical Society,* Vol. 355, pp. 3795-3811. (2003)

We begin by considering the graded vector space with a basis consisting of rooted trees, with grading given by the count of non-root vertices. We define two linear operators on this vector space, the growth and pruning operators, which respectively raise and lower grading; their commutator is the operator that multiplies a rooted tree by its number of vertices, and each operator naturally associates a multiplicity to each pair of rooted trees. By using symmetry groups of trees we define an inner product with respect to which the growth and pruning operators are adjoint, and obtain several results about the associated multiplicities.

Now the symmetric algebra on the vector space of rooted trees (after a degree shift) can be endowed with a coproduct to make a Hopf algebra; this was defined by Kreimer in connection with renormalization. We extend the growth and pruning operators to Kriemer's Hopf algebra and related them via the inner product. On the other hand, the vector space of rooted trees itself can be given a noncommutative multiplication: with the appropriate coproduct, this gives a Hopf algebra studied by Grossman and Larson. We show that the inner product on rooted trees can be used to define an isomorphism of the Grossman-Larson Hopf algebra with the graded dual of Kreimer's Hopf algebra, correcting an earlier result of Panaite.

JOYNER, W. David, Professor, "Toric Codes Over Finite Fields," *Applicable Algebra in Engineering, Communication and Computing*, Vol. 15, No. 1, pp. 63-79. (2004)

In this note, a class of error-correcting codes is associated to a toric variety associated to a fan defined over a finite field F, analogous to the class of Goppa codes associated to a curve. For such a "toric code" satisfying certain additional conditions, we present an efficient decoding algorithm for the dual of a Goppa code. Many examples are given. For small q, many of these codes have parameters beating the Gilbert-Varshamov bound. In fact, using toric codes, we construct a [n,k,d]=[49,11,28] code over GF(8), which is better than any other known code listed in Brouwer's on-line tables for that n and k.

KONKOWSKI, Deborah A., Professor (co-author), "Quantum Singularity of Levi-Civita Spacetimes," *Classical and Quantum Gravity*, Vol. 21, pp. 265-272. (2004)

Quantum singularities in general relativistic spacetimes are determined by the behavior of quantum test particles. A static spacetime is quantum mechanically singular if the spatial portion of the wave operator is not essentially self-adjoint. Here Weyl's limit point-limit circle criterion is used to determine whether a wave operator is essentially self-adjoint. This test is then applied to scalar wave packets in Levi-Civita spacetimes to help elucidate the physical properties of the spacetimes in terms of their metric parameters.

LIAKOS, Anastasios, Assistant Professor, "Discretization of the Navier-Stokes Equations with Slip Boundary Condition II," *Computers and Mathematics with Applications*, accepted for publication.

Multi-level methods are a common tool for solving non-linear systems arising from discretizations of elliptic boundary value problems (see e.g. Layton and Lenferink, Xu). Multi-level methods consist of solving the non-linear problem on a coarse mesh and then performing one or two Newton correction steps on each subsequent mesh thus solving only one or two larger linear systems. In a previous paper by the author, a two level method was proposed for the stationary Navier-Stokes equations with slip boundary condition. Uniqueness of the non-linear problem is guaranteed provided that the data of the problem (i.e. the Reynolds number, the forcing term and the bound on the non-linear form) is strictly bounded.

In practice, however, the aforementioned bound on the data is rarely satisfied. Using the abstract theory of Brezzi, Rappaz and Raviart, we bypass the uniqueness condition of small data to prove new error estimates for the multi-level method. As expected, the scalings of the meshwidths that guarantee optimal accuracy, in the H^1-norm, are less favorable to those in the uniqueness case.

Finally, we also propose and analyze the error of a two level modified Picard method without relying on a uniqueness condition. The scalings of the meshwidths that guarantee optimal accuracy, in the H^1-norm, are less favorable to those for the two-level Newton method in the non-uniqueness case.

LOCKHART, Jody M., Associate Professor and WARDLAW, William P., Professor, "Quickie Q993," and "Answer A993," *Mathematics Magazine*, Vol. 76, No. 4, p. 319 and p. 325. (October 2003)

Q933. Let f_k denote the kth Fibonacci number (with $f_0 = 0$ and $f_1 = 1$) and let [p] be a positive prime. Prove that if p divides f_k, then for each positive integer r, [p^{r+1}] divides $f_{p^r k}$. (Items in brackets were omitted in the published version.) A933 gives the solution to Q933.

MARUSZEWSKI, Richard F., Professor, "Launching a Projectile into Deep Space," *MACE Journal,* Volume 38, #1, pp. 5-10.

This article describes a classroom investigation into finding the amount of energy necessary for a two stage launch of a projectile into deep space. The first stage takes the projectile from the surface of the earth to a point outside of our atmosphere. The second goes from there to deep space.

MICHAEL, T. S., Associate Professor, "Alumni Profiles: United States Naval Academy," *Math Horizons,* Vol. 12, February 2004.

The Mathematics major and program at the Naval Academy is described. Accomplishments and professional activities of four recent alumni are given as evidence of the usefulness and power of a mathematics degree.

MICHAEL, T. S., Associate Professor, and Pinciu, Val, "Art Gallery Theorems for Guarded Guards," *Computational Geometry: Theory and Applications,* Vol. 26, pp. 257-258. (2003)

Let P be a polygon (the art gallery) with n sides. We wish to select a set G of points (the guards) so that (a) every point in the polygon is visible to some guard; and (b) every guard is visible to at least one other guard. What is the minimum number of guards g(n) that is guaranteed to suffice for all polygons with n sides? We prove that g (n) = floor [(3n-1)/7] for n at least 5. Also, if we restrict to rectilinear galleries the function is floor (n/3) for n at least 6.

MICHAEL, T. S., Associate Professor, "Impossible Decompositions of Complete Graphs into Three Petersen Subgraphs," *Bulletin of the Institute of Combinatorics and Its Applications*, Vol. 39, pp. 64-66. (2003)

We prove that it is impossible to decompose the complete graph on 9 vertices into three copies of the graph obtained from the Petersen graph by deleting a vertex. This result generalizes a theorem of Schwenk.

MICHAEL, T. S., Associate Professor and Bower, Richard M., "When can you tile a box with translates of two given rectangular bricks?" *Electronic Journal of Combinatorics,* Vol. 11, Note 7. (2004)

We give both arithmetic and geometric conditions that are necessary and sufficient for a d-dimensional rectangular box to be tiled (packed) by translating two given rectangular bricks. Our main theorem unifies a number of results in the tiling literature.

MICHAEL, T. S., Associate Professor and Quint, Thomas, "Sphere of Influence Graphs and the L-Infinity Metric," *Discrete Applied Mathematics*, Vol. 127, pp. 447-460. (2003)

We introduce sphere of influence graphs (SIGs) in the L-infinity metric and study their fundamental properties. We argue that SIGs defined with the L-infinity metric are superior to the Euclidean SIGs of Toussaint in capturing low-level perceptual information in certain dot patterns. Every graph without isolated vertices is a SIG in the L-infinity metric for all sufficiently high dimensions, and this allows us to define a graphical parameter, the SIG-dimension, that is akin to boxicity. We determine the SIG-dimensions for some classes of graphs and obtain inequalities for others.

MICHAEL, T. S., Associate Professor and TRAVES, William N., Associate Professor, "Independence Sequences of Well-Covered Graphs: Non-Unimodality and the Roller Coaster Conjecture," *Graphs and Combinatorics,* Vol. 19, pp. 403-411. (2003)

A graph G is well-covered provided every independent set of vertices has the same cardinality. The k-th term of the independence sequence of G equals the number of independent vertex sets of cardinality k in G. We disprove a recent unimodality conjecture for independence sequences of well-covered graphs and replace it by a more complicated conjecture. We prove our conjecture for several infinite families of graphs.

Overby, Jeff, TRAVES, William N., Associate Professor, and Wojdylo, J., "On the Keyspace of the Hill Cipher," *Cryptologica*, accepted for publication.

We count the number of invertible d-by-d matrices mod n, |Gld(Zn)| and then apply the formula to the Hill Cipher. This paper is largely expository in nature and it arises from Mr. Overby's undergraduate honors paper.

POPOVICI, Irina, Assistant Professor and WITHERS, W. Douglas, Professor, "The Eidochromatic Transform for Color Image Coding," *Institute of Electrical and Electronics Engineers* (*IEEE) Transactions on Image Processing*, accepted for publication.

We introduce the *eidochromatic transform* as a tool for improved lossy coding of color images. Many current image-coding formats (such as JPEG 2000) utilize both a color-component transform (relating values of different image components at a single location) and a wavelet or other spatial transform (relating values of a single image component at proximate, but different image locations). The eidochromatic transform further reduces redundancy by relating image values simultaneously across color components and in the two spatial dimensions.

Our approach is to introduce an additional transform step following the color-component and spatial transforms. In tests, this step reduced the overall static entropy of the chrominance components of quantized transformed images by up to 40\% or more. Combined with JPEG 2000's modeling and coding method, the eidochromatic transform was found to reduce the size of lossily coded color images by up to 27% overall.

RATCLIFFE, Colin, Associate Professor (Mechanical Engineering Department), BAGARIA, William, Professor Emeritus (Aerospace Engineering Department), GARCIA, Sonia M. F., Associate Professor, FAHEY, Richard, Visiting Professor (Aerospace Engineering Department), "Rapid binary gage function to extract a pulsed signal buried in noise," accepted for publication to the EURASIP, Journal on Applied Signal Processing (2004).

The type of signal studied in this paper is a periodic pulse, with the pulse length short compared to the period, and the signal is buried in noise. If standard techniques such as the Fast Fourier Transform are used to study the signal, the data record needs to be very long. Additionally, there would be a very large number of calculations. The Rapid Binary Gage Function was developed to quickly determine the period of the signal, and the start time of the first pulse in the data. Once these two parameters are determined, the pulsed signal can be recovered using a standard data folding and adding technique.

Saito, Mutsum and TRAVES, William N., Associate Professor, "Finite Generation of Rings of Differential Operators of Semi-group Algebras," *Journal of Algebra*, accepted for publication.

This paper investigates the algebraic and combinatorial structure of rings of differential operators on toric varieties and semigroup algebras. We prove that the ring of differential operators on semigroup algebras is finitely generated precisely when the semigroup is scored, a combinatorial condition related to the Cohen-Macaulayness of the coordinate ring. We also prove that the ring of differential operators is Noetherian under certain conditions.

VAN JOOLEN, Vincent J., Commander, USN, Givoli, D., and Neta, B., "High-Order Non-Reflecting Boundary Conditions for Dispersive Waves in Cartesian, Cylindrical, and Spherical Coordinate Systems," *International Journal Computation Fluid Dynamics*, pp. 263-274, 2003.

Among the many areas of research that Professor Kawahara has been active in is the subject of *open boundaries* in which linear time-dependent dispersive waves are considered in an unbounded domain. The infinite domain is truncated via an artificial boundary **B** on which an open boundary condition (OBC) is imposed. In this paper, Higdon OBCs and Hagstrom-Hariharan (HH) OBCs are considered. Higdon-type conditions, originally implemented as low-order OBCs, are made accessible for any desired order via a new scheme. The higher-order Higdon OBS is then reformulated using auxiliary variables and made compatible for use with finite element (FE) methods. Methodologies for selecting Higdon parameters are also proposed. The performances of these schemes are demonstrated in two numerical examples. This is followed by a discussion of the HH OBC, which is applicable to non-dispersive media on cylindrical and spherical geometries. The paper extends the OBC to the "slightly dispersive" case and is dedicated to Professor Mutsuto Kawahara on the occasion of his 60th birthday.

VAN JOOLEN, Vincent J., Commander, USN, Givoli, D., and Neta, B., "A Stratified Dispersive Wave Model with High-Order Non-Reflecting Boundary Conditions," *Computers and Mathematics with Applications*, accepted for publication.

A layered-model is introduced to approximate the effects of stratification on linearized shallow water equations. This time-dependent dispersive wave model is appropriate for describing geophysical (e.g. atmospheric or oceanic) dynamics. However, computational models that embrace these very large domains that are global in magnitude can quickly overwhelm computer capabilities. The domain is therefore truncated via artificial boundaries, and robust non-reflecting boundary conditions (NRBC) devised by Higdon are imposed. A scheme previously proposed by Neta and Givoli that easily discretizes high-order Higdon NRBCs is used. The problem is solved by Finite Difference (FD) methods. Numerical examples follow the discussion.

VAN JOOLEN, Vincent J., Commander, USN, Givoli, D., and Neta, B., "High-Order Boundary Conditions for Linearized Shallow Water Equations with Stratification, Dispersion, and Advection," *International Journal for Numerical Methods in Fluids*, accepted for publication.

The two-dimensional linearized shallow water equations are considered in unbounded domains with density stratification. Wave dispersion and advection effects are also taken into account. The infinite domain is truncated via a rectangular artificial boundary B, and a high-order Open Boundary Condition (OBC) is imposed on B. Then the problem is solved numerically in the finite domain bounded by B. A recently developed boundary scheme is

employed, which is based on a reformulation of the sequence of OBCs originally proposed by Higdon. The OBCs can easily be used up to *any* desired order. They are incorporated here in a finite difference scheme. Numerical examples are used to demonstrate the performance and advantages of the computational method, with an emphasis on the effect of stratification.

WARDLAW, William P., Professor, Problem 173, *Math Horizons*, Vol. XI, No. 1, p. 32. (September 2003)

Let A be an $n \times n$ matrix, B an $n \times r$ matrix, C an $r \times n$ matrix and I_r the $r \times r$ identity matrix, all over a commutative ring. Prove that

$$\det\begin{bmatrix} A & B \\ C & I_r \end{bmatrix} = \det(A - BC).$$

WARDLAW, William P., Professor, Problem 11034, *American Math. Monthly*, Vol. 110, No. 8, p. 772. (October 2003

Problem 11034 gives a 3×3 matrix B and asks (a) Find the smallest field **K** such that there is a matrix A over **K** with adj A = B; (b) Find one such matrix A over this field **K**; (c) How many different such A are there?, and then asks the same questions for another 3×3 matrix C.

WARDLAW, William P., Professor, Problem 178, *Math Horizons*, Vol. XI, No. 2, p. 32. (November 2003)

Let R be a commutative ring with identity and let P_R be the ring of all polynomial functions on R. Can P_R be
(a) an integral domain;
(b) finite;
(c) a finite integral domain?

WARDLAW, William P., Professor, Quickie Q938 and Answer A938, *Mathematics Magazine*, Vol. 77, No. 1, p. 70 and p. 75. (February 2004)

Q938**.** Let R be a ring, let G be a finite subset of R that forms a multiplicative group under the multiplication of R, and let s be the sum of the elements of G. Prove that if G has more than one element, then s is either zero or a zero divisor in R. Give examples in which s is a nonzero divisor of zero.

WARDLAW, William P., Professor, Problem 179, *Math Horizons*, Vol. XI, No. 3, p. 32. (February 2004)

Let F(q) denote the finite field with q elements. Two $n \times n$ matrices with elements in F(q) are chosen independently, so that any such matrix has the same probability of being chosen. What is the probability that the two matrices chosen are inverses of each other?

WARDLAW, William P., Professor, Problem 11083, *American Math. Monthly*, Vol.111, No. 5, p. 440. (May 2004)

Find the characteristic polynomial and the null ideal of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 2 & 1 \\ 9 & 1 & 9 & 0 & 9 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 2 & 1 & 3 & 1 \\ 0 & 2 & 0 & 2 & 0 \end{bmatrix}$$

over the ring of integers modulo 10. (The *null ideal* of a matrix A is the set of polynomials that annihilate A.)

Conference Proceedings

HOFFMAN, Michael E., Professor, "Algebraic Aspects of Multiple Zeta Values," *Proceedings of the International Symposium on Zeta Functions, Topology and Quantum Physics,* Osaka, 2003, accepted for publication.

Multiple zeta values have been studied by a wide variety of methods. In this article we summarize some of the results about them that can be obtained by an algebraic approach. This involves "coding" the multiple zeta values by monomials in two noncommuting variables x and y. Multiple zeta values can then be thought of as defining a map $\zeta: H^0 \rightarrow R$ from a graded rational vector space H^0 generated by the "admissible words" of the noncommutative polynomial algebra $Q<x,y>$. Now H^0 admits two (commutative) products making ζ a homomorphism - the shuffle product and the "harmonic" product. The latter makes H^0 a subalgebra of the algebra QSym of quasi-symmetric functions. We also discuss some results about multiple zeta values that can be stated in terms of derivations and cyclic derivations of $Q<x,y>$, and we define an action of QSym on $Q<x,y>$ that appears useful. Finally, we apply the algebraic approach to relations of finite partial sums of multiple zeta series.

POPOVICI, Irina, Assistant Professor and WITHERS, W. Douglas, Professor, "A New Transform for Lossy Image Coding," *Proceedings of the 2003 International Conference on Imaging Science, Systems, and Technology*, Vol. 2, pp. 679-684, 2003.

Luminance and chrominance components of a color image have redundancies often exploited for coding the image with a reduced number of bits, while maintaining good visual quality for the decoded image. Most existing image-coding algorithms use spatial correlation between pixels in each color-transformed image component via wavelet or other spatial transform, but applied separately to the luminance and to each chrominance component.

This paper introduces a non-linear predictive transform that exploits the interdependence between spatially proximate pixels of different color components. The eidochromatic transform reduces the entropy of a quantized chrominance component up to 40% or more over a wavelet transform alone. Combined with JPEG 2000 coding techniques, the eidochromatic transform reduces the bit-rate for the entire image at a specified mean-square error by up to 10% or more when compared to JPEG 2000 alone.

VAN JOOLEN, Vincent J., Commander, USN, Givoli, D., and Neta, B., "High-Order Non-Reflecting Boundary Conditions for Dispersive Wave Problems in Stratified Media" *Proceedings of the Sixth International Conference on Computer Modeling and Experimental Measurements of Seas and Coastal Regions*, Cadiz, Spain, (C.A. Brebbia, D. Almorze and F. Lopex-Aguayo, eds), pp. 73-82, 2003.

Problems of linear time-dependent dispersive waves in an unbounded domain are considered. The infinite domain is truncated via an artificial boundary B. A high-order Non-Reflecting Boundary Condition (NRBC) is imposed on B, and the problem is solved by a finite difference (FD) scheme in the finite domain. The sequence of NRBCs proposed by Higdon is used. However, in contrast to the original low-order implementation, a new scheme is devised which allow the easy use of Higdon-type NRBC of any desired order. In addition, the problem is considered for a stratified media. The performance of the scheme is demonstrated via numerical example.

Books, Book Chapters and Monographs

BUCHANAN, James L., Professor, Gilbert, R. P., Wirgin, A., and Xu, Y.S., Marine Acoustics: Direct and Inverse Problems, SIAM, Philadelphia, 2004.

This book presents current research trends in the field of underwater acoustic wave direct and inverse problems. Until very recently, little had been published concerning model-based inversions of the boundaries and material constants of finite-sized targets located in either the water column or the sediments. This text is the first to investigate inverse problems in an ocean environment, with a heavy emphasis placed on the description and resolution of the forward scattering problem.

BUCHANAN, James L., Professor, Gilbert, R. P., Wirgin, A., and Xu, Y.S., "Unambiguous Reconstruction of the 2D Boundary of a Half-space Probed by Acoustic Waves," Progress in Analysis, Vol. I and II (Berlin, 2001), pp. 1377-1390, World Science Publishing, River Edge, NJ, 2003.

It is shown that the use of: 1) of the physical optics (POA) approximation to simulate the response (i.e. predictor), on a line above the open cylindrical boundary in its cross-section plane, to two probe plane waves of different frequencies, 2) the intersecting canonical boundary approximation (ICBA) of the wave-boundary interaction for a trial boundary (i.e., estimator), and 3) an analysis of the comparison equation and/or cost function, employing the ICBA as estimator and either the ICBA or POA as predictor, enables a non-ambiguous identification of the scattering boundary.

JOYNER, W. David, Professor, Kreminski, Richard, and TURISCO, JoAnn S., Associate Professor, "Applied Abstract Algebra," The Johns Hopkins University Press, 2004.

This book is intended to be used as an upper level undergraduate or a graduate level textbook. It contains chapters on (1) Elementary Number Theory; (2) Polynomials, Rings, and Fields; (3) Error-Correcting Codes; (4) Permutations; (5) An Introduction to Groups; (6) Special Projects: Codes; and (7) Appendixes by David Joyner (Basics of GAP and Simple Exercises in MAGMA).

NAKOS, George C., Professor, "Modern Linear Algebra and Applications," accepted for publication.

This text is a thorough treatment of elementary linear algebra, which includes linear systems of differential equations with constant coefficients. It covers the following topics: Linear systems, vectors, matrices, vector spaces and linear transformations, determinants, eigenvalues, orthogonality, systems of differential equations, orthogonalization of symmetric matrices, and numerical linear algebra (iterative methods and numerical approximations of eigenvalues). Each chapter has projects, historical notes, and computer-aided exercises. Status: First draft of the entire manuscript is completed.

NAKOS, George C., Professor, "Instructor's Solutions Manual for Modern Linear Algebra and Applications," scheduled for publication with the text (above).

This supplement consists of the complete solutions manual of the above text.

NAKOS, George C., Professor, "Student's Manual for Modern Linear Algebra and Applications," scheduled for publication with the text (above).

This supplement consists of the student's solutions manual of the above text. It is essentially a subset of the instructor's solutions manual with some modifications.

PRICE, Geoffrey L., Professor, (editor-in-chief); BAKER, B. Mitchell, Professor, Jorgensen, Palle E. T., and Muhly, Paul S., (co-editors), "Advances in Quantum Dynamics," *Joint Summer Research Conference on Advances in Quantum Dynamics, AMS Contemporary Mathematics Series,* Vol. 335, 2003.

This volume contains the proceedings of the conference on *Advances in Quantum Dynamics* held at Mount Holyoke College June 16 through 20, 2002. The conference was one in a continuing series of conferences in mathematics at Mount Holyoke sponsored by the American Mathematical Society. In keeping with the general philosophy of this series, [the] conference brought together experts and initiates alike to assess the current state of knowledge of quantum dynamical semigroups on von Neumann algebras and to map out directions for future research.

The conference was organized by Professor Price, with Professor B. Mitchell Baker (USNA) and Professors P. E. T. Jorgensen and Paul Muhly (University of Iowa). The conference was funded by the American Mathematical Society, which also published these proceedings. Professor Price served as principal editor of this volume, which includes 20 research articles and is over 300 pages.

Technical Reports

ANDRE Peter P., Professor, "Notes on the Geometry of N-Dimensions with Applications to Economics," 2003.

The subject matter of this book forms the material for the course SM279 which is a link between the matrix theory and calculus in the mathematics department and microeconomics in the economics department. The book gives an elementary introduction to the world of n-dimensions and makes the connection between the mathematics and its applications in microeconomics. The text begins with a discussion of affine sets in R^n, the affine n-dimensional space. Functions are introduced and the calculus in n-dimensions is discussed from an intuitive point of view without the formal definitions. The book covers the chain rule, inverse function theorem, implicit function theorem, unconstrained optimization and constrained optimization.

HOFFMAN, Michael E., Professor, "Quasi-symmetric Functions and Mod *p* Multiple Harmonic Sums," Max-Planck-Institut für Mathematik Preprint 2004-03.

We present a number of results about (finite) multiple harmonic sums modulo a prime, which provide interesting parallels to known results about multiple zeta values (i.e., infinite multiple harmonic series). In particular, we prove a "duality" result for mod *p* harmonic sums similar to (but distinct from) that for multiple zeta values. We also exploit the Hopf algebra structure of the quasi-symmetric functions to do calculations with multiple harmonic sums mod *p*, and obtain, for each weight $n \leq 9$, a set of generators for the space of weight n multiple harmonic sums mod *p*.

HOFFMAN, Michael E., Professor, "Updown Categories," Max-Planck-Institut für Mathematik Preprint 2004-11.

A poset can be regarded as a category in which there is at most one morphism between objects, and such that at most one of Hom(c,c') and Hom(c',c) is nonempty for $c \neq c'$. If we keep in place the latter axiom but allow for more than one morphism between objects, we can have a sort of generalized poset in which there are multiplicities attached to the covering relations, and possibly nontrivial automorphism groups. We call such a category an "updown category." In this paper we give a precise definition of such categories and develop a theory for them, which incorporates earlier notions of differential posets and weight-relation posets. We also give a detailed account of ten examples, including the updown categories of integer partitions, integer compositions, planar rooted trees, and rooted trees.

MYLANDER, W. Charles, Professor, "The Plan for SM233, An Introduction to Applied Math," Part I: Operations Analysis & Probability and Statistics.

In this course the students will be presented with situations where Applied Mathematics has proven valuable and which later math majors' courses present theory and techniques that have proven useful in dealing with the situations. They will be given a toy version of the situation in the form a problem. Students will be assigned the problem to work on either as individuals or in a group. After they have had a chance to work on the problem and produce a brief report on their analysis of the problem using the tools and techniques at their disposal the instructor will lead a discussion of the problem and will present ideas, tools, and the results of some of the theory used to deal with the problem. There may be a follow up problem of a similar nature for the students to tackle. This is essentially a case study approach. The amount of development of theory and of the methods used to deal with the problem will be very limited and seldom will an attempt be made to develop a general attack on a class of problems.

PENN, Howard L., Professor, "The Impact of Calculus Reform on Student Performance in Subsequent Courses at the United States Naval Academy," January 2004.

The Naval Academy was one of six institutions of higher learning participating in a study of the effects of calculus reform. There were three portions of the study, 1) a study of grades in calculus and follow on courses comparing the class of 2002 with the class of 1996. While there were changes in some follow on courses, these were attributed to changes in those courses rather than changes in calculus I. 2) An attitude survey of graduating Midshipmen.

On the whole, the Naval Academy graduates were positive in their reaction to the various questions. 3) Interviews and a test given to 12 TAD ensigns. They did very well on most of the questions on this exam.

SANDERS, Thomas J, Professor, "RAT-STATS II Software," and "RAT-STATS II User's Guide," Department of Health & Human Services, OIG, Office of Audit Services, September 2003.

RAT-STATS is a suite of modules that was initially developed by the Regional Advanced Techniques Staff (RATS) in San Francisco to aid in auditing procedures using a mainframe computer. This software and companion User's Guide contains a portion of those modules that were converted to the MS Windows environment. The RAT-STAT modules include ratio and regression estimation for unrestricted and stratified samples, two-stage variable ratio and regression estimation, attribute discovery sample size, sample size for controlling alpha and beta risks, one-step acceptance sampling, two-step acceptance sampling, and two-step attribute appraisals.

Presentations at Professional Meetings and Conferences

ALEVRAS, Alexis, Assistant Professor, "Cocycles for One-parameter Flows of B(H)," Non-Commutative Geometry and Operator Algebras Seminar, Vanderbilt University, 4 April 2004.

ALEVRAS, Alexis, Assistant Professor, "Semigoups of Endomorphisms of B(H)," First East Coast Operator Algebras Symposium, Vanderbilt University, 20 September 2003.

GAGLIONE, Anthony M., Professor, "Discriminating and Squarelike Groups," AMS Special Session, October 2003.

GAGLIONE, Anthony M., Professor, "Notions of Discrimination," Albany Group Theory Conference, Albany, NY, October 2003.

GAGLIONE, Anthony M., Professor, "On an Example of V. H. Dyson," NY Group Theory Seminar, February 2004.

GAGLIONE, Anthony M., Professor, "On Finitely Presented Infinite Torsion Groups," Yale University, New Haven, CT, June 2004.

GAGLIONE, Anthony M., Professor, "On the Existence of Finitely Presented Infinite Torsion Groups and a Question of V. H. Dyson," Ohio State University/Dennison Group Theory Conference, Columbus, OH, June 2004.

GAGLIONE, Anthony M., Professor, "Notions of Discrimination," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 9 October 2003.

GARCIA, Sonia M. F., Associate Professor, "From Numerical Analysis to Analysis of the Numerics," NAVAIR Naval Air Station, Patuxent River, MD, 25 May 2004.

HANNA, Charles C., Professor, "Implementing Euler's Method for Solving Differential Equations," Teachers Teaching with Technology International Conference, New Orleans, LA, 13 March 2004.

HOFFMAN, Michael E., Professor, "Algebras Associated with Posets," Ruhr-Universität Bochum, Germany, 25 July 2003.

HOFFMAN, Michael E., Professor, "Categories, Groups, and Posets," U.S. Naval Academy Mathematics Colloquium, Annapolis, MD, 18 September 2003.

HOFFMAN, Michael E., Professor, Hopf Algebras and Multiple Harmonic Sums," Loops and Legs in Quantum Field Theory, Zinnowitz, Germany, 29 April 2004.

HOFFMAN, Michael E., Professor, "Quasi-symmetric Functions and Mod p Multiple Harmonic Series," Number Theory Seminar, Max-Planck-Institut für Mathematik, Bonn, Germany, 5 November 2003.

HOFFMAN, Michael E., Professor, "Categories, Groups, and Posets," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 18 September 2004.

HOFFMAN, Michael E., Professor, "Topology and Multiple Zeta Values," Oberseminar Topologie, ax-Planck-Institut für Mathematik, Bonn, Germany, 8 December 2003.

JOYNER, W. David, Professor, "Modular Representations on Riemann-Roch Spaces: Computational Aspects," Algebra & Discrete Mathematics Seminar, Clemson University, Clemson, SC, 23 October 2003.

JOYNER, W. David, Professor, "Modular Representations on Riemann-Roch Spaces: Computational Aspects," National Security Agency (NSA), Ft. Meade, MD, February 2004.

KIDWELL, Mark E., Professor, "From Linking Numbers to Lights Out," Mathematical Adventures and Diversions, Lafayette College, Easton, PA, March 2004.

KIDWELL, Mark E., Professor, "Two types of Amphichiral Links," Knots in Washington, Washington, D.C., December 2003.

KIDWELL, Mark E., Professor, "Two Types of Amphichiral Links," Lehigh Valley Geometry Seminar, Bethlehem, PA, February 2004.

KONKOWSKI, Deborah A., Professor, "Quantum Singularity of Levi-Civita Spacetimes," Quantum Gravity Session, Spring Meeting of the American Physical Society, Denver, CO, 1-4 May 2004.

KONKOWSKI, Deborah A., Professor, "Classical and Quantum Singularity of Levi-Civita Spacetimes with and without Cosmological Constant," Dynamics and Thermodynamics of Naked Singularities: An International Workshop, Dipartimento di Matematica, Politecnico di Milano, Milan, Italy, 13-15 May 2004.

KONKOWSKI, Deborah A., Professor, "Are Classically Singular Spacetimes Quantum-Mechanically Singular as Well?" 10th Marcel Grossmann Meeting on General Relativity, Rio de Janeiro, Brazil, 20-26 July 2003.

KONKOWSKI, Deborah A., Professor, "Definition and Classification of Singularities in GR: Classical and Quantum," 10th Marcel Grossmann Meeting in General Relativity, Rio de Janeiro, Brazil, 20-26 July 2003.

KONKOWSKI, Deborah A., Professor, "`Singularity' of Levi-Civita Spacetimes," 10th Marcel Grossmann Meeting in General Relativity, Rio de Janeiro, Brazil,20-26 July 3003.

KONKOWSKI, Deborah A., Professor, "Singularities in General Relativity: Classical and Quantum," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 25 March, 2004.

KSIR, Amy E., Assistant Professor, "Representations of Finite Groups on Riemann-Roch Spaces," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 22 January 2004.

KSIR, Amy E., Assistant Professor, "Functions on Riemann Surfaces – Parts I, II and III," U.S. Naval Academy, Annapolis, MD, 5, 12 and 19 April 2004.

KSIR, Amy E., Assistant Professor, "Geometry of the Lifts of Orientifold Six-planes From Type IIA String Theory to M-theory," American Mathematical Society Eastern Section Meeting, Lawrenceville, NJ, 18 April 2004.

KSIR, Amy E., Assistant Professor, "Introduction to Calabi-Yau Manifolds," University of Maryland String Theory Research Interaction Team Meeting, College Park, MD, 28 April 2004.

LEJARRAGA, Pablo, Adjunct Professor, "Moduli Schemes: Moduli of Elliptic Fibrations," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 11 September 2003.

LIAKOS, Anastasios, Assistant Professor, "A Posteriori Error Estimators for a Two-Level Finite Element Method for the Navier-Stokes Equations with Slip Boundary Condition, Part 1," U.S. Naval Academy Applied Mathematics Colloquium, Annapolis, MD, 29 January, 2004.

LIAKOS, Anastasios, Assistant Professor, "A Posteriori Error Estimators for a Two-Level Finite Element Method for the Navier-Stokes Equations with Slip Boundary Condition, Part I1," U.S. Naval Academy Applied Mathematics Colloquium, Annapolis, MD, 13 February, 2004.

LIAKOS, Anastasios, Assistant Professor, "A-posteriori Error Estimation for a Two-Level Method for the Navier-Stokes Equations with Slip Boundary Condition," SIAM Annual Meeting, Montreal, Canada, 16 June 2003.

MAHAR, Thomas J., Associate Professor, "The Analytic Ghost Fluid Method," Joint USNA – NSWC Indian Head Programs Review U.S. Naval Academy, Annapolis, MD, September 2003.

MALEK-MADANI, Reza, Professor, "Dynamical Systems Tools for Ocean Studies: Chaotic Advection vs Turbulence," U.S. Naval Academy Applied Mathematics Colloquium, Annapolis, MD, 2 September 2003.

McCOY, Peter A., Professor, and MALEK-MADANI, Reza, Professor, "Applications of Special Functions to the Atmospheric Propagation of Lasers," AMS-SIAM joint Special Session on "Classical and Nonlinear Special Functions," Joint AMS-MAA Annual Meetings, Phoenix, AZ, 9 January 2004.

McMATH, Stephen S., Midshipman 2/C, USN, "Continued Fractions, Quadratic Forms, and Factorization," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 22 April 2004.

MICHAEL, T. S., Associate Professor, "When Can You Pack a Box with Translates of Two Bricks?" SIAM Conference on Discrete Mathematics, Nashville, TN, June 2004.

MYLANDER, W. Charles, Professor, "Analyzing the Campaign Game," Institute for Operations Research and the Management Sciences (INFORMS) National Meeting, Atlanta, GA, 20 October 2003.

PENN, Howard L., Professor, "The Impact of Calculus Reform on Student Performance in Subsequent Courses at the United States Naval Academy," Joint Mathematics Meetings, Phoenix, AZ, 9 January, 2004

PENN, Howard L., Professor, "The Impact of Calculus Reform on Student Performance in Subsequent Courses at the United States Naval Academy," Mathematical Association of America, MD-DC-VA Sectional Meeting, Salisbury, MD, 23 April 2004.

PLATT, Charles R., Midshipman 1/C, USN, "Finding Solutions to the Traveling Salesman Problem," U.S. Naval Academy Operations Research Colloquium, Annapolis, MD, 13 April 2004.

POPOVICI, Irina, Assistant Professor, "A New Transform for Lossy Image Coding," The 2003 International Conference on Imaging Science, Systems, and Technology (CISST'03), Las Vegas, NV, 23-26 June 2003.

POPOVICI, Irina, Assistant Professor and WITHERS, W. Douglas, Professor, "The Eidochromatic Transform for Image Compression," International Conference on Multimedia and Expo, 8 July 2003.

POPOVICI, Irina, Assistant Professor, "A Geometric View of Complex Analysis Topics in Riemann Surfaces," Riemann Surfaces Seminar, U.S. Naval Academy, Annapolis, MD, January and February 2004.

POPOVICI, Irina, Assistant Professor, "A New Spatio-Chromatic Transform for Improved Lossy Coding of Color Images," ICNPAA 2004: Mathematical Problems in Engineering and Sciences, West University, Timisoara, Romania, 2-5 June 2004.

PRICE, Geoffrey L., Professor, "Comparison of CP-Flows," India – American Mathematical Society Joint Meeting, Bangalore, India, 20 December 2003.

SANDERS, Thomas J., Professor, "Agent Based Models and Markov Chains," Institute for Operations Research and the Management Sciences (INFORMS) National Meeting, Atlanta, GA, 20 October 2003.

SANDERS, Thomas J., Professor, "Simulation and Mathematical Models," Institute for Operations Research and the Management Sciences (INFORMS) National Meeting, Atlanta, GA, 20 October 2003.

SPOERL, David R., Commander, USN, "A Stochastic Generalized Assignment Problem," Institute for Operations Research and the Management Sciences (INFORMS) Annual Meeting, Atlanta, GA, October 2003.

TRAVES, William N., Associate Professor, "Differential Operators on Toric Varieties," AMS-RSME Meeting, Seville, Spain, 21 June 2003.

TRAVES, William N., Associate Professor, "Projective Curves I," Algebraic Curves Seminar, U.S. Naval Academy, Annapolis, MD, 1 March 2004.

TRAVES, William N., Associate Professor, "Projective Curves II," Algebraic Curves Seminar, U.S. Naval Academy, Annapolis, MD, 8 March 2004.

TRAVES, William N., Associate Professor, "The Algebra of Optimization," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 6 November 2003.

VAN JOOLEN, Vincent J., Commander, USN, "Application of Higdon Non-Reflecting Boundary Conditions to Shallow Water Models," International Conference on Spectral and High-Order Methods (ICOSAHOM'04), Brown University, Providence, RI, June 2004.

WARDLAW, William P., Professor, "The Cardinality of $GL(n, Z_m)$ and Several of Its Subgroups," Spring Meeting of MD-VA-DC Section of the Mathematical Association of America, Salisbury State University, Salisbury, MD, 24 April 2004.

WARDLAW, William P., Professor, "A Short Proof that Row Rank Equals Column Rank," Mathematical Association of America MathFest, Boulder, CO, 1 August 2003.

WARDLAW, William P., Professor, "Two Problems Regarding e," Fall Meeting of the MD-VA-DC Section of the Mathematical Association of America, The Johns Hopkins University, Baltimore, MD, 8 November 2003.

WARDLAW, William P., Professor, "Two Problems Regarding e," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 4 December 2003.

WARDLAW, William P., Professor, "The Cardinality of $GL(n, Z_m)$ and Several of Its Subgroups," U.S. Naval Academy Pure Mathematics Colloquium, Annapolis, MD, 15 April 2004.