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Mathematics Department

Professor Mark D. Meyerson
Chair

Mathematics provides a logical framework and a language indispensable to understanding the technical world in which we live. The following description summarizes the many contributions to this field of study made during the past academic year by the faculty and midshipmen majors of the Mathematics Department of the U.S. Naval Academy. The results cited reveal the great scope, diversity, and applicability of mathematics and offer glimpses of its intellectual beauty and appeal.

Several midshipmen conducted research projects as Trident Scholars, as Honors Mathematics Majors, or in specially created projects under the guidance of faculty members. Many faculty members spent numerous hours serving as mentors and readers for capstone projects required of all midshipmen majors. Listed below are the three Trident Scholars who were mathematics honors majors in the Class of 2005, their project titles, and their faculty mentors:

Midshipman 1/C Andrew C. Bashelor, USN, "Enumerative Algebraic Geometry: Counting Conics" (Assistant Professor Amy E. Ksir and Associate Professor William N. Traves)

Midshipman 1/C Nathan F. Brasher, USN, "Trajectory and Invariant Manifold Computation for Flows in the Chesapeake Bay" (Professor Reza Malek-Madani and Associate Professor Gary O. Fowler)

Midshipman 1/C Stephen S. McMath, USN, "Parallel Integer Factorization Using Quadratic Forms" (Professor W. David Joyner and Assistant Professor Frederick L. Crabbe of the USNA Computer Science Department)

In addition to these three midshipmen, there were six other mathematics honors majors in the Class of 2005, for a total of nine. The others are listed below with their project titles and their faculty mentors:

Midshipman 1/C Patrick Brooks, USN, "Classification of Minority Group Stability" (Associate Professor Gary O. Fowler)

Midshipman 1/C Jerry W. Irons, USN, "A Polynomial-time Probabilistic Algorithm for the Minimum Distance of an Arbitrary Linear Non-binary Error-correcting Code" (Professor W. David Joyner)

Midshipman 1/C Clifton G. Lennon, USN, "List-Decoding of Generalized Reed-Solomon Codes Using Sudan's Algorithm" (Professor W. David Joyner)

Midshipman 1/C Jason M. McGowan, USN, "Implementing Generalized Reed-Solomon Codes and a Cyclic Code Decoder in GUAVA" (Professor W. David Joyner)

Midshipman 1/C Brian Ross, USN, "Classical and Quantum Singularities in a Cylindrically Symmetric Spacetime" (Professor Deborah A. Konkowski and Professor B. Mitchell Baker)

Midshipman 1/C Matthew C. Welch, USN, "Compactification of Plane Curves" (Associate Professor William N. Traves)

Special notes are that Midshipman Brasher was the winner of the Harry E. Ward Trident Scholar Prize and that Midshipman McMath was the winner of the Naval Intelligence Foundation Award for the best honors project at USNA.

Regularly meeting throughout the year are four colloquia series: in pure mathematics, applied mathematics, operations research, and teaching methods. These bring in outside speakers and provide a forum for department members and mathematics majors to present their work. There are also weekly seminars in topics of special interest, such as Riemann Surfaces, Algebra, Applied Math and the Mathematics of Fluid Flows.

Once again, the Naval Academy's Mathematics Department produced a wide range of scholarly work that appeared as technical reports or as publications in refereed journals throughout the world. Dozens of articles appeared as applications of mathematics or as pure mathematical research. Topics that Mathematics Department faculty researched include:

> mathematics pedagogy, chaos and dynamical systems, mathematical physics and cosmology, algorithms for computers, wavelets, cryptology, ocean acoustics, image recognition, fluid flows, beam propagation, and basic mathematical research in areas such as algebra, analysis, combinatorics, computational geometry, differential equations, differential geometry, matrices, number theory, operator theory, and statistics.

In addition to many independent research projects, several research projects were sponsored in whole or part by a variety of sources, such as:

Department of Defense, High Performance Computing Initiative
Department of Health and Human Services
National Aeronautics and Space Administration
National Science Foundation
National Security Agency
Naval Research Laboratory
Naval Surface Warfare Center
Office of Naval Research

During the past year, members of the U.S. Naval Academy Mathematics Department presented the results of their scholarly activities on over fifty occasions at professional mathematical meetings and colloquia throughout the United States and abroad. This activity, along with publication, enhances the academic stature of the Naval Academy and promotes the professional growth and reputation of those individuals involved. Through research activity, the faculty expanded their intellectual horizons and stayed vital in their disciplines. They contribute to the discovery of new mathematics. And they develop new materials and ideas that they can share with midshipmen students in their mathematics courses and research projects.

Sponsored Research

Cocycles for One-parameter Flows of B(H)

Researchers: Assistant Professor Alexis A. Alevras, Professor Geoffrey L. Price,
and Dr. Robert Powers (University of Pennsylvania)
Sponsor: Naval Academy Research Council (NARC)

In this project, we are interested in invariants for E_0-semigoups of B(H), *i.e.,* for one-parameter semigroups of endomorphisms on the algebra of bounded operators on a Hilbert space H. E_0-semigoups model evolution of quantum systems that is irreversible in time. We completed the computation of the set of local cocycles for a large class of E0-semigroups.

The set of local cocycles is a natural cocycle conjugacy invariant for an E_0-semigroup α. It has a multiplicative structure as well as an order structure on its positive elements and provides in fact several invariants for α; in particular the unitary local cocycles form a topological group that may be appropriately viewed as the automorphism group of the E0-semigroup.

While local cocycles for the standard, type I examples of E_0-semigoups were computed early on by W. Arveson (unitary cocycles) and R. Bhat (positive contractive cocycles) nothing was known about other classes of E_0-semigoups. Ours is the first such computation for type II semigroups. The description of the cocycles is at the level of the so-called boundary weight associated with the E_0-semigroup.

This work has resulted in a paper that has been accepted for publication in the *Journal of Functional Analysis*.

An Assessment of the Biot-Stoll Model for a Poroelastic Seabed

Researcher: Professor James L. Buchanan
Sponsor: Naval Research Laboratory (NRL)

The purpose of this project is to assess the mathematical models currently available for predicting the acoustic behavior of a seabed at frequencies up to about 500kHz. For unconsolidated seabeds there are currently two rival models, the general model of a poroelastic medium due to Biot and particularized by Stoll to seabeds, and the model of Buckingham. The Biot-Stoll model treats a poroelastic medium as an elastic frame with interstitial pore fluid. It has two loss mechanisms, inter-granular friction and viscous friction due to the motion of the frame relative to the fluid. It depends upon thirteen parameters. The Buckingham model is derived from the linearized Navier-Stokes equations. Attenuation is due to grain-to-grain friction and strain hardening of the fluid between grains. It may be regarded as a causal version of the standard elastic model of the seabed in the sense that, like the elastic model, it predicts wave attenuations that are approximately linear in frequency, but unlike the elastic model, it predicts wave speeds that are logarithmically dispersive as required by the Kramers-Kronig causality.

Assessing the models requires addressing several questions:

To what extent are the predictions of the model in accord with experimental observations? In particular both models were formulated prior to Sediment Acoustics Experiment 1999 (SAX99) which is to date the most comprehensive attempt to measure such observables as wave speeds and attenuations for a sediment with known parameters.

Are the underlying assumptions used in deriving the models appropriate for an unconsolidated sediment? For instance does an aggregation of uncemented sand grains constitute a Hookean elastic frame in the sense assumed by the Biot model? To the extent that this is not the case what modifications are necessary?

Is the Biot model unnecessarily complicated? The conventional model Biot-Stoll model depends upon thirteen input parameters, some of which are not easy to measure. Are the viscous losses predicted by the Biot model due to the relative motion of the fluid and frame important for unconsolidated seabeds? If not might the simpler Buckingham model, which requires only five inputs, suffice?

Classroom Implementations of State-of-the Art Results in Applied Graph Theory

Researcher: Professor Carol G. Crawford
Sponsor: U. S. Naval Academy Sabbatical Program

This research was devoted to the design of classroom implementations of the investigator's own research results in creating and implementing graph models for image processing and neural network design. In particular, the project was devoted to two specific areas**:** (i) Aspect Graph Construction for Robotic Vision System Design and (ii) Sphere-of-Influence Graph Representations for Neural Network Design of an Automated Fingerprint Representation and Matching System. This prior research was funded by grants from The Federal Bureau of Investigation and The Office of Naval Research.)

The classroom implementations consist of 4 PowerPoint modules for use in upper level discrete mathematics courses or seminars. The modules provide students with an introduction to computer vision problems in robotics and in automated fingerprint identification. Next the modules present the role of high-level graph theoretic models in image processing and neural network design. In particular, the modules present Aspect Graphs and Sphere-of-Influence Graphs, (SIGs), relatively new classes of graphs that were previously studied only in theory. (My colleagues, Stuart Ullman, DTRC, and Eric Mjolsness, Yale, and I were the first to apply them to Robot Vision and to Automated Fingerprint Identification.)

These modules will be available online at the Academy for use by both instructors in the classroom and for use by individual midshipmen for their own research project reference.

Quantitative Population Models for Cell-Based Biosensor Arrays

Researcher: James D'Archangelo
Sponsor: Naval Research Laboratory (NRL)

The rapid detection and identification of potentially toxic substances has assumed new urgency given the increasing threat of weapons of mass destruction. The firing patterns of neurons can be potentially used to characterize the presence and effect of both toxins and benign stimulants. This is the continuation of an ongoing project.

The objectives of the project are as follows:

(1) To design, implement, and test realistic analytical mathematical models of individual cultured living neurons, and networks of cultured neurons,
(2) To refine and improve the neuron models based upon laboratory measurements and modeling of neuron geometry, electrical membrane characteristics (ionic channels), and synaptic conductivity (*i.e.,* network behavior) of cultured neurons, and
(3) To validate these models by predicting the behavior of living cultured neurons in response to diverse neurotoxins (*e.g.,* channel blockers, synaptic blockers).

MATLAB® programs were written to model a variety of neural networks. For example, one program models a two-neuron closed-loop network of neocortical cells with both excitatory and inhibitory post synaptic connections. The differential equations are based on a paper by Wilson in the 1999 Journal of Theoretical Biology titled "Simplified Dynamics of Human and Mammalian Neocortical Neurons". The program incorporates for each neuron a synaptic current input representing the effects of all other synaptic connections (from other than these two neurons). This synaptic current can be constant or can be stochastic in nature. Each neuron is modeled by a system of 4 Hodgkin-Huxley type differential equations, and each post synaptic connection is modeled with two differential equations. So the entire network model involved sixteen differential equations and is solved using a RK4 algorithm.

Data Assimilation in the Analysis of Fluid Flow - Especially Flow in the Chesapeake Bay

Researcher: Associate Professor Gary O. Fowler
Sponsor: Defense Modeling and Simulation Office (DMSO)

The transport of fluid particles is typically modeled by partial differential equations (*e.g.,* Navier-Stokes). Of course, uncertainty associated with the coefficients and initial conditions produces uncertainty in predicted paths. Complementing the mathematical models are actual observations. Both Eulerian (at fixed positions) and Lagrangian (along fluid paths) observations are available. This research investigates techniques for incorporating observations into the modeling process.

The current goal is to predict with error bounds the trajectory of an autonomous drifter. This drifter would be programmed before launch to alter its depth. If this can be accomplished, the drifter would be able to explore and return data associated with "dead" zones in the Chesapeake Bay. Possible methods of analysis included mesh-free interpolation methods, Kalman filter, stochastic Gaussian and Levy processes, and statistical functional data analysis.

On the Development of an Analysis of the Comparison of MODIS Data with HARLIE and MPLNET Data

Researchers: Associate Professor Sonia M. Garcia,
Geary Schwemmer and Yoram J. Kaufman, National Aeronautics and Space Administration (NASA)
Sponsor: National Aeronautics and Space Administration (NASA)

The proposed research has two components: continuation of the study of calibration techniques for data from the Holographic Airborne Rotating Lidar Instrument Experiment (HARLIE) as an extension of a collaboration over for the last three summers between Mr. Geary Schwemmer and the applicant. And, in a new collaboration with Dr. Yoram J. Kaufman, Climate and Radiation Branch, Code 913 a study for comparison and analysis of the results of the Moderate Resolution Imaging Spectroradiometer (MODIS) satellite view of aerosols and data from ground-based measurements from HARLIE, and again from MODIS and the Micro-pulse Lidar Network - MPLNET a network of lidar instruments, is proposed.

Representations of Finite Groups on Riemann-Roch Spaces of Artin-Schreier Covers

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir
Sponsor: National Security Agency (NSA)

We continue our earlier work, concentrating on the case where the curve is defined over a field of ``bad'' characteristic (*i.e.,* the characteristic of the ground field divides the order of the automorphism group of the curve). This work is still in progress.

RUI: Classical Singularities, Quantum Singularities and Cauchy Horizons

Researcher: Professor Deborah A. Konkowski
Sponsor: National Science Foundation (NSF)

This project is a study of spacetimes with mild singularities and Cauchy horizons. Mild singularities include quasiregular and nonscalar curvature singularities. In the case of quasiregular singularities, particle paths end suddenly with no warning from infinitely increasing tidal forces, while in the case of nonscalar curvature singularities some, but not all, particles moving near the singularity feel infinite tidal forces. A Cauchy horizon is the boundary of the causal development of spacetime. Cauchy horizons and singularities are intimately connected, including through the cosmic censorship conjecture, which continues to be debated.

In particular, this work is focusing on:

(1) a quantum-mechanically singular spacetimes,
(2) two previously developed stability conjectures, and
(3) the global structure of various singular spacetimes.

Some of this work has been done in collaboration with T.M. Helliwell (Harvey Mudd College) and students at both Harvey Mudd and USNA.

In the first case, following Horowitz and Marolf and work begun by T.M. Helliwell and D.A. Konkowski, classical test particles are replaced with quantum test wave packets to test the singularity structure of spacetimes which classically possess mild singularities. Results were presented at conferences in London (England), Dublin (Ireland) and Tampa (Florida). A colloquium talk at the University of North Carolina (Wilmington) also mentioned these results amongst other things about singularities.

In the second case, conjectures proposed by T. M. Helliwell and D. A. Konkowski to predict whether various mild singularities and Cauchy horizons are stable are being investigated further. Thus far, the singularity conjecture always holds true but the Cauchy horizon conjecture misses purely Weyl singularities that occur in the plane wave and exact mass-inflation spacetimes.

In the third case, spacetimes with nonscalar and quasiregular singularities are under analytical investigation. This includes work on: (1) spacetimes with nonscalar singularities due to Siklos, (2) quasiregular singularities in the context of dislocations and disclinations and in the context of Columbeau's extended theory of distributions, and (3) the structure of some spacetimes with directional singularities. In addition, numerical studies with B. Berger have been under consideration.

Automorphisms of Curves and Codes

Researchers: Assistant Professor Amy E. Ksir and Professor W. David Joyner
Sponsors: Naval Academy Research Council (NARC to AEK)

Given a projective algebraic curve X and two divisors D and E on X, one can use the Riemann-Roch space L(D) to construct a code C(D,E), called the Goppa code or AG code. If G is a finite subgroup of the automorphism group of X, and D is invariant under G, then there is a natural action of G on the code. One goal was to determine when a group of automorphisms of the code could be extended to act on the curve. We were able to prove a theorem relating the two automorphism groups. This theorem has been written up and submitted for publication. Our second goal was to compute the module structure of the Riemann-Roch space in some large classes of examples. We had previously obtained a very simple formula for the module structure, under some conditions on the group G. This formula was submitted for publication last year; it has now been accepted and is being revised for publication. We used the GAP and MAGMA computer programs to investigate a large class of examples, the modular curves. We found some patterns and were able to prove a theorem explaining the module structure completely for modular curves. This has been accepted for publication.

Geometry of Superstrings

Researchers: Assistant Professor Amy E. Ksir and Paul Green (University of Maryland)
Sponsor: Naval Academy Research Council (NARC to AEK)

We studied the conformal anomaly, a physical quantity calculated in the process of solving conformal field theories. While its definition comes from physics, it is a geometric quantity: for a given curve of genus g, one can compute the determinant of the period matrix. It seems a priori that this determinant will depend heavily on the choice of a Lagrangian subspace of homology cycles for the period integrals. However there is evidence both from conformal field theory and from small genus examples that there is a canonical choice of subspace, which will allow this determinant to be well-defined.

Our objective was to give a mathematical definition of the conformal anomaly, and to show that it is additive with respect to a connected sum, which is also expected from the physics.

Our original method was to begin by studying the conformal anomaly on hyperelliptic curves, where everything can be described very concretely. We were then able to compute the determinants of the period matrices in terms of hyperelliptic integrals. However, this concreteness did not help us in proving that the anomaly was independent of the choice of homology basis or that it would behave well with respect to connected sums. During this year, the researchers spoke with several physicists and began to change the conjectured mathematical definition of the conformal anomaly to more closely reflect its definition in physics, which involves taking a limit as the ends of the curve are stretched infinitely far from each other.

Algebraic Surfaces and Seiberg-Witten Curves

Researchers: Assistant Professor Amy E. Ksir and Stephen G. Naculich (Bowdoin College)
Sponsors: Naval Academy Research Council (NARC to AEK)
and National Science Foundation (NSF to SGN)

The idea of the Seiberg-Witten approach to gauge theories is that all of the data of the gauge theory can be computed from one function, the holomorphic prepotential, which can itself be computed using period integrals over an algebraic curve, the Seiberg-Witten curve. In the past, we have been using a technique known as "geometric

engineering" in type IIA string theory to find Seiberg-Witten curves for various gauge theories. Nikita Nekrasov has proposed an alternative way to compute instanton terms of the holomorphic prepotential, using equivariant cohomology on the moduli space of instantons. His method led to a conjectured generating function for the series of instanton terms. Two recent papers, one by Nekrasov and Okounkov, and the other by Nakajima and Yoshioda, have each proven the conjecture by very different methods.

The purpose of this investigation is to compute higher instanton terms of the holomorphic prepotential for gauge theories where the geometric engineering description in type IIA string theory involves two orientifold planes. These cases have not yet been computed either by period integrals or using Nekrasov's method.

Our previous work led to precise descriptions of the Seiberg-Witten curves as hyperelliptic curves, which make the period integrals possible, if not easy, to compute. This year we were able to formulate and begin to compute these integrals. At the same time, we began to study Nekrasov's method and modify it to fit this case.

Twin Screw Extrusion of Energetic Materials

Researchers: Assistant Professor Anastasios Liakos and Dr. H. Bruck (University of Maryland)
Sponsors: Naval Surface Warfare Center - Indian Head Division (NSWC-IH)
and Naval Academy Research Council (NARC to AL)

This project is still in progress. The Naval Surface Warfare Center in Indian Head funded my summer salary. A second-generation three-dimensional program has been written in C to try to predict the values of the flow variables (velocity, pressure, stress). In addition, research was done to mesh the complex geometry of the flow domain of the extruder. A white paper proposal was submitted to the Office of Naval Research to obtain funding on an initiative to advance the frontier of twin-screw extrusion research. The proposal was co-authored by six other scientists from the University of Maryland and from NSWC-Indian Head.

The Mathematics of Entanglement

Researcher: Professor Robert B. Lockhart
Sponsors: Naval Research Laboratory (NRL)

This was a continuation of past work with Michael Steiner at the Naval Research Laboratory. The goal was to establish a mathematical description of a measurement devise. The goal was not obtained, though some results, which are currently being written up, are worthy of publication. They include the fact that generically the matrices in a POVM are full rank and the fact that one never needs separable pure states to form a mixture.

Conservation Laws in Generalized Coordinates

Researcher: Associate Professor T. J. Mahar
Sponsor: Naval Surface Warfare Center - Indian Head Division (NSWC-IH)

Euler's equations for compressible flows were derived for arbitrary generalized coordinates and finite volume methods were applied in a general form. A slightly modified version of the 1-D Euler equations in generalized coordinates was shown to be transformable to a problem in rectangular coordinates. The transformed problem provides a strong test for numerical methods for flow problems in generalized coordinates.

Asymmetric Beam Propagation

Researcher: Professor Peter A. McCoy
Sponsor: Joint Technical Office-Office of Secretary of Defense (OSD), HEL

The aim of this research was to discuss some of the characterizations of an asymmetric beam propagating through a uniform medium with zero conductivity in the absence of free charge. Higher order atmospheric effects such as thermal blooming and motion of the medium are to be considered in subsequent modeling and simulation.

The method used in the current analysis is a combination of normal mode analysis and function theoretic methods that are inherently integral transform based. Normal mode analysis is one of the best ways of determining numerical stability of a solution; whereas, the function theoretic method has demonstrated great utility in the analysis and characterization of solutions to a broad range of problems in mathematical physics. This research is the first application of these methods in this venue and leads to some new and interesting results.

Reciprocal integral transforms based on normal mode expansions are constructed that link analytic solutions of the Paraxial Wave Equation (PWE) with associated analytic functions of two complex variables. Analytic continuation of these formulae allows characterization of solutions to the PWE that are radiating into a half-space, as well as, the construction of a new class of solutions to the PWE with singularities. We establish that the Hermite-Gauss solutions are a complete set for analytic solutions in a half-space that decay to zero in the transverse direction as the solution advances along the optical axis.

Research is being completed on the analysis of normal mode expansions in terms of the Laguerre-Gauss functions. Several of the results are "sharper" than the Hermite-Gauss expansions. Additional work is moving forward on two parallel fronts. First, the third and final set in the triad of normal mode expansions, the Ince-Gauss normal modes. And, a study of the paraxial wave equation without the paraxial approximation is continuing to bear fruit.

Characterizing Surface Current Features in the Chesapeake Bay using Lagrangean Flow Methods

Researchers: Professor Reza Malek-Madani, Associate Professor John F. Pierce, and Associate Professor Gary O. Fowler
Sponsor: Defense Modeling and Simulation Office (DMSO)

Over the last decade, methods of dynamical systems have been used to examine fluid flow from a Lagrangean perspective to extract information about the transport of materials by surface currents. The models developed have been applied successfully to study the dynamics of transport in the Monterey Bay, producing results that were unexpected, highly provocative, quite promising and imminently verifiable. Previously, we identified the factors that support the application of these methods to the Chesapeake Bay. We designed two projects to serve as a proof of concept that these methods could be applied to the Chesapeake Bay: formulating normal modes for the Chesapeake Bay, and computing distinguished hyperbolic trajectories from simulated flow data for the Chesapeake Bay and analyzing material transport in the bay using lobe dynamics. The designs lead to successful research projects by two Trident scholars. In addition, the work demonstrated how, with this level of analysis, the USNA could enhance the efforts of and add value to the program of the Chesapeake Bay Consortium, an independent regional research consortium.

Naval Academy Chesapeake Bay Research

Researcher: Professor George C. Nakos
Sponsor: Defense Modeling and Simulation Office (DMSO)

In summer of 2004 and partially continuing throughout the Academic year, the investigator was part of an effort to study the water movement and residence time in the Chesapeake Bay. The specific research participation in this project was the installation in a linux environment of several libraries and the QUODDY software, as well as the use of the above. The results of the QUODDY runs were crucial in Midshipmen research, especially in Midshipman Nathan Brasher's Trident Scholar Thesis project. The ability to run these programs locally and fast enhances the overall Chesapeake Bay Research program at the Naval Academy.

A Multi-scale Edge Transform for Image Coding

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers
Sponsors**:** Office of Naval Research (ONR) and Naval Academy Research Council (NARC to IP)

This project concerns lossy compression of either color or black-and white images, using an approach radically different from the popular JPEG and JPEG 2000 standards. Our objective is to exploit the edges and shadows in an image such that a smaller number of bits can be used to represent it, while maintaining an "acceptable" visual quality for the decompressed image. The correlation between spatial locations of pixels has been exploited by almost all existing algorithms, but the driving principle of these algorithms is elimination of data in a simple (say linear or cubic) dependency from proximate pixels. We propose a recurrent coding technique that also exploits the discontinuities in data near the contours of an image.

The popular wavelet approach to image compression describes an image in terms of a linear basis of elements which are self-similar and which also reflect the smooth nature of many image regions. In this basis, however, a sharp, straight edge must be described as a superposition of many wavelet elements at all scales. The first step in this project is development of a linear basis that shares the wavelet property of self-similarity but is reflective of sharp, straight edge structures rather than smoothness. Such a basis has potential applications in pattern analysis and image enhancement as well as image coding. Following the development of this basis, the remainder of the coding problem can be approached in a fashion similar to standard wavelet image coders.

A Dynamical Systems Approach to Cardiac Arrhythmias

Researcher: Assistant Professor Irina Popovici and Professor B. M. Baker
Sponsor: Naval Academy Research Council (NARC to IP)

This project concerns the properties of an iterated system proposed by B. M. Baker and R. Kline to model the behavior of the heart, based on the connection between membrane current kinetics, action potential duration and cardiac rhythm.

The starting point is a dynamical system incorporating one or two kinetic parameters which control repolarization, and hence action potential duration. A simply formulated family of even one-dimensional dynamical systems is mathematically capable of producing restitution curves, bifurcation schemes, rhythmic response and chaotic behavior mimicked by cardiac preparations. The family of one- and two-dimensional maps in which the kinetic parameters are modeled follow three basic assumptions: 1) exponential increase of the parameter until a threshold for repolarization is reached, 2) exponential decay of this parameter immediately after threshold is reached, 3) a periodic constraint.

The parameter sets we study consist of points where the dynamical system transitions from nearly-periodic behavior to chaos for the models that include a refractory period and for the two-dimensional (two loader) model. This work is in progress. We have theorems describing the existence and uniqueness of periodic orbits, and a characterization of the stability of escalator-type orbits of the two-loader cardiac model. We have bifurcation results for large-period and small-period systems.

Cocycles for One-parameter Flows of B(H)

Researchers: Professor Geoffrey L. Price Assistant Professor Alexis Alevras
and Dr. Robert Powers (University of Pennsylvania)
Sponsor: National Science Foundation (NSF)

The theory of operator algebras was developed by von Neumann and other mathematical physicists in the 1930's as a mathematical framework to study the time evolution of quantum mechanical systems. Although time reversible systems of observables on operator algebras have been studied for many years, only recently has a systematic investigation begun into the study of time-irreversible dynamical systems. An E_0-semigroup is a one-parameter semigroup of *-endomorphisms on B(H) (the algebra of bounded linear transformations on a Hilbert space H.) that may be viewed as a quantum dynamical system in which the observables in a quantum mechanical

system are allowed to evolve forward but not backward in time. The work of W. B. Arveson and R. T. Powers has provided invariants that distinguish the cocycle conjugacy classes of quantum dynamical systems. One invariant is the structure of the family of cocycles corresponding to a quantum dynamical semigroup. Given such a semigroup $\alpha = \{\alpha_t: t > 0\}$, a cocycle is a one-parameter family $\{V_t: t > 0\}$of operators in B(H) satisfying the cocycle identities $V_{t+s} = V_t\,\alpha(V_s)$ for all non-negative real numbers t and s. In joint research with Assistant Professor Alexis Alevras of USNA and Professor Powers of the University of Pennsylvania, Price has studied the structure of the family of local cocycles associated with E_0-semigroups of a certain type known as type II_0.

Although these E_0-semigroups are rather mysterious the authors have succeeded in determining both the order structure and the multiplicative structure of their local cocycles. In fact they have shown that the structure of these objects can been reduced to the study of some questions on the ordering of a certain family of 2×2 matrices. This work has recently been accepted for publication in the Journal of Functional Analysis.

Alevras, Powers and Price expect to continue working on this subject by looking at the cocycle structure of quantum dynamical semigroups of type II_n for $n \geq 1$.

Invariant Subalgebras of the Hyperfinite II_1 Factor

Researchers: Professor Geoffrey L. Price Assistant Professor Alexis Alevras
and Dr. Robert Powers (University of Pennsylvania)
Sponsor: National Science Foundation (NSF)

In the early 1980's Vaughan F. R. Jones developed an index theory for subfactors of the hyperfinite II_1 factor R that is closely related to the Galois theory of groups and field extensions. Since the late 1980's Price has studied a family shifts on R of (Jones) index 2 known as the binary shifts. More specifically, α is a binary shift of index 2 on R if α is a unital *-endomorphism of R whose image $\alpha(R)$ satisfies $[R: \alpha(R)] = 2$ and if there is a sequence of hermitian unitary operators u_n that generate the algebra, pairwise commute or anticommute, and satisfy the property $\alpha(u_n) = u_{n+1}$ It is natural to try to determine what are the invariant von Neumann subalgebras of R under the shift α This question seems difficult even if one restricts attention to the invariant subfactors of index 2 in R. There are some very interesting examples that are related to invariant subspaces of Fredholm index 2 of the unilateral shift on the subspace H^2 of the Hilbert space of L^2–functions on the unit circle (these subspaces are completely classified). The question is: Are there more?

Differential Operators on Toric Varieties

Researchers: Associate Professor William N. Traves and Mutsumi Saito, Hokkaido University, Japan
Sponsor: Naval Academy Research Council (NARC to WNT)

This project investigated the algebraic and combinatorial structure of rings of differential operators on toric varieties and semigroup algebras. It was a long-term project; this was its third year. This year we proved that the ring of differential operators on semigroup algebras is finitely generated precisely when the semigroup is scored, a combinatorial condition related to the Cohen-Macaulayness of the coordinate ring. We also proved that the ring of differential operators is Noetherian under certain conditions. The paper describing our results appeared in the *Journal of Algebra*.

Independent Research

A Class of E_0-semigroups on the Hyperfinite II_1-factor

Researcher: Assistant Professor Alexis Alevras

This project is in progress. We are interested in a class of E_0-semigroups on the hyperfinite II_1-factor arising from ergodic transformations. Specifically, starting with a unilateral Markov shift T on a probability space (Y,p), and an appropriate step function $h:Y \rightarrow R$ we construct the flow (T_t) under h, acting on the space X under the graph of h. We introduce on X certain equivalence relations R that are ergodic, measure preserving, countable and standard. Hence, the von Neumann algebra $M(R)$ associated with R is the hyperfinite II_1 factor; and the transformations T_t induce a semigroup of endomorphisms on $M(R)$. We are in the process of computing invariants introduced in our earlier work, in the hope of classifying these flows.

An Assessment of the Biot-Stoll Model for a Poroelastic Seabed

Researcher: Professor James L. Buchanan

Human bone is a poroelastic medium consisting of an outer cortical layer surrounding a high porosity cancellous interior with an interstitial lipid fluid. An important question is can the porosity of the cancellous bone be ascertained by acoustic interrogation, thereby determining whether the bone is osteoporotic. This year the investigator submitted two articles for publication. One focused on using numerical contour integration to calculate the acoustic field arising from cancellous bone in the frequency range 100-500 kHz, and the other on presented an algorithm which uses parallel processing to solve the problem of ascertaining the poroelastic parameters of cancellous bone.

Non-Parametric Density Estimation of Streaming Data using Orthogonal Series (Dissertation Topic)

Researcher: Lieutenant Commander Kyle Caudle, USNR

Computer technology in the 21st Century has allowed us to gather and collect data at rates that would have seemed impossible less than a decade ago. As such, typical data base management systems (DBMS) are having great difficult storing and analyzing data in the traditional way. Systems that receive large amounts of data in a transient data streams generally need to analyze the data immediately without storing it on a disk. These systems are referred to as data stream management systems (DSMS). This emerging field has been pushed to the forefront by technology that demands analyzing data in realtime. Babcock et. el. [2002] analyzed the issues involved in mining rapid time-varying data streams. To date most of the work in the area of DSMS has primarily been concerned with querying the data streams. These queries provide estimates of parameters, such as the mean, and then continuously update them as more data arrives. Recently, Heinz and Seeger [2004] used data streams to provide an estimate of the underlying probability density function by dividing the data up into bins or windows containing the most recent data. An estimate of the density is then created using the standard wavelet cascading algorithm on the binned data.

This dissertation provided an alternative approach to finding the probability density function of streaming data, by estimating the density by using an orthogonal series. Obtaining a density estimate by orthogonal series has several advantages that will be discussed throughout this dissertation. Although the approach is applicable to a myriad of basis functions, the density estimation problem will be studied by using wavelets as the basis functions. The history of wavelets as a mathematical tool dates back to the early 1900s. In the 1990s Donoho and Johnstone really established wavelets as a scientific discipline by applying them in the areas of image compression, denoising and density estimation. Devroyer [1985], Silverman [1986] and Scott [1992] provide excellent background material on density estimation in general. The first paper that used wavelets in density estimation is attributed to Doukhan and Leon [1990]. This work was followed by Walter [1990] and Kerkyacharian and Picard [1992]. As a mathematical tool for representing functions, and specifically probability densities, wavelets work especially well. This is due in part, to the fact that they form an orthonormal basis for L_2. Another pioneer in the field of wavelet density estimation was Vidakovic [1994,] who constructed density estimations based on the square root of the density.

Successful, Interactive Math WEB Design

Researcher: Professor Carol G. Crawford

This project is an extension of the author's work with Dr. Mark Meyerson, of the Naval Academy Mathematics Department. Their work resulted in the creation of an innovative, Real World Calculus Laboratory for the 3- semester Calculus sequence taken by all midshipmen at the United States Naval Academy. The Curriculum Development Program at the Academy funded the research for the initial two-year design project. The website received two honors for excellence this past year.

The Eisenhower National Clearinghouse (ENC) recently chose their site as one of a dozen, exemplary websites for educators for November 2004. The ENC is funded by the United States Department of Education and administered by The Ohio State University. In this paper the author shares the design characteristics that contributed to the success of this online, learning tool for college and university calculus programs.

In addition, the web based calculus site was chosen this past spring 2005 for EEVL**:** the UK Internet guide for Engineering, Mathematics, and Computing at: http://www.eevl.ac.uk/ .

The author's project explores and develops design characteristics to help researchers and educators design successful online, learning tools. A paper has been accepted for publication and the paper will be presented at the joint meeting of the 3rd International Conference on Education and Information Systems, Technologies and Applications (EISTA 2005) and the International Conference on Cybernetics and Information Technologies, Systems and Applications (CITSA 2005).

Discriminating and Squarelike Groups

Researcher: Professor Anthony M. Gaglione

The concept of *discriminating groups* arose out from the study of solving equations over groups and also the study of logic in group theory. It turns out that these groups arise in many diverse settings. We need to fix some terminology. Let G and H be groups. We say that G *separates* H if for any nontrivial element of H there is a homomorphism from H to G that does not map the element to the identity of G. If it is the case that for any finite set of nontrivial elements of H there is a homomorphism from H to G which does not map any element of the set to the identity of G, then G is said to *discriminate* H. A group G is *discriminating* if every group separated by G is discriminated by G. It is not hard to prove that the group G is discriminating if and only if for every finite nonempty subset S of nontrivial elements of the Cartesian square, $G\times G$, there is a homomorphism $\phi_S : G\times G \rightarrow G$ such that $\phi_S(s) \neq 1$ for all s in S, i.e., G discriminates $G\times G$. These notions were introduced by Baumslag, Myasnikov and Remeslennikov and studied in several papers by A.M. Gaglione (et al). In particular in a paper by A.M. Gaglione (et al) entitled *Discriminating Groups* (*Journal of Group Theory*, 2001, 463-474) it was shown that a discriminating group has the same universal theory (with respect to a first-order language appropriate for group theory) as its direct square. In an effort to futher capture this property *squarelike* groups were introduced. These are groups universally equivalent to their direct squares, i.e., a group G is squarelike if G and $G\times G$ satisfy precisely the same first-order sentences. Thus any discriminating group is squarelike. In this project, A.M. Gaglione together with D. Spellman and B. Fine, tackled many of the open questions concerning discriminating and squarelike groups. Two of the theorems we proved were:

(1) Any squarelike group is elementarily equivalent to a countable discriminating group.

(2) The theory of the class of discriminating groups is computably enumerable but undecidable.

We also introduced an explicit set of universal-existential axioms, developed by Verena Huber-Dyson and Oleg Belegradek, for the class of squarelike groups. The questions that we solved as usual led to others, so that this research project is still on going.

Finitely Presented Infinite Torsion Groups and Logic

Researcher: Professor Anthony M. Gaglione

A group G is said to be *periodic* or a *torsion group* if all of its elements are of finite order. A well known and very difficult question is (see Baumslag, Myasnikov and Shiplrain, "Open problems in combinatorial group theory," *Contemporary Math*., AMS, Providence, 2002, 1-38)

Q1: *Does there exist an infinite finitely presented torsion group?*

In this project, the investigator together with his colleagues (B. Fine and D. Spellman) has attempted to show a relationship between two seemingly unrelated fields. In particular, we relate the above Q1 to the following question of Verena Huber-Dyson:

Q2: *Does the universal theory of the torsion groups coincide with the universal theory of the finite groups?*

Let L be the first-order language with equality (which we write as = and is interpreted as identity) containing a binary operation symbol (which we write as $\bullet$ or simply juxtaposition), a unary operation symbol (which we write as $^{-1}$)and a constant symbol (which we write as 1). This is what is meant by saying that L is a language appropriate for group theory. A universal sentence of L is one of the form

$$\forall x_1 \forall x_2 \ldots \forall x_n \varphi(x_1, x_2, \ldots, x_n).$$

Here φ is a formula of L containing at most the variables $x_1, x_2, \ldots, x_n$, and containing no quantifiers. If X is a class of groups, then the universal theory of X is the set of all universal sentences of L that are true in every group in X. In this project, we showed that if the universal theory of the torsion groups coincides with the universal theory of the finite groups (by the way, it is clear that the universal theory of the torsion groups is contained in the universal theory of the finite groups), then there can not exit a finitely presented infinite group of any finite exponent. (Recall that the exponent of a group (if it exists) is the least positive integer n such that $x^n = 1$ for all x in the group.) We can not say anything yet about the converse. Also we still don't know whether or not such groups exist. Thus this project is also still on going.

Updown Categories

Researcher: Professor Michael E. Hoffman

In earlier work on the Hopf algebra of rooted trees introduced by Dirk Kreimer to provide an algebraic framework for renormalization in quantum field theory, the researcher introduced a particular combinatorial structure on the partially ordered set (poset) of rooted trees. He later noticed that the same type of structure can be given to many other posets of interest in combinatorics, including subsets of a finite set, polynomials in a finite set of variables, necklaces, integer partitions, integer compositions, and planar rooted trees. Further, he noticed that category theory could be used to give a succinct definition of this structure, leading him to define an "updown category." This definition generalizes combinatorial structures defined earlier, including R. Stanley's notion of a differential poset. The researcher set out to develop a general theory of updown categories, with the objective of extending and unifying earlier results.

The researcher has been able to define a number of interesting constructions for updown categories, including a definition of complementation, a construction of universal covers, and two types of generating functions. Any updown category carries a pair of linear operators D and U on an associated graded vector space, and conditions on the commutator of D and U imply various results about the updown category. The researcher worked out twelve examples of updown categories in some detail.

A paper including the results mentioned above has been submitted for publication, and research on properties of updown categories continues.

A Question about *Pic(X)* as a *G*-module

Researchers: Professor W. David Joyner,
Daniel Goldstein, Center for Communications Research, San Diego, CA, and
Robert Guralnick, University of Southern California

Let G be a finite group acting faithfully on an irreducible non-singular projective curve. Does every G-invariant divisor class contain a G-invariant divisor? We show that the answer depends only on G and not on the curve. We addressed the same question for degree-0 divisor (classes). This work has been accepted for publication.

Decomposing Representations of Finite Groups on Riemann-Roch Spaces

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir

We studied the action of a finite group on the Riemann-Roch space of certain divisors on a curve. If G is a finite subgroup of the automorphism group of a projective curve X and D is a divisor on X left stable by G then we show the natural representation of G on the Riemann-Roch space $L(D)=L_{X(D)}$ is a direct sum of irreducible representations of dimension m_i, where m_i is determined explicitly in many cases. (What the cases are and what the value of m_i is are rather technical, and therefore is not provided here) This work has been submitted for publication, and the article is in revision.

Conjectural Permutation Decoding of Some AG codes

Researcher: Professor W. David Joyner

We studied the action of a finite group on the Riemann-Roch space of certain divisors on a specific hyperelliptic curve X defined over a finite field with ``large'' automorphism group G. If D and $E = P_1+...+P_n$ are G-equivariant divisors on X ($P_i \in X(F)$) then G acts on associated AG code $C = C(D,E)$ by permuting coordinates. This note discusses the permutation decoding of these AG codes. The main "results" are conjectures regarding the complexity of the permutation decoding of these hyperelliptic codes. The open source GAP error-correcting codes package GUAVA is used to compute examples. This work has been accepted for publication.

Modular Representations on Some Riemann-Roch Spaces of Modular Curves X(N)

Researchers: Professor W. David Joyner and Associate Professor Amy E. Ksir

We computed the $PSL(2,N)$-module structure of the Riemann-Roch space $L(D)$, where D is an invariant non-special divisor on the modular curve $X(N)$, with $N \geq 7$ prime. This depends on a computation of the ramification module, which we give explicitly. These results hold for characteristic p if $X(N)$ has good reduction mod p and p does not divide the order of $PSL(2,N)$. We give as examples the cases $N = 7$ and 11, which were computed using GAP. Applications to AG codes associated to this curve are considered, and specific examples are computed using GAP and MAGMA. This work has been accepted for publication.

Frequentist Sequential Inference

Researcher: Professor Harold M. Kaplan

Frequentist statisticians are usually under the impression that they are using fixed-sample tests and fixed-sample confidence sets. In practice they often add more data to their samples as the data become available. This is unsound practice unless the decision boundaries are adjusted. Correct frequentist sequential methods include the Wald test, use of Doob's martingale inequality, and the Gambler's Ruin. In some ways they are easier to understand and use than fixed-sample methods. Their arithmetic is heavy, but computer programs written in, say, Java, can do it easily. An example is finding a one-sided non-parametric confidence set for the population mean of a non-negative population.

Infiqs

Researcher: Professor Harold M. Kaplan

The Infiqs freeware macro-expander is a very-old-style operator-precedence compiler for assignment statements. It is meant to help programmers writing Java programs that use the BigDecimal class. It allows them to use infix operators for the arithmetic of BigDecimal. Infiqs is written in Java. The length of the Infiqs.java file is a little under ten thousand bytes.

The Web site for Infiqs is www.toad.net/~jkaplan2/infiqs . There is a link to it at http://www.robert-tolksdorf.de/vmlanguages.html .

Intrinsic Knotting and Linking

Researchers: Professor Mark E. Kidwell, Associate Professor T.S. Michael and Brenda Johnson (Union College)

The complete graphs K_1, K_2, K_3, and K_4 can be embedded in a plane. K_5 is a minor minimal non-planar graph, but can be embedded in three-space with all cycles unknotted and unlinked. Sachs proved that K_6 is intrinsically linked: however it is embedded in three-space it contains a pair of linked cycles. Conway and Gordon proved that K_7 (with 21 edges) is intrinsically knotted. The researchers believe they have proved that any intrinsically knotted graph must contain at least 21 edges.

Classification of Singularities with SHEEP/CLASSI

Researcher: Professor Deborah A. Konkowski

SHEEP and CLASSI - two algebraic computation programs for general relativity - were used to help classify spacetimes with singularities. During the summer breaks, the researcher has visited Queen Mary, University of London, which is known for its developmental work of both SHEEP and CLASSI. She has been studying the utility of these programs in classifying spacetimes with curvature singularities. During the past two summers she has looked at spacetimes with quantum singularities and used SHEEP/CLASSI to describe their classical curvature singularities. The analyzed spacetimes include Levi-Civita and cylindrical spacetimes that reduce to Levi-Civita in some limits. The advantages and disadvantages of using SHEEP/CLASSI in place of MAPLE have been noted. The investigator also helped a colleague at Harvey Mudd College use all three algebraic programs for singularity classification. The results of these computations have been used in their collaborative work on quantum singularities that is still in development. In particular, they have studied the computation of the 16 curvature scalars of general relativity for these spacetimes.

Competition in Two-Dimensional Heterogeneous Environments

Researchers: Assistant Professor Aurelia Minut and Daniel Kern (University of Nevada, Las Vegas)

Competition in a heterogeneous environment was studied in two spatial dimensions. We examined the case of two competing populations (one native, one invasive) expanding in a random manner through a domain alternating between two patch types. We focus on a small population of the invasive species being introduced in a limited portion of the environment. We have the model equations and among the main goals are: examining boundaries, especially between patch type condition(s) for successful invasion - stability analysis traveling waves and wave propagation speed directions for more realistic models.

Investigations into M&m Sequences

Researchers: Professor Mark E. Kidwell and Professor Mark D. Meyerson

This project is an investigation of the properties of a certain class of sequences defined by letting the next term be such that the new mean equals the previous median.

Modeling Gas Diffusion in Nanocomposites

Researchers: Assistant Professor Anastasios Liakos
C. Swannack (Massachusetts Institute of Technology),
and C.L. Cox and D. Hirt (Clemson University)

The food packaging industry is continually looking for ways to increase the shelf life of organic products. Product freshness can be maintained with the use of films that exhibit low oxygen permeability. Examples of such films are polymer-silicate layered nanocomposites (PSLN's). PSLNs are a class of reinforced plastic formed by dispersing nano-sized clay particles throughout a polymer matrix. In addition to their low oxygen permeation properties, PSLNs also promise stronger packaging films because of their structures. The term "nanocomposite" describes a two-phase material where one of the phases is dispersed in the second one on a nanometer ($10^{-9} m$) scale. In this work, we present a new model for the estimation of the average diffusion coefficient of a nanocomposite film.

Do Differential Equations Swing?

Researcher: Professor Richard F. Maruszewski

This work is a look at forcing functions and resonance using both analytic techniques and MAPLE. The example of the playground swing is used to illustrate the ideas. The results of this project have been submitted for publication.

Explicit Construction of Complete Kahler Metrics of Saper Type by Desingularization

Researchers: Associate Professor Caroline G. Melles
and Dr. Pierre Milman (University of Toronto)

This project is a continuation of earlier work. The authors have added a section to this paper, showing that their local metrics satisfy a boundedness condition of Ohsawa. This result is a further step towards their goal of showing that the L2-cohomology of these metrics is isomorphic to the intersection cohomology of Goresky-MacPherson. The authors plan to resubmit this paper for publication in the near future.

Ryser's Embedding Problem for Hadamard Matrices

Researcher: Associate Professor T. S. Michael

The influential combinatorialist H. J. Ryser asked for the smallest order of a Hadamard matrix that contains a given r-by-s matrix of all 1's. We obtain the strongest upper and lower bounds on the order of such a Hadamard matrix.

Grappling with Mathematics: A New Method to Distribute Byes at the Eastern Intercollegiate Wrestling Association Championship Tournament

Researcher: Associate Professor T. S. Michael

The Naval Academy wrestling team is a member of the EIWA (Eastern Intercollegiate Wrestling Association), which has held a championship tournament annually for a century to determine qualifiers for the national championship wrestling meet. Because the number of teams in the EIWA (13) is not a power of 2, byes must be distributed to individuals in the opening round of the tournament. The byes are distributed randomly in each of the ten weight classes. In some years the random distribution happens to place some teams at a disadvantage as far as winning the Eastern team title. We propose and study a new method to distribute byes that treats the teams more fairly, while preserving fairness to the individuals, too.

Sphericity of Graphs

Researchers: Associate Professor T. S. Michael and Thomas Quint (University of Nevada, Reno)

The sphericity of a given graph G is the minimum dimension d such that G can be represented as the intersection graph of a family of unit spheres in d-dimensional space. We are developing new upper and lower bounds for the sphericity of G in terms of the edge-clique cover number of G.

Knots in Graphs

Researchers: Associate Professor T. S. Michael, Professor Mark E. Kidwell
and Brenda Johnson (Union College)

We seek to prove that no graph with fewer than 15 edges is intrinsically knotted.

Diagonally Dominant Matrices and the Ranking of Bridge Players

Researchers: Associate Professor T.S. Michael and Thomas Quint (University of Nevada, Reno)

The current method of rating duplicate bridge players in the United States is unsatisfactory because it is cumulative; a player's rating never goes down, even after an extensive period of poor performance. We propose a new method that uses recent performances and accounts for varying strengths of a player's partners. Our method is based on a least squares estimate and reduces to solving a linear system of equations. Singularity of the system turns out to be an entirely combinatorial matter. We appeal to deep theorems in matrix theory to explain the behavior of our proposed rating method.

Even Order Tournament Matrices with Minimal Rank over Finite Fields

Researchers: Associate Professor T. S. Michael,
Bryan Shader (University of Wyoming), and Liz Boyer (Lyon College)

In a series of paper by de Caen and Michael in the 1990's a lower bound for the rank of a tournament matrix over a finite field was obtained, and the case of equality was characterized. The order of such a rank-minimizing tournament matrix must be odd. We have obtained a similar but more complicated characterization for tournament matrices of even order with minimum rank.

Bresenham's Algorithm and Quadratic Reciprocity

Researcher: Associate Professor T. S. Michael

Bresenham's algorithm determines which pixels to darken on a computer monitor to represent a straight line segment. The algorithm is noteworthy because it relies only on integer arithmetic and can thus be carried out efficiently. We have discovered a surprising relationship between Bresenham's algorithm and the famous Gauss-Eisenstein proof of the quadratic reciprocity law in number theory. We seek a unified treatment of these two disparate results.

Orthogonal Matrices with Zero Diagonal, Revisited

Researcher: Associate Professor T.S. Michael

Two classical papers in design theory focus on combinatorial properties of orthogonal matrices whose diagonal entries are all 0's and whose off-diagonal entries are +1 or -1. We show how many of the basic theorems in those papers can be obtained without the high-powered algebraic machinery typically used.

Nonlinear Interaction of Light with Liquid Crystals

Researcher: Assistant Professor Aurelia Minut

In this research, we propose a new model of Partial Differential Equations (PDE's) in the field of nonlinear optics of liquid crystals to model the evolution of nematic liquid crystals in the presence of an electromagnetic field. The current research in the field of liquid crystals uses the Oseen-Frank model, the Ericksen model or the Ericksen-Leslie theory and no electromagnetic fields are present. There are indications from this research that the Oseen-Frank theory needs modifications in order to fit real physical phenomena. We provide modifications by coupling this equation with Maxwell's Equations.

One particular application we have in mind is to use this theory to model the propagation of light through liquid crystals and compare it with the propagation of light through optical fibers. We are currently writing the system of equations and then we will study them.

Arc Length and Pythagorean Triples

Researcher: Associate Professor Courtney Moen

In this pedagogical note, we show that, surprisingly, infinitely many of the curves in the family given by the parametric equations $x = t^i$, $y = t^{(i+1)}$, for t between 0 and 1, have rational length. The proof uses the fact that there are infinitely many Pythagorean triples of the form $(x, x+1, z)$.

On a New Notion of Ellipticity

Researchers: Assistant Professor Christopher W. Brown (USNA Computer Science Department) and Professor George C. Nakos

In this work, we generalize the notion of the Douglis-Nirenberg ellipticity (DN-ellipticity) for square systems of partial differential equations. DN-ellipticity was introduced in 1955 for the following reason: under the new notion ellipticity can be preserved when a higher order elliptic equation or system reduces to a lower order one. Unfortunately, DN-ellipticity involves the choice of a weight system that makes the definition noninvariant under singular transformations. This lack of good functoriality properties of DN-ellipticity is a serious obstacle in developing properties of solutions for such systems. In this research we propose a new notion of ellipticity that has all the desired properties and includes all DN-elliptic systems both square and rectangular and does not involve any choice of weights. The research on this project is nearly complete, and writing of a manuscript has begun.

Modern Linear Algebra and Applications

Researcher: Professor George C. Nakos

The first draft of the manuscript for this text was completed in July 2004, and the timeline for publication of this text (550 pages) has been moved up by two years. As a result of this acceleration in the publication date, substantial time has been devoted to revisions this year. The manuscript is a thorough treatment of elementary linear algebra, which includes linear systems of differential equations with constant coefficients. It covers the following topics: Linear systems, vectors, matrices, vector spaces and linear transformations, determinants, eigenvalues, orthogonality, systems of differential equations, orthogonalization of symmetric matrices, and numerical linear algebra (iterative methods and numerical approximations of eigenvalues). The second draft of the entire manuscript is about 80% complete.

Instructor's Solutions Manual for Modern Linear Algebra and Applications and Student's Manual for Modern Linear Algebra and Applications

Researcher: Professor George C. Nakos

These two supplements, in preparation, are the complete solutions manual for the instructor and a student's solution manual of the above text.

Mathematical Foundations for Statistical Learning

Researchers: Associate Professor John F. Pierce and Associate Professor Gary O. Fowler

In the interaction with empirical researchers on matters of forecasting change in oceanographic features in the Chesapeake Bay, the lack of information on the protocol by which one could validate data products, verify their prediction, and assess bounds on the level of confidence of decisions based upon them was of interest. In addition, the researchers were intrigued by the lack of information on the reliability of methods of forecasting change in the values of estimators, such as Monte Carlo methods of simulation, or by using trained "machines", such as neural nets or Kalman filters.

Recently, there has been a bloom of intellectual through in mathematics on the rational foundation of this form of empirical reasoning, which is called "statistical learning". The bloom is due the translation into English and of the provocative work of V. Vapnik, (1991), and its amplification. His work inspired seminal papers by S. Smale et al (2002, 2004, 2005) that link Vapnik's perspective on learning and a-priori bounds on convergence to a-priori bounds functional analysis. Papers by R. Devore et al (2003, 2004, 2005) expand Smale's lines of reasoning to include links to functional approximation theory that specify more subtle bounds on rates of convergence for principles of learning, upon which design of learning machines are based.

We are undertaking the task of comprehending these papers and creating a suite of empirical estimators for features of the Chesapeake Bay that would concretely realize the models developed by S. Smale, R. Devore et al. We anticipate the exercise will highlight how one might assess the validity and value of an estimator and its forecast, in a manner applicable to a project like the Chesapeake Bay enterprise.

Custom-Built Moments for Edge Location

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers

We present a general construction of functions whose moments serve to locate and parametrize step edges within an image. A characteristic of the moment approach to edge location is that the result is specified parametrically rather than as a set of pixels; thus our method performs best on sharp, straight edges. Previous use of moments to locate edges was limited to functions supported on a circular region, but our method allows the use of ``custom-designed" functions whose support is circular, rectangular, or any desired shape, and with graphs whose shape may be chosen with great freedom. We present analyses of the sensitivity of our method to pixelization errors or discrepancy between the image and an idealized edge model. The parametric edge description provided by our method makes it especially suitable as a component of wedgelet image coding. A paper describing this work has been submitted to *IEEE Transactions on Pattern Analysis and Machine Intelligence*, October 2004.

Locating Edges and Removing Artifacts in JPEG Images by Frequency-domain Analysis

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers

We present a method of removing, in linear time, image compression artifacts. Our technique is tailored to remove spurious oscillations in the vicinity of major edges at low-bit rate JPEG compression caused by large-bin quantization of the high frequencies. At the heart of the technique is a moment-based, efficient identification of edges and edge parameters (of the original image) directly in frequency domain. Once an edge is identified, we substitute the de-quantization and inverse Fourier transform for that block with pixel values derived from the parameters of the edge. The technique is stable, computationally fast, requires limited memory, and can be parallelized.

Using Off-the-shelf Games for Analysis in the Classroom

Researcher: Professor Thomas J. Sanders

The researcher continues to seek ways of incorporating off-the-shelf games into classroom instruction. He reviewed the wargame named *Empire Earth*, and although he could think of ways to use the game for analysis and optimization of force structure, it does not appear to be a good vehicle for use in the analysis of tactics. The tactics learning tends to be in how to beat the game, not in good military strategies. The investigator expects to review the game *Dangerous Waters* next. This game is also being reviewed in the Naval Academy's Professional Development Division for possible use in helping to teach tactics to the midshipmen.

Representations of Finite Groups on Riemann-Roch Spaces

Researchers: Associate Professor William N. TRAVES and Professor W. David Joyner

Let X be a curve defined over a finite field k and let G be a finite group of automorphisms of X. Suppose that D is a divisor on X that is stable under the action of G. Then G acts on the Riemann-Roch space L(D). We investigated the structure of L(D) as a G-module and discussed an application to coding theory. We are currently investigating a conjecture that relates to permutation decoding and might produce codes with excellent parameters and coding-decoding properties.

Counting Conics

Researchers: Associate Professor William N. Traves, Assistant Professor Amy E. Ksir
and Midshipman 1/C Andrew C. Bashelor, USN

This investigation considers a classical problem in enumerative geometry that asks for the number of plane conics tangent to five fixed conics. Steiner gave an incorrect answer in the mid-nineteenth century. We developed the tools of algebraic geometry necessary to correctly answer the enumerative question. Along the way, we treated the Chow ring, the blow-up operation and several simpler problems involving conics tangent to lines. This work is based on Midshipman Bashelor's Trident Scholar project.

Quadratic Forms and Cohomology

Researcher: Associate Professor JoAnn S. Turisco

In the project, cohomology theory is applied to the study of quadratic forms. The relatively new methods of "motivic" homotopy, developed by V. Voevodsky, (for which he received the Fields Medal in 2002) has lead to many advances in the algebraic theory of quadratic forms. This theory in algebraic geometry is similar to homotopy theory in algebraic topology. The motivic homotopy category provides a motivic cohomology, K-theory, and algebraic cobordism.

One part of this project is to apply the new methods of motivic homotopy theory to quadratic forms. Another aspect of it is to use the more established methods using Galois cohomology. There are many open problems in this area, *e.g.,* classification of forms, computations of invariants. A third part of the study involves the application of these methods to systems of quadratic forms.

Flat Earth Guide to Probability and Statistics

Researcher: Professor John C. Turner

Students in introductory probability and Statistics courses have difficulty that stems from their lack of familiarity with the process of mathematical modeling. The investigator is developing a presentation of both probability and statistics that makes clear how the principles of mathematical modeling apply in these fields. This includes (1) clearly stating assumptions, (2) asserting why these assumptions are appropriate in this particular case, (3) determine at least the direction of the impact of violations of the assumptions, (4) closing the feedback loop between results and further investigations.

Application of High-Order Higdon Non-Reflecting Boundary Conditions to Linear Shallow Water Models

Researcher: Commander V. Van Joolen, USN

In the late 1980s, Robert Higdon proposed a sequence of high-order non-reflecting boundary conditions (NRBCs) as a means to artificially truncate a very large domain. Due to algebaic complexity he limited his discussion to orders up to three. In 2002 Givoli/Neta devised a scheme that showed that Higdon's NRBCs could be successfully employed for higher orders. They demonstrated a 90% reduction in reflection, however computational constraints limited their scheme to orders of nine. Here the Givoli/Neta scheme as refined and Higdon NRBCs with orders greater 9 are successfully employed. An additional four-fold decrease in reflection is achieved. A shallow water model with linear time-dependent dispersive waves propagating in a semi-infinite channel truncated by an artificial boundary is considered. A numerical example is presented with finite difference schemes used throughout. The results of this study have been submitted for publication.

A Fast Method for Determining Taylor Series Weights for Finite Difference Approximations of Derivatives on Uniform Grids

Researcher: Commander V. Van Joolen, USN

Finite difference approximations for derivatives provide an important means for numerically solving systems modeled with ordinary and partial differential equations. Taylor series polynomials are a major conveyance by which finite difference expressions are generated. Though basic expressions for the lower derivatives are well known, easily derived, or readily available in mathematical tables, expressions involving higher order derivatives or that sample a greater number of grid points are more difficult to come by. Here a scheme is proposed that will allow the reader to theoretically calculate finite difference weights to approximate any derivative using any combination of grid points. The results of this study have been submitted for publication.

Matrices over Communicative Rings

Researcher: Professor William P. Wardlaw

For years the researcher has been interested in matrices over commutative rings. He has continued his research into such matrices with special attention given to (1) the various definitions of the rank of a matrix over a commutative ring and (2) the characterization of the classical adjoint (adjugate) of a matrix over a commutative ring. So far no publishable results have been obtained.

Trident Scholar Projects

Enumerative Algebraic Geometry: Counting Conics

Researcher: Midshipman 1/C Andrew C. Bashelor, USN
Advisers: Assistant Professor Amy E. Ksir and Associate Professor William N. Traves

Projectiles follow parabolic paths and planets move in elliptical orbits. Circles, hyperbolas, parabolas and ellipses are curves that are so abundant in nature, engineering, and art that one cannot help but notice them. Each of these curves is an example of a conic. In 1848, the mathematician Jacob Steiner posed a famous question: “How many conics are tangent to five fixed conics?” Steiner claimed to have solved the problem and he gave the answer “7776”. This solution was accepted as valid for sixteen years. When the problem was revisited in 1864, the mathematician Michel Chasles realized that Steiner had miscounted the true number of conics that satisfied the conditions.

Not all conics are smooth plane curves. Singular conics are curves whose defining polynomials are reducible to the product of two linear factors. These conics can be represented as either a pair of crossed lines or a

line of multiplicity two. Steiner failed to account for the degenerate conics that can be represented as a double line. He had fallen victim to what algebraic geometers call excess intersection. This Trident project centered on understanding how excess intersection affects problems of enumeration involving plane conics.

This research focused on finding the solutions to twenty-one variations of Steiner's problem. These problems were solved by examining the *blowup* of the space of conics along the set of double lines and executing computations in what is known as the *Chow Ring*. These methods provided not only tangible numerical results but also helped to illuminate the rich underlying geometry of these fundamental problems.

Trajectory and Invariant Manifold Computation for Flows in the Chesapeake Bay

Researcher: Midshipman 1/C Nathan F. Brasher, USN
Advisers: Professor Reza Malek-Madani and Associate Professor Gary O. Fowler

The field of mathematics known as dynamical systems theory has seen great progress in recent years. A number of techniques have been developed for computation of dynamical systems structures based on a data set of a given flow, specifically distinguished hyperbolic trajectories (DHTs) and their invariant manifolds.

In this project, algorithms in MATLAB® have been successfully implemented and applied to a number of test problems, as well as to the Chesapeake Bay flow data generated by the QUODDY shallow-water finite-element model. A number of interesting discoveries have been made including instabilities of convergence of the DHT algorithm and evidence of lobe dynamics in the Chesapeake.

Additionally, MATLAB® code has been developed to compute Synoptic Lagrangian Maps (SLMs). When applied to an oceanographic flow, SLMs produce plots of the time that it takes particles in various regions to encounter the coast or escape to the open ocean. Such maps are of interest to the oceanography community. A new algorithm for SLM computation has been developed resulting in orders of magnitude increase in efficiency. Previously SLM computation for a week of flow data was a problem limited to massively parallel supercomputers. With the new algorithm, similar data is computed in a few days on a single processor machine.

The development of platform-independent MATLAB® implementation of the algorithms for computation of DHTs, invariant manifolds and SLMs should prove valuable tools for studying the dynamics of complex oceanographic flows.

Parallel Integer Factorization Using Quadratic Forms

Researcher: Midshipman 1/C Stephen S. McMath, USN
Advisers: Professor W. David Joyner
and Assistant Professor Frederick L. Crabbe (USNA Computer Science Department)

This research analyzed algorithms for integer factorization based on continued fractions and binary quadratic forms, focusing on runtime analysis and comparison of parallel implementations of the algorithm. In the process it proved several valuable results about continued fractions.

Factorization is important for both practical and theoretical reasons. In secure digital communication, security of the commonly used RSA public key cryptosystem depends on the difficulty of factoring large integers. In number theory, factoring is of fundamental importance.

In 1975, Daniel Shanks used class group infrastructure to modify the Morrison-Brillhart algorithm and develop Square Forms Factorization, but he never published his work on this algorithm or provided a proof that it works. This research began by analyzing Square Forms Factorization, formalizing and proving the premises on which the algorithm is based and considering several variations, including two different parallel implementations, one of which was considered by Shanks and one of which is original. The results demonstrate that the new implementation, which utilizes composition of quadratic forms, is slower for small numbers of processors, but is more efficient asymptotically as the number of processors grows.

Shanks' Square Forms Factorization, including a concept he called Fast Return, has been implemented in C and Magma and some experimental runtime analysis has been done. A parallel version in C has been implemented and tested extensively.

Midshipman Research Course Projects

Modeling the Effects of Anti-terrorism Policy on the Actions of Terrorists

Researcher: Midshipman 1/C Peter Barkley, USN
Advisers: Professor Thomas J. Sanders
and Assistant Professor Pamela M. Schmitt (USNA Economics Department)

In a world where the U.S. government and its military are increasingly threatened by transnational terrorism, sound antiterrorism policy is imperative. Various policies must be empirically compared and knowledgeably selected in order to maximize our national security at minimal cost in both lives and money. The intention of this project is to bridge a gap in existing literature on mathematical models of antiterrorism policy, linking together several existing models in a larger composition model. The composition model will join several game theoretic models of interaction between terrorist groups and nation-states to models that describe the effect antiterrorism policy choices upon the composition of the terrorist groups themselves.

To begin the development of this model, a study of models described by Todd Sandler and Daniel Arce (2004), Mark Lichbach (1991), Walter Enders and Todd Sandler (2004), Franz Dietrich (2004), and Frey and Luechinger (2003) has been initiated. Once a working composite model is developed, data that is used in these models will be accumulated in order to generate a baseline case study on current US antiterrorism policy. Much of this data is available in several established terrorism databases such as the MIPT Terrorism Knowledge Base, the RAND-St. Andrews University Databases on Terrorism and Low-Intensity Conflict, ITERATE 3 (International Terrorism: Attributes of Terrorist Events, 1978-2002), and the U. S. State Department's Pattern of Global Terrorism, as well as the journals Terrorism and Political Violence and Studies in Conflict and Terrorism. The model would then be implemented to study the effect of various quantifiable policy indicators upon terrorists and their preferences and actions. Ideally, this implementation will be able to compute the stand-alone results of any of the models used as well as the composite, generating the equilibrium solutions as well as best and worst case scenarios and their relative probabilities.

To further verify the model, a proposed laboratory economics experiment in the same vein as a Prisoner's Dilemma game may be conducted. This game would test the payoff matrices associated with one of the game theoretic models or one of the model's assumptions about government-terrorism. While this is not necessary for the project, it would add to the value of the project by introducing some original data and empirically testing some of its assumptions.

Classification of Minority Group Stability

Researcher: Midshipman 1/C Patrick Brooks, USN
Adviser: Associate Professor Gary O. Fowler

Classification trees have the ability to "learn" by identifying similarities between identically classified items in a dataset. Then, by asking simple binary inequality questions, a tree can classify an unrelated set with a known accuracy. Using trees built with the Minorities at Risk (MAR) dataset, this paper attempts to replicate the work of political scientist Harff and Gurr, who classify the stability of minority groups by determining if a minority group will violently oppose its government. The building of the classification tree with the MAR data (referred to as the MAR tree) occurred by following the procedure outlined by Martinez [2002] to build, prune, and cross validate a tree with MATLAB®. Using this tree and procedure, the investigator calculated the percentage of misclassifications to exceed 36% with all values of alpha. The high misclassification rate and the shape of the final MAR tree provide evidence of the tree's inability to replicate the classification process utilized by professional political scientists. This inability occurs because the method employed by political scientists relies heavily upon the collection of background information to uniquely weigh each variable depending upon the minority group.

Unmanned Vehicle Search Planning Incorporating Communications Reliability

Researcher: Midshipman 1/C Matthew D. Collinsworth, USN
Advisers: Professor Charles Mylander and Lieutenant Commander Kyle Kliewer, USN
Sponsor: Mr. Jeffrey Crawford, Office of the Chief of Naval Operations, OPNAV N81

This research developed a nonlinear programming model for use in planning an open ocean search by an Unmanned Vehicle (UV) searching for a target known to be in a region. Impact of communications degradation between the UV and a mother ship processing the data gathered by the UV's sensor is accounted for in the model. The communications degradation varies in different parts of the region. The UV sensor is characterized in terms of sweep width. The model optimizes the allocation of search effort in the various parts of region. The objective is to maximize the probability of detection of the target in a fixed amount of time. The random search formula was adapted for use in the model. The model is implemented in a spreadsheet environment and Excel's Solver is used to solve the nonlinear program.

Simulation of a Sensor Field Localizing a Target

Researcher: Midshipman 1/C Matthew D. Collinsworth, USN
Advisers: Professor Charles Mylander and Lieutenant Commander Kyle Kliewer, USN

A model was coded in MATLAB® simulating the detection of a single stationary target in a field of omni-directional sensors. The model was build to study the performance of localization rules in the presents of false targets and serial correlation between glimpses by made the field of sensors. This problem was motivated by a paper by Bland et al (2003). In attempting to code the localization rules given in that paper it was decided they were flawed. New rules were developed and preliminary testing indicates they work well in localizing the target within a circle of a couple hundred meters radius.

A Polynomial-time Probabilistic Algorithm for the Minimum Distance of an Arbitrary Linear Non-binary Error-correcting Code

Researcher: Midshipman 1/C Jerry W. Irons, USN
Adviser: Professor W. David Joyner

The goal of the project was to examine a polynomial-time probabilistic algorithm to compute the minimum distance of an arbitrary, non-binary, linear error-correcting code. Only the binary case was done in Midn A. Foster's honors project. For the non-binary case a new function was needed in the GAP kernel. We extended Foster's work to the non-binary case but with a new feature making it faster. The paper ends with applications to the non-binary McEliece public key cryptosystem and trace codes.

List-Decoding of Generalized Reed-Solomon Codes Using Sudan's Algorithm

Researcher: Midshipman 1/C Clifton G. Lennon, USN
Adviser: Professor W. David Joyner

This project required writing a program to perform list-decoding of Reed-Solomon codes using the Sudan Algorithm and to implement this program into the GAP coding-theory package known as GUAVA.

Implementing Generalized Reed-Solomon Codes and a Cyclic Code Decoder in GUAVA

Researcher: Midshipman 1/C Jason M. McGowan, USN
Adviser: Professor W. David Joyner

This project involved:

(a) Implementing evaluation codes.
(b) Writing a decoding algorithm for Generalized Reed-Solomon codes.
(c) Writing a decoding algorithm for cyclic codes.
(d) Discuss the mathematics behind these algorithms.

Computer Classification of Surface Targets

Researcher: Midshipman 1/C Brian A. Ross, USN
Advisers: Professor Charles Mylander and Lieutenant Commander Kyle Kliewer, USN

Throughout the military, it is understood that there is significant value in minimizing distractions in combat. The purpose of this project was to create a computer program to classify surface targets using data gathered by Inverse Synthetic Aperture Radar (ISAR). A method for identifying the most likely surface ships being illuminated by ISAR and providing the operator with a measure of fit of potential targets is developed. Recursive use of Bayes Theorem improves the measure of fit when a sequence of glimpses of the target is made. A simulation of the procedure coded in MATLAB® demonstrates the impressive performance of the procedure.
This paper is being submitted for publication in Military Operations Research.

Classical and Quantum Singularities in a Cylindrically Symmetric Spacetime

Researcher: Midshipman 1/C Brian Ross, USN
Advisers: Professor Deborah A. Konkowski and Professor B. Mitchell Baker
Sponsor: Bowman Scholar Program

In this project, the singularity structure of a wiggly cosmic string was studied. It is represented by a cylindrically symmetric metric. It differs from an ordinary cosmic string because it has electromagnetic "wiggles" along its length.

Both the classical and quantum singularities in wiggly cosmic string spacetimes were studied. A classical singularity exits in a spacetime that is geodesically incomplete and cannot be extended. Classical singularities may be divided into three categories: quasiregular, nonscalar curvature and scalar curvature. Classical singularities are probed with classical point particles and thus one studies the behavior of classical timelike and null geodesics together with the curvature scalars of the spacetime. Quantum singularities are determined by the behavior of test quantum wave packets. A static spacetime with timelike singularity is quantum mechanically singular if the spatial portion of a wave operator is not essentially self-adjoint. One thus solves the wave equation and tests the operator for essential self-adjointness to determine if a spacetime is quantum mechanically singular.

Wiggly cosmic string spacetimes were found to have classical quasiregular singularities through an analysis of their topological properties and MAPLE calculations of curvature scalars. The spacetimes were also found to have generic quantum singularities. These were found by considering the behavior of scalar waves described by a Klein-Gordon wave equation and studying the essential self-adjointness of the wave operator using a well-known Weyl limit point – limit circle criterion.

The results of this research were presented by Midshipman Ross at the National Conference on Undergraduate Research (NCUR) at VMI in Virginia in April 2005. In addition, these results will be included in a more extensive paper on wiggly cosmic strings that will be posted on the gr-qc Web ArXiv and possibly submitted to a research journal.

Compactification of Plane Curves

Researcher: Midshipman 1/C Matthew Welch, USN
Adviser: Associate Professor William N. Traves

This project examined two different compactifications of the real plane: the one-point compactification (a sphere) and the real projective plane. The behavior of curves in the real plane at infinity is examined and criteria for smoothness are developed. The relationship between the sphere and the real projective plane is examined.

Professional Publications

Journal (Refereed) Manuscripts

ALEVRAS Alexis, Assistant Professor, "Cocycles for One-parameter Flows of B(H)", *Journal of Functional Analysis*, accepted for publication.

> The set of local cocycles is a natural invariant for an E_0-semigroup. It has a multiplicative structure, as well as a partial order structure among its positive elements. In particular, the unitary local cocycles form a topological group that may be appropriately viewed as the automorphism group of the E_0-semigroup, while the set of positive contractive local cocycles is order isomorphic to the set of flows of completely positive maps dominated by the semigroup.
>
> The local cocycles have been computed for the standard, type I examples of the CAR/CCR flows by W. Arveson and R. Bhat. In this paper we compute for the first time the local cocycles for a class of type II E_0-semigroups of B(H) with index zero, and describe the order structure as well as the multiplication in terms of the boundary weight associated with such a semigroup.

BUCHANAN, James L., Professor, Gilbert R. P., and Khashanah, K., "Determination of the *P*arameters of Cancellous Bone using Low Frequency Acoustic Measurements", *Journal of Computational Acoustics*, 12(2), pp. 99-126, June 2004.

> The Biot model is widely used to model poroelastic media. Several authors have studied its applicability to cancellous bone. In this article the feasibility of determining the Biot parameters of cancellous bone by acoustic interrogation using frequencies in the 5-15 kHz range is studied. It is found that the porosity of the specimen can be determined with a high degree of accuracy. The degree to which other parameters can be determined accurately depends upon porosity.

GAGLIONE, Anthony M., Professor (co-author), "The Axiomatic Closure of Discriminating Groups," *Archiv der Mathematik*, Vol. 83, pp. 106-112, 2004.

> We show that the class of squarelike groups is first-order axiomatizable while the class of discriminating groups is not. We then show that the class of squarelike groups is the least axiomatic class containing the class of discriminating groups, i.e. its axiomatic closure. This answers a previously open question of the authors.

Goodman, R., and KSIR**,** A.E., Assistant Professor, "The Mathematics of Fox Trot", *Math Horizons*, accepted for publication.

> *Fox Trot* is a syndicated comic strip appearing in newspapers across the country and around the globe. Its creator, Bill Amend, was a physics major in college and often includes mathematical references in the strips. We examine some of these references and conduct an interview with Amend.

HOFFMAN, Michael E., Professor, "The Hopf Algebra Structure of Multiple Harmonic Sums," *Nuclear Physics B (Proc. Suppl.),* Vol. 135, pp. 215-219, 2004.

Multiple harmonic sums appear in the perturbative computation of various quantities of interest in quantum field theory. In this article we introduce a class of Hopf algebras that describe the structure of such sums, and develop some of their properties that can be exploited in calculations.

JOYNER, W.D. Professor and KSIR, A.E. Assistant Professor, "Modular Representations on Some Riemann-Roch Spaces of Modular Curves X(N)," *Lecture Notes in Computing on Computational Aspects of Algebraic Curves*, accepted for publication.

We compute the PSL(2,N)-module structure of the Riemann-Roch space L(D), where D is an invariant non-special divisor on the modular curve X(N), with N > 5 prime. This depends on a computation of the ramification module, which we give explicitly. These results hold for characteristic p if X(N) has good reduction mod p and p does not divide the order of PSL(2,N). We give as examples the cases N=7, 11, which were also computed using GAP. Applications to AG codes associated to this curve are considered, and specific examples are computed using GAP and MAGMA.

KIDWELL, Mark E., Professor, "Two Types of Amphichiral Links," *Journal of Knot Theory and its Ramifications*, accepted for publication.

We classify two-component links as component preserving amphichiral (CPA) or component switching amphichiral (CSA). We use the two-variable Alexander polynomial to prove that CSA links with linking number a multiple of four do not exist, answering a question of Livingston. We give the first examples of CPA links with linking number 12, 20, or any number of the form four times an odd number.

LIAKOS, Anastasios, Assistant Professor, "Finite Element Approximation of Viscoelastic Fluid Flow with Slip Boundary Condition," *Computers and Mathematics with Applications,* accepted for publication.

We study the steady-state, isothermal flow of a viscoelastic fluid obeying an Oldroyd-type constitutive law with slip boundary condition. The slip boundary condition is appropriate for problems that involve free boundaries and other examples where the usual no-slip condition **u**=0 is not valid, such as fiber spinning and microfluidics.

First, we study the Newtonian problem with slip boundary condition where the viscoelastic stress is added into the list of unknowns. In addition, the normal elastic stress component associated with the slip boundary condition is introduced. In order to balance its effects, a second inf-sup condition is proven.

To treat the discrete case, we assume that the continuous solution of the Non-Newtonian problem exists and is small and smooth. Approximating the extra stress, velocity, pressure and normal elastic stress component via P_1 discontinuous, P_2 continuous, P_1 continuous and P_0 discontinuous elements respectively, yields a stable finite element scheme. Finally, via a fixed point argument, we establish the existence of an approximate solution and derive error estimates.

LIAKOS, Anastasios, Assistant Professor, "A Three-dimensional Simulation of Barrier Properties of Nanocomposite Films", *Journal of Membrane Science*, accepted for publication.

> Monte Carlo simulations are conducted in two and three dimensions to compute the diffusion coefficients of gas molecules permeating membranes containing oriented platelets. The effects of platelet aspect ratio, relative separation, loading and spatial dimension are examined. The model of Aris (1986) is shown to correlate well with a 2D Monte Carlo simulation. However, the Aris model, when compared to a 3D simulation, over-predicts the barrier-effect of filler, by 50% in some cases. Additionally, flaws in idealized geometries for 3D models are addressed and a new 3D geometry is presented. The results have particular value for nanocomposite films at low platelet loadings.

MARUSZEWSKI, Richard, Professor, and CAUDLE, Kyle, Lieutenant Commander USN, "Approximating Integrals Using Probability," *MACE Journal*, Volume 39, #2, pp 143-147, Spring, 2005.

> Monte Carlo techniques and expectations from probability are used to approximate the value of a definite integral. Programs in Visual Basic are presented to implement the algorithms.

MICHAEL, T. S., Associate Professor, and Quint, Thomas, "Optimal Strategies for Node Selection Games on Oriented Graphs: Skew Matrices and Symmetric Games", *Linear Algebra and Its Applications*, accepted for publication.

> The node selection game on an oriented graph D is a zero-sum, two person game. The two players simultaneously select a node from D, and there is a payoff from the dominated player to the dominating player. When D is a tournament (complete oriented graph), it is known that an optimal strategy is always unique. We prove that when the underlying undirected graph is bipartite (with at least two nodes in each part), then an optimal strategy is never unique.

MICHAEL, T. S., Associate Professor, "Ryser's Embedding Problem for Hadamard Matrices", *Journal of Combinatorial Designs,* accepted for publication..

> We obtain the strongest known upper and lower bounds for the minimum order of a Hadamard matrix that contains a given *r*-by-*s* submatrix of all 1's.

MINUT, Aurelia, Assistant Professor (co-author), "On the Regularity of Solutions to a Parabolic System related to Maxwell's Equations", *Journal of Math. Anal. Appl.*, Vol. 299, 2004, pp. 89-99.

> The goal of this paper is to establish Holder estimates for the solutions of a certain parabolic system related to Maxwell's equations arising in a quasi-stationary electromagnetic field.

Overby, J., Wojdylo, J., and TRAVES, W. N., Associate Professor, "On the Keyspace of the Hill Cipher", *Cryptologica*. Volume 29, Number 1, 2005, pp. 59 - 72.

> We count the number of invertible *d*-by-*d* matrices mod *n*, $|\mathrm{Gl}_d(Z_n)|$ and then apply the formula to the Hill Cipher. This paper is largely expository in nature and arises from Jeff Overby's undergraduate honors paper.

POPOVICI, Irina, Assistant Professor, and WITHERS, W. Douglas, Professor, "The Eidochromatic Transform for Color Image Coding," *IEEE Transactions on Image Processing*, 14 (2005), no. 3, pp. 334-342.

We introduce the *eidochromatic transform* as a tool for improved lossy coding of color images. Many current image-coding formats (such as JPEG 2000) utilize both a color-component transform (relating values of different image components at a single location) and a wavelet or other spatial transform (relating values of a single image component at proximate, but different image locations). The eidochromatic transform further reduces redundancy by relating image values simultaneously across color components and in the two spatial dimensions.

Our approach is to introduce an additional transform step following the color-component and spatial transforms. In tests, this step reduced the overall static entropy of the chrominance components of quantized transformed images by up to 40\% or more. Combined with JPEG 2000's modeling and coding method, the eidochromatic transform was found to reduce the size of lossily coded color images by up to 27% overall.

PRICE, Geoffrey L., Professor, "On Shifts of Minimal Index on the Hyperfinite II_1 Factor", *Operator Algebras, Quantization and Noncommutative Geometry*, American Mathematical Society Contemporary Mathematics, Vol. 365, 2004, pp. 299-314.

In this paper shifts are shown to exist on the hyperfinite II_1 factor of subfactor index 2 and commutant index 2 that are distinct from the binary shifts. The conjugacy classes and the Connes-Størmer entropy of a family of these shifts are determined.

Saito, Mutsumi and TRAVES, William N., Associate Professor, "Finite Generation of Rings of Differential Operators of Semigroup Algebras", *Journal of Algebra,* Volume 278, Number 1, August 2004, pp. 76 – 103.

This paper investigates the algebraic and combinatorial structure of rings of differential operators on toric varieties and semigroup algebras. We prove that the ring of differential operators on semigroup algebras is finitely generated precisely when the semigroup is scored, a combinatorial condition related to the Cohen-Macaulayness of the coordinate ring. We also prove that the ring of differential operators is Noetherian under certain conditions.

Swannack, C. H., LIAKOS, A., Assistant Professor**,** Cox, C. L., and Hirt, D. E., "A Three Dimensional Simulation of Barrier Properties of Nanocomposite Films", *Journal of Mathematical Fluid Mechanics,* accepted for publication.

Monte Carlo simulations were conducted in two and three dimensions to compute the diffusion coefficients for membranes containing oriented platelets. The effects of platelet aspect ratio, relative separation, loading and spatial dimension were studied. The model of Aris (1986) was found to accurately predict these effects in two dimensions. However, in 3D this model over-predicts the effects of the filler by 50% when compared to simulations. The results have particular value for nanocomposite film currently under development for use in food packaging applications.

VAN JOOLEN, V., Commander, USN, D. Givoli, and B. Neta, "High-Order-Like Boundary Conditions for Exterior Transient Wave Problems", *Int. J. Numer. Meth. Engng*, published online at: http://www3.interscience.wiley.com/cgi-bin/fulltext/110431948/PDFSTART.

> Recently developed non-reflecting boundary conditions (NRBCs) are applied for exterior time dependent wave problems in unbounded domains. The linear time-dependent wave equation, with or without a dispersive term, is considered in an infinite domain. The infinite domain is truncated via an artificial boundary *B*, and a high-order NRBC is imposed on *B*. Then the problem is solved numerically in the finite domain bounded by *B*. The new boundary scheme is based on a reformulation of the sequence of NRBCs proposed by Higdon. We consider here two reformulations: one that involves high-order derivatives with a special discretization scheme, and another that does not involve any high derivatives beyond second order. The latter formulation is made possible by introducing special auxiliary variables on *B*. In both formulations the new NRBCs can easily be used up to *any* desired order. They can be incorporated in a finite element or a finite difference scheme; in the present paper the latter is used. In contrast to previous papers using similar formulations, here the method is applied to a fully exterior two-dimensional problem, with a rectangular boundary. Potential difficulties with corner instability are shown not to arise. Numerical examples in infinite domains are used to demonstrate the performance and advantages of the new method. In the auxiliary variable formulation long-time corner instability is observed, that requires special treatment of the corners (not addressed in paper). No such difficulties arise in the high-derivative formulation.

Conference Proceedings

BROWN, Christopher W., Assistant Professor (USNA Computer Science Department), de VLAMING, Sjaak, Midshipman 1/C, NAKOS, George C., Professor, "Quantifier Elimination and the Ellipticity of Systems of Partial Differential Equations", Algorithmic Algebra and Logic 2005, *Proceedings of a Conference in Honor of the 60th Birthday of Volker Weispfenning*, April 3-6 2005, Passau, Germany.

> This work considers the problem of computing conditions on parameters and independent variables in a system of linear partial differential equations for the ellipticity of the system. When the coefficients of the system are algebraic or rational functions, real quantifier elimination can be used to compute such conditions. This work seeks to improve the effectiveness of this application of quantifier elimination.

BUCHANAN, James L., Professor, and Gilbert, R.P., "Measuring Osteoporosis using Ultrasound", Advances in Scattering and Biomedical Engineering, *Proceedings of the 6th International Workshop*, September 2003, D.I. Fotiadis and C.V Massalas, Eds., pp. 484-494, World Scientific.

> We simulated ultrasonic data to determine whether bones are osteoporotic. Ultrasound allows the clinician to measure various structural coefficients for bone in addition to testing for density. This fact alone means that ultrasound may lead to more accurate assessments of osteoporosis. With regard to density the ultrasound measurements are in general accurate.

CRAWFORD, Carol G., Professor, "6 Steps to Successful, Interactive Math WEB Design - An Innovative Real World Calculus Laboratory Designed with Java for the United States Naval Academy", *Proceedings of the Joint Meeting of the 3rd International Conference on Education and Information Systems, Technologies and Applications (EISTA 2005) and the International Conference on Cybernetics and Information Technologies, Systems and Applications (CITSA 2005)*, accepted for publication.

> The author presents "6 Steps" to help in the design of an effective, interactive mathematics resource for higher education. These steps reflect collaborative work with Dr. Mark Meyerson (USNA Mathematics Department) in the creation of an innovative, online Real World Calculus Laboratory for the 3-semester Calculus sequence taken by all midshipmen at The United States Naval Academy.

The Eisenhower National Clearinghouse recently chose the site as one of a dozen, exemplary websites for educators for November 2004. The ENC is funded by the United States Department of Education and administered by The Ohio State University. In this paper the author shares the design characteristics that contributed to the success of this online, learning tool for college and university calculus programs.

JOYNER**,** W.D. Professor, and KSIR, A.E. Assistant Professor, "Decomposing Representations of Finite Groups on Riemann-Roch Spaces," *Proceedings of the American Mathematical Society*, accepted for publication.

If G is a finite subgroup of the automorphism group of a projective curve X and D is a divisor on X stabilized by G, then under the assumption that D is nonspecial, we compute a simplified formula for the trace of the natural representation of G on the Riemann-Roch space L(D).

KONKOWSKI, Deborah A., Professor (co-author), "Mining Metrics for Buried Treasure," *Proceedings of Malcolm@60 Festschrift Conference:* to appear as a special volume of General Relativity and Gravitation, accepted for publication.

The same but different: That might describe two metrics. On the surface CLASSI may show two metrics are locally equivalent, but buried beneath one may be a wealth of further structure. This was beautifully described in a paper by M.A.H. MacCallum in 1998. In this paper, the author illustrates the effect with two flat metrics - one describing ordinary Minkowski spacetime and the other describing a three-parameter family of Gal'tsov-Letelier-Tod spacetimes. The researcher describes the beautiful hidden classical singularity structure of the latter (a structure first noticed by Tod in 1994) and then shows how quantum considerations can illuminate the riches. Finally, the researcher discusses how quantum structure can help us understand classical singularities and metric parameters in a variety of exact solutions mined from the Exact Solutions book.

KONKOWSKI, Deborah A., (co-author), "Classical and Quantum Singularity of Levi-Civita Spacetimes with and without Cosmological Constant," *Proceedings of Workshop on Dynamics and Thermodynamics of Black Holes and Naked Singularities*, Milan, Italy, accepted for publication.

Levi-Civita spacetimes have classical naked singularities. They also have quantum singularities. Quantum singularities in general relativistic spacetimes are determined by the behavior of quantum test particles. A static spacetime is said to be quantum mechanically singular if the spatial portion of the wave operator is not essentially self-adjoint on a C_0^∞ domain in L_2, a Hilbert space of square integrable functions. Here we summarize how Weyl's limit point-limit circle criterion can be used to determine whether a wave operator is essentially self-adjoint and how this test can then be applied to scalar wave packets in Levi-Civita spacetimes with and without a cosmological constant to help elucidate the physical properties of these spacetimes.

Technical Reports

D'ARCHANGELO, James, Professor, (co-author), "An Extended Stochastic Model for Synaptic Activity in Cultured Spinal Neurons", Naval Research Laboratory (NRL) Technical Memorandum Report, AIT-05-035.

> This paper describes the results of a biosensor modeling effort which was a joint collaboration between NRL Code 5583 and NRL 6920. The program objectives are to design, implement, and test realistic analytic models of both individual cultured living spinal cord neurons and of networks comprised of such cultured neurons. This report investigates the applicability of a stochastic model for synaptic activity, as an alternative or complementary approach to building large network models composed of discrete models of individual neurons.

MELLES, Caroline G., Associate Professor, "USNA Panel Discussion on Advanced Calculus and Fundamentals of Math Courses for Math Majors," *Mathematica Militaris,* Vol. 15, No. 1, pp. 18 – 24, Spring 2005.

> This paper reports on a USNA Math Department panel discussion on three courses formerly required of all math majors: Fundamentals of Math and Advanced Calculus I and II. Recent instructors commented on texts, goals of the courses and how well they were met, need for a bridge course, and whether all three courses should be required of all math majors. This discussion and others like it contributed to the design of the new two-track math major.

Books, Book Chapters, and Book Reviews

GAGLIONE, Anthony M., Professor, (coauthor), "Discriminating and Squarelike Groups I: Axiomatics," Group Theory, Statistics, and Cryptography, Editors: A.G. Myasnikov and V. Shpilrain, Contemporary Math. 360, 2004, 35-46.

> A group G is squarelike provided that the direct square G×G is universally equivalent to G. Every group which is discriminating in the sense of Baumslag, Myasnikov and Remeslennikov is squarelike but not necessarily conversely. The class of squarelike groups is axiomatic and is the closure under elementary equivalence of the class of discriminating groups. This paper shows that the theory of squarelike groups is neither decidable nor finitely axiomatizable. Further we exhibit a set of axioms due to O. Blegradek and V.H. Dyson for the class of squarelike groups.

HOFFMAN, Michael E., Professor, "Algebraic Aspects of Multiple Zeta Values," in Zeta Functions, Topology and Quantum Physics, Developments in Mathematics, Vol. 14, T. Aoki *et al.* (eds.), Springer-Verlag, New York, 2005, pp. 51-74.

> Multiple zeta values have been studied by a wide variety of methods. In this article we summarize some of the results about them that can be obtained by an algebraic approach. This involves "coding" the multiple zeta values by monomials in two noncommuting variables x and y. Multiple zeta values can then be thought of as defining a map $\zeta: H^0 \rightarrow \mathbf{R}$ from a graded rational vector space H^0 generated by the "admissible words" of the noncommutative polynomial algebra $\mathbf{Q}\langle x,y\rangle$. Now H^0 admits two (commutative) products making ζ a homomorphism—the shuffle product and the "harmonic" product. The latter makes H^0 a subalgebra of the algebra QSym of quasi-symmetric functions. We also discuss some results about multiple zeta values that can be stated in terms of derivations and cyclic derivations of $\mathbf{Q}\langle x,y\rangle$, and we define an action of QSym on $\mathbf{Q}\langle x,y\rangle$ that appears useful. Finally, we apply the algebraic approach to relations of finite partial sums of multiple zeta series.

MCCOY, Peter A. Professor, book review, Sampling, Wavelets and Tomography, John Benedetto and Ahmed Zayed (Editors), Applied and Numerical Analysis Series, Birkhauser, Boston, 2004, ISBN 0-8176-4303-4.

Reviewed the subject volume for the American Mathematical Reviews AMS MR 2-037-639.

Mathematical Monthly and Mathematics Problems

LOCKHART, JODY, Associate Professor, "Solution to Problem 11001", American Mathematical Monthly, Volume 111, Number 10, December 2004, p.920.

WARDLAW, William P., Professor, Solution to Problem 179, *Math Horizons*, Vol. XII, No. 1, September 2004, p. 33.

Solves Problem 179 by calculating the probability that two randomly chosen $n \times n$ matrices over the finite field F(q) with q elements are inverses of each other.

WARDLAW, William P., Professor, Problem 189, *Math Horizons*, Vol. XII, No. 2, November 2004, p. 32.

This problem wants a proof of two compound matrix (a matrix of matrices) determinant identities.

WARDLAW, William P., Professor, Problem 793, *College Mathematics Journal*, Vol.36, No. 1, January 2005, p. 74.

Problem 793 asks: When does conjugation by an element of the symmetric group result in an inner automorphism on the alternating subgroup?

WARDLAW, William P., Professor, Problem 1712, *Mathematics Magazine*, Vol. 78, No. 1, February 2005, p. 68.

Problem 1712 asks for the sum S(m) and for the product P(m) of all elements in the multiplicative subgroup of units in the ring of integers modulo m.

WARDLAW, William P., Professor, Problem 798, *College Mathematics Journal*, Vol.36, No. 2, March 2005, p. 161.

Problem 798 asks when the characteristic polynomial of a matrix A determines the rank of A.

WARDLAW, William P., Professor, Quickie Q950 and Answer A950, *Mathematics Magazine*, Vol. 78, No. 2, April 2005, p. 159 and p. 164.

Quickie Q950 asks for a proof that the classical adjoint of a matrix cannot have $x^3 - x$ as its characteristic polynomial. A950 gives the requested proof.

WARDLAW, William P., Professor, Problem 802, *College Mathematics Journal*, Vol.36, No. 3, May 2005, p. 239.

Let n and k be fixed positive integers, and let A and B be randomly and independently chosen $n \times n$ matrices whose entries are integers modulo k. Problem 802 asks for the probability that $AB = I$.

WARDLAW, William P., Professor, "A Short Proof that Row Rank is Equal to Column Rank", *Mathematics Magazine*, Vol. 78, No. 3, June 2005.

This note gives a short (perhaps the shortest) proof that row rank is equal to column rank and continues to derive some properties of matrix rank.

Presentations at Professional Meetings and Conferences

ALEVRAS, Alexis, Assistant Professor, "Cocycles for One-parameter Flows of B(H)", Great Plains Operator Theory Symposium, University of Central Florida, 7 – 12 June 2005.

BUCHANAN, James L., Professor, "Determination of the Parameters of a Poroelastic Seabed", Naval Research Laboratory, Washington, D.C., 3 August 2005.

FOWLER, Gary O., "A.A. Michelson's Measurement of the Speed of Light 1879", MAA/AMS Joint Mathematics Meetings, Atlanta, GA, 8 January 2005.

GAGLIONE, Anthony M., Professor, "On the Existence of Finitely Presented Infinite Torsion Groups and a Question of V.H. Dyson," Conference in Honor of Gerhard Rosenberg's 60th Birthday, Fairfield Univ., Fairfield, CT, 10 December 2004.

GAGLIONE, Anthony M. Professor, "On Groups Universally Equivalent to Free Nilpotent Groups," special session of CMS, Montreal, Canada, 12 December 2004.

HANNA, Charles C., Professor, "Thirteen Ways to Compute Derivatives on the Voyage 200," Teachers Teaching with Technology International Conference, Washington, D.C., 19 March 2005.

HOFFMAN, Michael E., Professor, "Higher Mathematics Meets High-Energy Physics," Sigma Xi Luncheon talk, U.S. Naval Academy, Annapolis, MD, 23 September 2004.

HOFFMAN, Michael E., Professor, "Updown Categories and their Generating Functions," Mathematics Colloquium, U.S. Naval Academy, Annapolis, MD, 10 February 2005.

KAPLAN, Harold M., Professor, "Kolmogorov-Smirnov-type Confidence for Finite Intervals", Mid-Atlantic Regional Probability and Statistics Day, The Johns Hopkins University, Applied Physics Laboratory, Laurel, MD, 13 November 2004.

KIDWELL, Mark E., Professor, "Intrinsic Knotting and Linking", Mathematics Colloquium, U.S. Naval Academy, Annapolis, MD, September 2004.

KLIEWER, Kyle, Lieutenant Commander, and Mylander, W. Charles, Professor, "The State of OR at the Naval Academy", Military Operations Research Society Educational and Professional Development Colloquium, U.S. Air Force Academy, Colorado Springs, CO, 5-6 April 2005.

KONKOWSKI, Deborah A., Professor, "Mining Metrics for Buried Treasure," Malcolm@60 Festschrift Conference, London, England, 17 July 2004.

KONKOWSKI, Deborah A., Professor, "Are Classically Singular Spacetimes Quantum Mechanically Singular As Well?", 17th International Conference on General Relativity and Gravitation, Dublin, Ireland, 19-23 July 2004.
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KONKOWSKI, Deborah A., Professor, "`Singularity' of Levi-Civita Spacetimes," 17th International Conference on General Relativity and Gravitation, Dublin, Ireland, 19-23 July 19-23, 2004.

KONKOWSKI, Deborah A., Professor, "Generalized Spacetimes with Quantum Singularities," American Physical Society April Meeting, 16-19 April 2005.

KONKOWSKI, Deborah A., Professor, "Black Holes, Big Bangs and Cosmic Strings," World Year of Physics public lecture, University of North Carolina at Wilmington, Wilmington, NC, 21 April 2005.

KONKOWSKI, Deborah A., Professor, "Classical and Quantum Singularities in General Relativistic Spacetimes", University of North Carolina at Wilmington, Wilmington, NC, 22 April 2005.

KSIR, Amy E., Assistant Professor, "Choosing the Right Job for You", Mathematical Association of America MathFest, 5 August 2004.

KSIR, Amy E., Assistant Professor, "Automorphism Groups of Curves and Codes", American University Mathematics Department Colloquium, Washington, D.C., 9 November 2004.

KSIR, Amy E., Assistant Professor, "Elliptic Surfaces in N=2 Seiberg-Witten Theory", University of Pennsylvania Math/Physics Workshop on Algebraic Geometry, Symplectic Geometry, and Theoretical Physics: a Conference Celebrating the Contribution of Women Researchers, 6 May 2005.

LIAKOS, A., Assistant Professor, "Slip with Friction Boundary Condition for the Navier-Stokes Equations – Numerical Studies for Time Dependent Laminar Flows", AMS Eastern Section Meeting, 6 November 2004.

LIAKOS, A., Assistant Professor, "A Posteriori Error Estimators for a Two-Level Finite Element Discretization of Viscoelastic Fluid Flow", SIAM Conference on Computational Science and Engineering, Orlando, FL, 13 February 2005.

LIAKOS, A., Assistant Professor, "Numerical Simulation of fluid flow in a Twin-Screw Geometry", technical briefing, NSWC-Indian Head, 29 March 2005.

MAHAR, T. J., Associate Professor, "Euler's Equations for Compressible Flow", technical briefing, NSWC-Indian Head, March 2005.

McCOY, Peter A., Professor, "Applications of Special Functions in Laser Propagation," special session on Special Functions and Orthogonal Polynomials, Midwest Regional Meeting of the AMS, Northwestern University, 23 October 2004. (Invited presentation)

McCOY, Peter A., Professor, "Laser Propagation in Free Space," AMS-MAA Joint Annual Meetings, session on Partial Differential Equations (Part I), Atlanta, GA, 6 January 2005.

McCOY, Peter A., Professor, "Progress in Laser Propagation, Modeling and Simulation at the USNA", HEL JTO M&S TAWG Review, WPAFB, Dayton, OH, 14-15 June 2005.

MELLES, Caroline G., Associate Professor, "Ohsawa's Boundedness Condition for Hermitian Metrics," U. S. Naval Academy, Annapolis, MD, 21 October 2004.

MICHAEL, T. S., Associate Professor, "When Can You Pack a Box with Translates of Two Bricks?" National Mathematics Joint Meetings, Atlanta, GA, January 2005.

MICHAEL, T. S., Associate Professor, "Matrices, Graphs, and How to Rate Bridge Players" Brualdi-Fest, Madison, WI, April 2005.

PENN, Howard L., Professor, "Making Faces and Other Parametric Equations Art", MAA Mathfest Providence, RI, 12 August 2004.

PENN, Howard L. Professor, "Making Faces and Other Parametric Equations Art", Mathematical Association of America MD-DC-VA Sectional Meeting, Baltimore, MD, 6 November 2004.

PENN, Howard L. Professor, "Which Ballparks are Homer Friendly?", Mathematical Association of America, MD-DC-VA Sectional Meeting, Charlottesville, VA, April 2, 2005.

POPOVICI, Irina, Assistant Professor, "The Eidochromatic Transform for Color Image Coding", The 2005 Joint Meeting of the Mathematical Societies, Atlanta, GA, January 2005.

POPOVICI, Irina Assistant Professor, "The Eidochromatic Transform for Image Compression," Center for Scientific Computation and Mathematical Modeling, College Park, MD, March 2005.

POPOVICI, Irina, Assistant Professor, "Topics in Riemann Surfaces", Riemann Surfaces Seminar, U.S. Naval Academy, Annapolis, MD, September 2004.

POPOVICI, Irina, Assistant Professor, "Thermo-dynamical Formalism in Dynamical Systems", Chesapeake Bay Seminar, U.S. Naval Academy, Annapolis, MD, October and November 2004

TRAVES, William N., Associate Professor, "Conics on the Cubic Surface, George Mason University, Fairfax, VA, 14 November 2004.

TRAVES, William N., Associate Professor, "Singularities", Algebraic Curves Seminar, U.S. Naval Academy, Annapolis, MD, January 2005. (four lectures)

TURNER, John, Professor, "The Flat Earth Guide to Probs & Stats", USCOTS, Columbus, OH, 18 May 2005.

VAN JOOLEN, V., Commander, USN, "Application of Higdon Non-reflecting Boundary **C**onditions to Shallow Water Models", International Conference on Spectral and High-Order Methods (ICOSAHOM-2004), 23 June 2004.

WARDLAW, William P., Professor, "For which C and D does AB = C imply that BA = D?", Mathematical Association of America MathFest, Providence, RI, 14 August 2004.

WARDLAW, William P., Professor, "For which C and D does AB = C imply that BA = D?", USNA Mathematics Colloquium, U.S. Naval Academy, Annapolis, MD, 28 October 2004.

WARDLAW, William P., Professor, "Good and Square Matrices are Invertible", Annual Joint Meeting of the AMS and Mathematical Association of America, Atlanta, GA, 8 January 2005.

WARDLAW, William P., Professor, "Good and Square Matrices are Invertible", Spring meeting of MD-VA-DC Section of Mathematical Association of America, Charlottesville, VA 2 April 2005.

WARDLAW, William P., Professor, "Good and Square Matrices are Invertible", USNA Mathematics Colloquium, U.S. Naval Academy, Annapolis, MD, 25 April 2005.