Mathematics Department

Professor Thomas J. Sanders
Chair

Mathematics provides a logical framework and a language indispensable to understanding the technical world in which we live. The following description summarizes the many contributions to this field of study made during the past academic year by the faculty and midshipmen majors of the Mathematics Department of the U.S. Naval Academy. The results cited reveal the great scope, diversity, and applicability of mathematics and offer glimpses of its intellectual beauty and appeal.

Several midshipmen conducted research projects as Honors Mathematics Majors, or in specially created projects under the guidance of faculty members. There were eight mathematics honors majors in the class of 2006. They are listed below with their project titles and their faculty mentors:

Midshipman 1/C Peter Barkley, "Justice in Liver Transplant Allocation, an Integer Programming Approach" (Assistant Professor Sommer Gentry)

Midshipman 1/C Timothy B. Brock, "Linear Feedback Shift Registers and Cyclic Codes in Sage" (Professor W. David Joyner)

Midshipman 1/C Greg Coy, "Long Quadratic Residue Codes" (Professor W. David Joyner)

Midshipman 1/C Gregory Dietzen, "Singular Analysis of an N-dimensional Cosmic String" (Professor Deborah A. Konkowski, Professor Mitch Baker and Associate Professor Alexis A. Alveras)

Midshipman 1/C John H. Doherty, "Dynamic Systems: Spring, Cycloid and Pendulum Systems" (Professor Mark Kidwell)

Midshipman 1/C Robert B. Irving, "Higher Dimensional Linear Regression as a Mathematical Foundation for Data Classification" (Associate Professor Gary O. Fowler)

Midshipman 1/C Gordon R. McDonald, "Mathematics Honors Project Summary: Low Density Parity Check Codes" (Professor W. David Joyner)

Midshipman 1/C Daniel C. Ryan, "God's Algorithm on the Edges and Corners of the Rubik's Cube" (Assistant Professor Amy E. Ksir)

A special note is that Midshipman Dietzen was the winner of the Naval Intelligence Foundation Award for the best honors project at the Naval Academy.

Regularly meeting throughout the year are four colloquia series, in pure mathematics, applied mathematics, operations research, and teaching methods. These both bring in outside speakers and provide a forum for department members and mathematics majors to present their work. There are also weekly seminars in topics of special interest, such as Riemann Surfaces, Algebra, Applied Math and the Mathematics of Fluid Flows.

Once again, the Mathematics Department produced a wide range of scholarly work that appeared as technical reports or as publications in refereed journals throughout the world. Dozens of articles appeared as applications of mathematics or as pure mathematical research. Topics that Mathematics Department faculty researched include:

mathematics pedagogy, chaos and dynamical systems, mathematical physics and cosmology, algorithms for computers, wavelets, cryptology, ocean acoustics, image recognition, fluid flows, beam propagation, and basic mathematical research in areas such as algebra, analysis, combinatorics, computational geometry, differential equations, differential geometry, matrices, number theory, operator theory, and statistics.

In addition to many independent research projects, several research projects were sponsored in whole or part by a variety of sources, such as:

Defense Modeling and Simulation Office
Joint Technical Office, High Energy Laser
National Aeronautics and Space Administration
National Science Foundation
Naval Research Laboratory
Naval Surface Warfare Center
Office of Naval Research
The Johns Hopkins University School of Medicine, Department of Surgery

During the past year, members of the USNA Mathematics Department presented the results of their scholarly activities on over forty occasions at professional mathematical meetings and colloquia throughout the United States and abroad. This activity, along with publication, enhances the academic stature of the Naval Academy and promotes the professional growth and reputation of those individuals involved. Through research activity, the faculty expanded their intellectual horizons and stayed vital in their disciplines. They contribute to the discovery of new mathematics. And they develop new materials and ideas that they can share with midshipmen students in their mathematics courses and research projects.

Sponsored Research

Endomorphism-semigroups of B(H)

Researcher: Assistant Professor Alexis A. Alevras
Sponsor: Naval Academy Research Council (NARC)

In joint work with Professor R. T. Powers ,University of Pennsylvania and G. L. Price, US Naval Academy, we initiated the study of non-conservative (i.e. non-unital) endomorphism semigroups on type I factors. Cocycle conjugacy among such semigroups has many of the properties of cocycle conjugacy among (conservative) E_0-semigroups. We introduced and studied a natural relationship between endomorphism semigroups: $\beta \subset \alpha$ if β is cocycle conjugate to a subordinate of α . Using techniques developed in our previous work, we proved that this relation is antisymmetric, at least among semigroups corresponding to one-dimensional boundary weights: if $\beta \subset \alpha$ and $\alpha \subset \beta$ then α and β are cocycle conjugates. This is work in progress.

Geo-Spacial Data Analysis

Researcher: Professor James D'Archangelo
Sponsor: Naval Research Laboratory (NRL)

The Counter Insurgency Pattern Assessment (CIPA) program seeks, among other objectives, to predict adversarial events as a function of geo-spatial variables correlated with historical events. Large numbers - in the order of hundreds or more - of variables may be under consideration as corollaries of these events.

The current task is two-fold:

(1) To collaboratively explore methods of multivariate data reduction (e.g. principal factors analysis) for finding collinear relationships among the provisional independent variables. The objective is to map the large set of independent variables into a reduced space of low dimensionality.

(2) To present the results of this analysis geospatially using the Matlab Mapping Toolbox. It should be possible to represent the predicted events geospatially, as a function of either the full or reduced set of independent variables.

The author investigated the feasibility of applying various spatial statistical and computational methods including Bayesian analysis with Gibbs sampling as methods for predicting adversarial events.

On the Visualization of Large Point Cloud Data

Researchers: Associate Professor Sonia M. F. Garcia and David Harding
Sponsor: Naval Research Laboratory (NRL)

Visualization is the act of making a visible presentation of numerical data, particularly a graphical one. The goal is to help the viewer to form a mental image. Our work was to produce a scientific visualization, i.e. using analysis and graphing tools to classify, summarize, explore, and present large data sets.

Visualization tools are needed for many reasons for example; reactions to future management plans will be influenced by experiences of current and past activities. Visualization can help with understanding these issues: Habitat Disturbance, Visual impacts, Operational considerations and Highlighting specific features.

Our particular study used a three-dimensional mathematical morphology, a machine vision technique, to extract measurements of individual seismic activity presented at the Mount St. Helens, Washington State. Mount St. Helens caused a dreadful eruption in 1980 with many dead, innumerous houses destroyed, and hundred of beautiful forest completely ruined. Mount St. Helens became active again in autumn 2004, indicated initially by hundreds and then thousands of localized earthquakes, and followed by several significant emissions of steam and ash. Since then small earthquakes (maximum about magnitude 1) continue to occur at a rate of about 1 per 5 to 10 minutes. Visual observations and thermal imaging of the crater, the 1980-86 lava dome, and the intensely deforming and uplifting area on the south side of the dome are occurring constantly. These observations were consistent with new lava having reached the surface of the uplift. Additional visual and thermal observations are being used to further evaluate this interpretation.

While geologists warn that an eruption similar to the May 1980 eruption is still possible, the chances are low. More studies are needed to monitor the behavior of this nature beast.

Microaltimeter Flight Data Post Processing

Researchers: Jan McGarry, Phil Dabney and Associate Professor Sonia M. F. Garcia
Sponsor: Laboratory for Terrestrial Physics NASA / Goddard Space Flight Center

The Microaltimeter System is a prototype for a new concept in laser altimeter systems, using a low energy per pulse (<10 MicroJoule), high repetition rate (~10khz) laser and a small diameter telescope (15 cm). This system requires less mass and power than traditional altimeter designs, but has significant noise content in the data stream. Signal processing is required to distinguish the signal from the noise.

Navigational (differential GPS and INS) and image data is collected along with the altimeter ranging data to allow transformation of the range time of flight information into terrain heights at the geolocated laser footprints. Calibration files (ranging to ground targets) to determine system delays, and flight data over bay water to determine offsets in the INS attitudes are also needed to remove biases from the data. We refer to this transformation of range times into latitudes, longitudes and heights, along with extracting the signal from noise, as the post processing.

To validate the Microaltimeter performance, we proposed to verify our geolocated terrain heights derived from the ranges, against an independent Digital Elevation Model (DEM) of the Ocean City and Assateague Island areas. The DEM covers roughly the upper half of Assateague Island and the lower half of Ocean City (from the Route 90 bridge south).

We are, unfortunately, not finished with the post processing effort - it has proven a more challenging effort than originally anticipated. A paper presenting the details of the Microaltimeter post processing will be written when the effort reaches maturity.

Optimizing the Utilization of Live Donor Kidneys through Kidney Paired Donation

Researchers: Dorry Segeg and Assistant Professor Sommer Gentry
Sponsor: The Johns Hopkins University School of Medicine, Department of Surgery

Members of an end-stage-renal-disease patient's family are often willing to be live kidney donors, but at least a third of such offers must be rejected because of blood-type or tissue-matching incompatibility. In kidney paired donation, two such patients and their donors exchange kidneys in simultaneous operations so that both patients receive a compatible kidney: the donor of the first family gives to the patient of the second, and vice versa. In a large pool of patients and their incompatible donors, deciding which pairs should exchange with which other pairs is equivalent to matching on a graph. We have used computational trials of proposed interventions on simulated patient databases to design and test these novel organ allocation systems in advance of implementation.

Because we provided some of the first numerical estimates of the extent to which this could alleviate the organ shortage, this work kicked off an effort to create a national kidney paired donation registry. A bill explicitly making kidney paired donation legal (its legality is now unclear) was introduced in February and is expected to pass in the U.S. Senate. Our research has also generated excitement in the popular press, which can only help encourage organ donation in general. Articles and programs on this research have appeared in TIME magazine, Reader's Digest, MIT Technology Review, the Baltimore Sun, CBS News, and National Public Radio's Diane Rehm Show. The TV show NUMB3RS also formulated an episode around this idea.

This grant focused our research more directly on the many questions that policy-makers, clinicians, and especially patients will have about paired donation as it becomes more widely available. The grant funded computer programming / support services for a 20- node Linux cluster housed at Johns Hopkins University, and to date 17 abstracts have been accepted / presented at national transplant meetings, from four different investigators working on this project.

Combinatorics of Updown Categories

Researcher: Professor Michael E. Hoffman
Sponsor: Naval Academy Research Council (NARC)

In earlier work the investigator developed the notion of an "updown category," which provides a uniform framework for numerous (graded) partially ordered sets in combinatorics, including integer partitions, integer compositions, rooted trees, and planar rooted trees. The definition naturally provides the graded vector space generated by the set of objects of an updown category with operators U and D, and the coefficients of iterates of these operators applied to objects of the category have an enumerative interpretation.

The investigator has singled out a number of properties of updown categories, based on his extensive study of examples. There is a hierarchy of "commutation conditions," based on properties of the commutator DU-UD. There are also "even covering conditions," which can be defined in terms of the effect of the operators U and D on objects in a given rank. There are some interesting relations between the two sets of conditions, which can be expressed in terms of generating functions associated to the updown category.

Automorphisms of Curves and Codes

Researchers: Professor W. David Joyner and Assistant Professor Amy E. Ksir
Sponsor: Naval Academy Research Council (NARC)

Given a projective algebraic curve X and two divisors D and E on X, one can use the Riemann-Roch space $L(D)$ to construct an error-correcting code $C(D,E)$, called the Goppa code or AG code. If G is a finite group of automorphisms of X, and D is invariant under G, then there is a natural action of G on the code. Our main goal this year was to find families of examples of curves with a finite group action where the representation of G on the Riemann-Roch space could be explicitly calculated. We found two such families of examples. The first, found with the help of Roger Vogeler from Ohio State University, is the family of Hurwitz curves with an action of the finite group $PSL(2,q)$. Building on our previous work with modular curves, which have an action of the same group, we were able to compute the representation structure of both the ramification module and $L(D)$, for any G-equivariant divisor *D*. Our second family of examples is the Artin-Schreier curves $y^2 = x^p - x$, where *p* is prime and is the characteristic of the field. We realized that the techniques we were developing to compute Riemann-Roch spaces for these curves could be generalized to any hyperelliptic curve $y^2 = h(x)$. We then turned our focus to finding explicit bases for Riemann-Roch spaces of hyperelliptic curves. This will have applications in computational algebraic geometry beyond our project. We have written up and submitted both results.

Twin Screw Extrusion of Energetic Materials

Researchers: Assistant Professor Anastasios Liakos
and Dr. H. Bruck (Mechanical Engineering Department, University of Maryland)
Sponsor: Naval Surface Warfare Center in Indian Head and Naval Academy Research Council (NARC)

This project is still in progress. A second-generation three dimensional program has been written in C to try to predict the values of the flow variables (velocity, pressure, stress). In addition, research was done to mesh the complex geometry of the flow domain of the extruder.

Modeling and Analysis of Laser Propagation IV

Researcher: Professor Peter McCoy
Sponsor: Joint Technical Office, High Energy Laser

This research considers the basic physics and foundations of mathematical analysis modeling beam propagation followed by computational-numerical simulation via code to be developed and adopted at the Naval Academy.

The Modeling, Mathematical and Numerical Analysis includes the Paraxial Wave Equation $\left(\nabla_t^2 - 2ik\,\partial/\partial z\right)\psi = 0$ where ∇_t^2, the transverse Laplacian, is the basic partial differential equation describing laser propagation through a uniform medium. It arises by making the so-called paraxial approximation to Helmholtz's equation. A cylindrical coordinate based Gauss-Laguerre normal mode analysis of solutions of the PWE initiated by Reza Malek-Madani (RMM) and Peter McCoy (PMc) is to be completed. An elliptical coordinate based analysis of the PWE will be conducted by RMM and PMc based on Gauss-Ince normal modes. These normal modes are state of the art in modeling Laser Propagation as well as in the study of orthogonal polynomials and special functions.

Modeling of MEMS Gyroscopes

Researcher: Assistant Professor Aurelia Minut
Sponsor: Institute for Scientific Research/Office of Naval Research (ONR)

The MEMS (microelectromechanical systems) gyroscopes can be used in navigation systems in cases where the GPS is denied. The MEMS gyroscopes are small devices that can be mounted on a wide range of platforms, such as UAVs, guided missiles. This research attempts to improve the performance of current gyroscopes by weakly coupling an array of MEMS gyroscopes.

Most MEMS vibratory gyroscopes are modeled by the mass spring system. A single proof mass with the same x and y-axis spring constants is suspeded above a plate and it is supported by anchored comb-drive flexible structures. These structures make the mass oscillate freely in two orthogonal directions: the drive (y-axis) and the sense (x-axis). If an external drive force (usually a sinusoidal force) is applied, the proof mass oscillates in the drive direction. The drive force is an electrostatic force induced by the comb-drive structures. The driving force in the sense direction is the Coriolis force which is induced by rotation and causes dynamic coupling between the oscillation axes.

We developed a model for the equations of motion and solved the equations numerically. Then we tried coupling two, four, nine and twelve gyroscopes in different ways and compared the results. The goal of this work is to study the stability and equilibrium of the system of equations for forty gyroscopes and find the best way to couple them.

United States Naval Academy Chesapeake Bay Research

Researcher: Professor George Nakos
Sponsors: Defense Modeling and Simulation Office (DMSO)

In summer of 2005 and partially continuing throughout the Academic year, the investigator was part of an effort to study the water movement and residence time in the Chesapeake Bay. The specific research participation in this project was the installation in a linux environment of several libraries and the QUODDY software, as well as the use of the above. The results of the QUODDY runs were crucial in Midshipmen research. The ability to run these programs locally and fast enhances the overall Chesapeake Bay research program at the Naval Academy.

A Multi-Scale Edge Transform for Image Coding

Researchers: Assistant Professor Irina Popovici and Professor Wm. Douglas Withers
Sponsors: Office of Naval Research (ONR)

This project concerns lossy compression of either color or black-and white images, using an approach radically different from the popular JPEG and JPEG 2000 standards. Our objective is to exploit the edges and shadows in an image such that a smaller number of bits can be used to represent it, while maintaining an ``acceptable'' visual quality for the decompressed image. The correlation between spatial locations of pixels has been exploited by almost all existing algorithms, but the driving principle of these algorithms is elimination of data in a simple (say linear or cubic) dependency from proximate pixels. We propose a recurrent coding technique that also exploits the discontinuities in data near the contours of an image.

The popular wavelet approach to image compression describes an image in terms of a linear basis of elements which are self-similar and which also reflect the smooth nature of many image regions. In this basis, however, a sharp, straight edge must be described as a superposition of many wavelet elements at all scales. The first step in this project is development of a linear basis which shares the wavelet property of self-similarity but is reflective of sharp, straight edge structures rather than smoothness. Such a basis has potential applications in pattern analysis and image enhancement as well as image coding. Following the development of this basis, the remainder of the coding problem can be approached in a fashion similar to standard wavelet image coders.

Cocycles for One-parameter Flows of B(H)

Researcher: Professor Geoffrey L. Price
Sponsor: National Science Foundation (NSF)

In the 1930s, E. Wigner proved that one-parameter groups of automorphisms on B(H), the algebra of bounded operators on a Hilbert space H, are implemented by one-parameter group of unitary operators. Many years later R. T. Powers began a systematic study to find the analogue of Wigner's result for the time-irreversible systems on B(H) known as E_0-semigroups. Price has collaborated with Powers and A. Alevras in this work. A recent paper by the three authors includes a study of the multiplicative structure of the family of local cocycles of certain E_0-semigroups that are called spatial, which means that they are intertwined with the identity on B(H) by a one-parameter semigroup of isometries. The local cocycles provide an invariant under the equivalence of E_0-semigroups known as cocycle conjugacy. In current work Alevras, Powers and Price are turning this problem around in a sense by trying to classify, for a given family of local cocycles, all spatial E_0-semigroups sharing this family.

Marginal Tracial States on Tensor Products of Matrix Algebras

Researcher: Professor Geoffrey L. Price
Sponsor: National Science Foundation (NSF)

A tracial state ϕ on an operator algebra *A* is a positive linear functional on *A* which satisfies 1) $\phi(I) = 1$, where *I* is the identity on *A*, and 2) $\phi(xy) = \phi(yx)$ for all *x*, *y* in *A.* It is well-known that there is a unique tracial state Tr_B on *B*, the algebra of $n \times n$ matrices over the complex numbers. Suppose $B_1 = B = B_2$. A *marginal* tracial state on the tensor product $B_1 \otimes B_2$ is a state ρ that satisfies $\rho(x \otimes I) = Tr_{B_1}(x)$ and

$\rho(I \otimes y) = Tr_{B_2}(y)$ for all *x*, *y* in *B*. In 2003 K. R. Parthasarathy showed that if *B* is the algebra of 2×2 matrices the extremal marginal states on $B_1 \otimes B_2$ are all pure states, i.e., they are extremal among the convex set of all states on $B_1 \otimes B_2$.

Recently Price and Shoichuro Sakai have made some progress in their attempt to generalize this result to the case where *B* is the algebra of $n \times n$ matrices over the complex numbers.

Using Off-the-shelf Software in the Classroom

Researcher: Professor Thomas J. Sanders
Sponsor: Defense Modeling and Simulation Office (DMSO)

The objective of this project was to further investigate the possibility of using off-the-shelf software in the classroom. In particular, I was interested in the possibilities of using the computer game *Empire Earth* as a vehicle to teach tactical analysis to Midshipmen. After becoming somewhat familiar with the play of the game and the tools included to build scenarios, I concluded that the software was not suitable for this purpose. The fidelity of the game and the amount of control over the units is not sufficient for the intended use.

Independent Research

Endomorphism Semigroups Parametrized by the Forward Light Cone

Researcher: Assistant Professor Alexis A. Alevras

This project is in progress. We are interested in the study of strongly continuous semigroups of endomorphisms of B(H), $\alpha=\{\alpha_\lambda:\lambda\in C\}$, where C is the forward light cone in Minkowski space. Such semigroups occur naturally, for example when one is given a system of local observables which is acted upon by the Poincare group in such a way that the Haag-Kastler axioms are satisfied, and a subalgebra which is invariant under the action of translations by elements of the forward light cone. Specific examples of this situation have been given using the construction of Free Hermitian Scalar Fields. The immediate goal is the classification of these examples up to cocycle conjugacy through index theory.

Seabed Acoustics

Researcher: Professor James L. Buchanan

Chotiros and Isakson [*J. Acoust. Soc. Am*., 116 (4), 2011-2022 (2004)] recently proposed an extension of the Biot-Stoll model for poroelastic sediments that makes predictions for compressional wave speed and attenuation which are in much better accord with the experimental measurements of these quantities extant in the literature than either those of the conventional Biot-Stoll model or the rival model of Buckingham [*J. Acoust. Soc. Am*., 108 (6), 2796-2815 (2000)]. Using a local minimizer, the Nelder-Mead simplex method, it is shown that there are generally at least two choices of the Chotiros-Isakson parameters which produce good agreement with experimental measurements. Since one postulate of the Chotiros-Isakson model is that, due to the presence of air bubbles in the pore space, the pore fluid compressibility is greater than that of water, an alternative model based on a conjecture by Biot [*J. Acoust. Soc. Am.,* 34 (5), 1254-1264 (1962)], air bubble resonance, is considered. While this model does as well or better than the Chotiros-Isakson model in predicting measured values of wave speed and attenuation, the Rayleigh-Plesset theory of bubble oscillation casts doubt on its plausibility as a general explanation of large dispersion of velocity with respect to frequency.

Non-Parametric Density Estimation of Streaming Data Using Orthogonal Series

Researcher: Lieutenant Commander Kyle Caudle, USNR

Computer technology in the 21st century has allowed us to gather and collect data at rates that would have seemed impossible less than a decade ago. As such, typical data base management systems (DBMS) are having great difficult storing and analyzing data in the traditional way. Systems that receive large amounts of data in a transient data streams generally need to analyze the data immediately without storing it on a disk. These systems are referred to as data stream management systems (DSMS). This emerging field has been pushed to the forefront by technology that demands analyzing data in realtime. Babcock et. el. [2002] analyzed the issues involved in mining rapid time-varying data streams. To date most of the work in the area of DSMS has primarily been concerned with querying the data streams. These queries provide estimates of parameters, such as the mean, and then continuously update them as more data arrives. Recently, Heinz and Seeger [2004] used data streams to provide an estimate of the underlying probability density function by dividing the data up into bins or windows containing the most recent data. An estimate of the density was created using the standard wavelet cascading algorithm on the binned data.

Designing Digital Tools for Communicating Mathematics in Higher Education

Researcher: Professor Carol G. Crawford

The author extended prior research into the design and development of online tools for communicating mathematics at the university and college level. A paper was published in September, 2005: *Proceedings of the Joint Meeting of the 3rd International Conference on Education and Information Systems, Technologies and Applications (EISTA 2005),* Volume I, pages 223-228. This work reflects work with Professor Mark Meyerson and Professor Michael Chamberlain (USNA Mathematics Department) and Commander Thomas Logue, USN (USNA Computer Science Department) in cooperative Curriculum Development Projects at the Naval Academy.

Nontrivially Discriminating Groups

Researcher: Professor Anthony M. Gaglione

Discriminating groups were first introduced by Baumslag, Myasnikov and Remeslennikov with an eye toward applications to the universal theory of various groups and solving equations in groups. We need to fix some terminology. Let G be a group. We say that G is discriminating provided that for every finite nonempty subset S of nontrivial elements of the Cartesian square, G×G, there is a homomorphism ϕ_S:G×G $\rightarrow$ G such that ϕ_S (s) $\neq$1 for all s in S. It is clear that if G×G embeds in G then G is discriminating. If this is the case, we term G trivially discriminating**.** The objectives of this project were to try to find examples of nontrivially discriminating groups and to address many of the open questions which have recently been put forth concerning them. This research was started in a paper by A.M. Gaglione (et al) entitled *Discriminating Groups* **(***Journal of Group Theory*, 2001, 463-474.(Abbreviated FGMS here) At the time of the writing of FGMS, the only examples of discriminating groups known were of two types: abelian groups and groups which embed their direct squares (i.e., trivially discriminating groups). It is easy to show that any torsion free abelian group is discriminating. Thus nontrivially discriminating groups do exist. The real open question was did they have to be abelian. Together with my collaborators, we have discovered several classes of nonabelian nontrivially discriminating groups, i.e., nonabelian groups which are discriminating but do not embed their direct squares. The methods used here go back to classical work of B.H. Neumann on certain two-generator subgroups in an unrestricted direct product of an infinite family of finite alternating groups. Here we were able to produce an uncountable family of two-generator nonabelian nontrivially discriminating groups. This work will appear in a recently accepted paper in the *Journal of Group Theory*. Another main question put forth in FGMS was whether finitely generated (f.g.) nilpotent groups are discriminating. We proved that f.g. nilpotent groups are discriminating if and only if they are torsion free abelian. This was done using the method of Malcev completions. As a matter of fact we proved more, i.e., the Malcev completion of a f.g. torsion free nilpotent group is discriminating if and only if it is abelian. This work will appear in an article recently accepted for publication in a volume of the American Mathematical Society (AMS), series *Contemporary Mathematics*. We have answered several of the open questions which arose in FGMS, but these have given rise to new questions concerning discriminating groups. Thus this research project is on going.

Maximum Matching Inequalities in Kidney Paired Donation

Researchers: Assistant Professor Sommer Gentry and Associate Professor T. S. Michael

Arranging kidney exchanges requires matching on a graph. Maximum cardinality matchings are desirable, but the relative utility of kidney exchanges must be represented by edge weights. We have proved that setting edge weights within a precisely described range guarantees that a maximum edge weight matching will also have maximum cardinality. This gives a method for choosing edge weights in a national kidney paired donation program, so as to guarantee that no opportunity for a kidney transplant will be lost but still allow for distinguishing between preferred and non-preferred edges. We are extending these results to special cases in which some exceptions to this maximum cardinality rule are allowed to give extraordinary preference to a small number of patients.

Designing and Evaluating Weighting Functions for Matching on Graphs

Researcher: Assistant Professor Sommer Gentry

In medical resource allocation applications, objective functions do not necessarily simplify to a dollar figure or even a life-years gained outcome. For instance, ethical considerations are paramount in deciding which of two needy patients should receive a single available kidney transplant. In deceased donation the objective function is essentially equivalent to rank-ordering, and allocation is done via an established points system. In kidney paired donation, however, which is equivalent to maximum edge weight matching on graphs, designing an objective function giving weights for different paired donation opportunities is presently more of an art than a science. Whether increasing the weight on a certain type of edges by a given amount will change the optimal solution depends greatly upon the structure of the graph. The goal of this work is to make the tradeoffs inherent in setting edge weights comprehensible to the transplant professionals and ethicists who must set these weights in practice. The gold standard would be proofs of simple statements about weights, but where these are impossible one may provide guidance using simulations.

Hopf Algebras of Trees and Symmetric Functions

Researcher: Professor Michael E. Hoffman

Several years ago, Dirk Kreimer introduced a Hopf algebra of rooted trees to describe renormalization in quantum field theory. This has stimulated much algebraic work, both on Kreimer's original Hopf algebra and on similar objects, including a Hopf algebra of planar rooted trees described by Loic Foissy. In fact, the relation between Kreimer's Hopf algebra and Foissy's Hopf algebra is analogous to that between the Hopf algebra of symmetric functions and the Hopf algebra of quasi-symmetric functions; this analogy can be made precise in the form of a commutative diagram, whose study may produce some interesting results.

The investigator has given two talks about his work on this topic, including one at the Mathematisches Forschungsinstitut in Oberwolfach, Germany. A short account of his work will be published in the Oberwolfach Reports, and he plans a longer publication on this topic later.

Combinatorics of Updown Categories

Researcher: Professor Michael E. Hoffman

We define the notion of an updown category, which generalizes Stanley's notion of a sequentially differential poset. The vector space generated by the set of objects of an updown category naturally admits two operators U and D, and the coefficients of iterates of these operators applied to objects of the category have an enumerative interpretation. We consider some conditions on the operators U and D, and establish their combinatorial implications.

A Stability Criterion for Waves in a Coupled Optical System

Researchers: Assistant Professor Russell Jackson and Christopher K.R.T. Jones (University of North Carolina)

In my thesis, I identified a new bifurcation that led to many novel pulses within a coupled optical system, including a family of N-pulses for all positive integer N. The geometry of this bifurcation provides a key component (which we have now analyzed) in understanding the stability of these same waves, but this (geometric) component needs to be incorporated into a more general (analytical) framework before the whole picture can be understood.

Bifurcation and Stability of Standing Waves in a Gross-Pitaevskii Equation

Researcher: Assistant Professor Russell Jackson

The nonlinear Schrodinger Equation with potential provides a general model for Bose-Einstein condensation. In most experimental setups, this potential is produced either by a magnetic trap (yielding a single-welled potential) or an optical lattice (yielding a periodic or many-welled potential). Using phase-plane techniques, a number of standing waves can be identified within this system, and once again, the geometric construction of these pulses provides much immediate information concerning the stability properties of each pulse. We have completed the analysis of the pulse construction in the *N*-well case, but of continuing interest is the relationship between the stability properties between the system of *N* wells with *N* large (but finite) and the limiting periodic system with an infinite number of wells.

A Numerical Package for Computation of the Evans Function

Researcher: Assistant Professor Russell Jackson

Numerical tools to help with the determination of stability in problems in nonlinear waves are being prepared. Once a wave has been identified within a system, its stability can theoretically be determined by the evaluation of the Evans Function. However, in practice this function is difficult to compute, except in special limiting cases (like a singular limit in parameter space). I am working on developing robust numerical tools for the evaluation of the Evans Function for pulse solutions within general nonlinear PDE. This involves linearizing the PDE about the pulse solution and following the stable and unstable subspaces of the asymptotic states along the pulse. These computations are most readily performed in the Space of Exterior Powers – for the coupled system in (Project 1) this becomes a 70 (8 *choose* 4) dimensional system; for the single system in (Project 2) this is a 6 (4 *choose* 2) dimensional system.

Pulses in Nonlinearly Coupled Schrodinger Equations: I. A Homoclinic Flip Bifurcation

Researcher: Assistant Professor Russell Jackson

In this work, we describe a new mechanism for the generation of multi-pulse solutions in a class of nonlinearly coupled Schrodinger equations. Many novel pulses have been observed in such systems both numerically and experimentally but, until now, an understanding of their origins had been lacking. The particular bifurcation studied here is spurred by the passage through degeneracy of a one-component pulse in orbit-flip configuration. We provide a straightforward geometric analysis, demonstrating the production not only of a multi-component 1-pulse nearby the original one-component pulse, but also of an entire family of alternating *N*-pulses, for all positive integers *N*.

Pulses in Nonlinearly Coupled Schrodinger Equations: II. An Instability Criterion

Researcher: Assistant Professor Russell Jackson

In this work, we derive a geometric criterion for the instability of pulses in a class of nonlinearly coupled Schrodinger equations. Many novel pulses have been observed in such systems both numerically and experimentally, and recent work has provided an understanding of the generation of such pulses. The criterion given here allows an immediate transfer of the information now known about the mechanisms and geometry of pulse creation into a new description of the stability of these same pulses. Additionally, we report the discovery of a secondary bifurcation of (potentially stable) asymmetric pulses that occurs when the value of this criterion changes.

Bases for Riemann-Roch Spaces of Hyperelliptic Curves

Researchers: Professor W. D. Joyner and Assistant Professor Amy E. Ksir

Let X denote a hyperelliptic curve and let D be an effective divisor on X. We give an explicit constructive method for computing a basis B of $L(D)$. In particular, it is done in such a way that if $E < D$ is another effective divisor, the basis B_E of $L(E)$ so constructed will be a subset of B. We plan to use this in future work to compute quotient representations, with applications to AG codes. We used SAGE to compute an example in some detail.

Group Actions on Riemann-Roch Spaces of Some Hurwitz Curves

Researchers: Professor W. D. Joyner, Assistant Professor Amy E. Ksir
and Dr. R. Ross Vogeler (The Ohio State University)

Let $q > 1$ denote an integer relatively prime to 2,3,7 and for which $G = PSL(2,q)$ is a Hurwitz group for a smooth projective curve X defined over the complex numbers. We compute the G-module structure of the Riemann-Roch space $L(D)$, where D is an invariant non-special divisor on X. This depends on a computation of the ramification module, which we give explicitly. In particular, we obtain the decomposition of $H^1(X,\mathbf{C})$ as a G-module.

Ramified Artin-Schreier

Researcher: Professor W. D. Joyner and Assistant Professor Amy E. Ksir

We consider the curve $y^2 = x^p - x$ over the field $GF(p)$, which is both an Artin-Schreier curve and a hyperelliptic curve. This curve has $SL(2, p)$ as its automorphism group. We compute the $SL(2, p)$-module structure of the ramification module and Riemann-Roch spaces for this curve.

Classification of Singularities with SHEEP/CLASSI and/or MAPLE

Researcher: Professor Deborah A. Konkowski

This research is an investigation into ways to apply computer algebra programs (e.g. SHEEP/CLASSI and MAPLE) to the problem of singularity classification. This computer algebra research was begun during a recent sabbatical in London at Queen Mary and Westfield College with Professor Malcolm MacCallum's group and has been continued during periodic visits to London. Although it is impossible to classify topological singularities such as quasiregular singularities in this manner, scalar curvature and non-scalar curvature singularities should yield to analysis. Ways to study scalar and nonscalar curvature singularities using a complete listing of C^0- curvature invariants and frames related to parallel propagated orthonormal ones are currently under consideration.

The use of MAPLE was just begun last summer whereas the use of SHEEP/CLASSI has been ongoing. MAPLE is very promising for singularity classification because it can algebraically evaluate the invariants needed to test for curvature singularities. I used it for special cases including power-law metrics. It would be nice if some numerical evaluation could be obtained but this requires knowing where the singularity may be through geodesic computation which is not automatic. This research together with the SHEEP/CLASSI work will be continued.

Classical and Quantum Singularities in General Relativistic Spacetimes

Researcher: Professor Deborah A. Konkowski

This researcher's main research area is the analytical study of spacetimes with singularities. She is particularly interested in studying spacetimes with mild singularities and Cauchy horizons. Mild singularities include quasiregular and nonscalar curvature singularities. In the case of quasiregular singularities, particle paths end suddenly with no warning from infinitely-increasing tidal forces, while in the case of nonscalar curvature singularities some, but not all, particles moving near the singularity feel infinite tidal forces. A Cauchy horizon is the boundary of the causal development of spacetime. Cauchy horizons and singularities are intimately connected, including through the cosmic censorship conjecture, which continues to be debated.

D. A. Konkowski is studying (1) a quantum-mechanically singular spacetimes, (2) two previously-developed stability conjectures, and (3) the global structure of various singular spacetimes. Some of this work is done in collaboration with T.M. Helliwell (Harvey Mudd College) and students at both Harvey Mudd and at the Naval Academy.

In the first case, following Horowitz and Marolf and work begun by T.M. Helliwell and D.A. Konkowski, classical test particles are replaced with quantum test wave packets to test the singularity structure of spacetimes which classically possess all types of classical singularities. The research began with mildly classically singular spacetimes but has branched out in the past few years. The most recent research has been in answering the question: "Can quantum mechanics heal classical singularities?" Broad classes of spacetimes with power-law metrics near the origin are under study. These classes can be nonsingular or singular classically. The classically singular ones have then been studied for quantum singularities and further analyzed. Whole parameter spaces of classically singular but quantum mechanically nonsingular spacetimes have been discovered. This research is almost complete and a paper will soon be in preparation. Whereas completed results for early research on primarily quasiregular spacetimes were presented at summer 2005 conferences in Bern (Switzerland) and Paris (France), preliminary results for the latter work was presented more recently in Dallas (Texas).

In the second case, conjectures proposed by T. M. Helliwell and D. A. Konkowski to predict whether various mild singularities and Cauchy horizons are stable are being investigated further. Thus far, the singularity conjecture always holds true but the Cauchy horizon conjecture misses purely Weyl singularities that occur in the plane wave and exact mass-inflation spacetimes.

In the third case, spacetimes with nonscalar and quasiregular singularities are under analytical investigatigation. This includes work on: (1) spacetimes with nonscalar singularities due to Siklos, (2) quasiregular singularities in the context of dislocations and disclinations and in the context of Columbeau's extended theory of distributions, and (3) the structure of some spacetimes with directional singularities.

Rationality of *G*-Modules

Researchers: Dr. Ted Chinburg (University of Pennsylvania) and Assistant Professor Amy E. Ksir

Let X be a projective algebraic curve and let G be a finite group of automorphisms of X. The ramification module is a G-module induced from the action of the stabilizer subgroup on the cotangent space at the ramification points of X over X/G. A recent paper of Joyner and Ksir established a simple formula for the structure of the ramification module in the case that it is rational. This has led to the natural question: when will such a module will be rational? More generally, when will a G-module induced from a cyclic subgroup be rational? This project has two goals. The first is to determine, in general, when induced modules from cyclic subgroups will be defined over the rational numbers. The second is to determine when such a G-module is defined over the p-adic numbers. The answers to these questions are of interest both in the theory of error-correcting codes and in number theory. Research on this project was started in April 2006; one theorem giving a criterion in the case that G is $PSL(2,q)$ has been proven and is being written up.

Expository Notes on Quantum Field Theory

Researcher: Assistant Professor Amy E. Ksir

In the past ten years, algebraic geometry has been deeply affected by results in theoretical physics, especially the physics surrounding string theory. Quantum mechanics and quantum field theory are the basis for many of these results, but these fields are unknown to many algebraic geometers. These notes aim to explain the ideas behind quantum mechanics, quantum field theory, supersymmetry, and string theory to the audience of algebraic geometers.

A Posteriori Error Estimators for a Two-Level Finite Element Discretization of Viscoelastic Fluid Flow

Researchers: Assistant Professor A. Liakos and H.K. Lee (Clemson University)

We derive locally calculable a posteriori error estimators for a two-level method of discretizing the equations of steady-state flow of a viscoelastic fluid obeying an Oldroyd-type constitutive equation with no-slip boundary condition.

The two-level algorithm consists of solving a small non-linear system of equations on the coarse mesh and then using that solution to solve a larger linear system on the fine mesh. Specifically, following Najib and Sandri, we linearize the Oldroyd-type constitutive equation about the coarse mesh solution thus nullifying the difficulties brought by the advection term. Our theoretical error estimates show that it has optimal order accuracy provided the true solution is smooth and its norm is sufficiently small. In addition, our computational error estimates exhibit the validity of our analysis.

Modeling Gas Diffusion in Nanocomposites

Researchers: Assistant Professor A. Liakos, C.S. Swannack (Massachusetts Institute of Technology) and C.L. Cox (Clemson University)

Polymer-silicate layered nanocomposites (PSLNs) have received the attention of the food packaging industry since they have exhibited enhanced barrier properties such as decreased oxygen permeability. In this report, we present a model for the estimation of the average diffusion coefficient of a nanocomposite film.

No-Slip Boundary Conditions for the Navier-Stokes Equations by Penalty-penalty Method

Researchers: Assistant Professor A. Liakos and A. Caglar

We prove convergence of the finite element method for the Navier-Stokes equations in which the no-slip condition and the no-penetration condition on flow boundary are both imposed by penalty methods. This approach has been studied for the Stokes problem but it has not progressed beyond the linear Stokes problem. Since the inertial effects dominate the motivating application, it is crucial to extend the validity of the method to the non-linear Navier-Stokes case. We have started this extension, analyzed the method, and given numerical results. We have shown that optimal order of convergence can be achieved if the computational boundary follows the real flow boundary exactly.

Atiyah-Singer Index Theorem

Researcher: Professor Robert Lockhart

I am currently studying the Atiyah-Singer Index Theorem. I hope to eventually use it in nonlinear analysis – in particular in dealing with computability of various problems in the calculus of variations.

Seasonality Methods with Statistical Measure

Researcher: Lieutenant Commander Marc D. Lucas, USN

Current methods used by the US Government and standard actuarial practice for smoothing and interpreting seasonal data lack statistical interpretation. Use of a linear regression on transformed data yields identical factors with statistical measure.

Controls for Complete Controllability of Linear, Autonomous Systems of ODE

Researcher: Associate Professor T. J. Mahar

There is a well known theorem in control theory which gives a necessary and sufficient condition for a system of ODE to be completely controllable. The proof is not constructive. An approach has been devised which generates controls which steer the system to any desired target. The controls do not appear to be optimal in any sense, though this has not been proven. The approach extends to systems of PDE in a natural way.

Classical Poincaré Metric Planted off Singularities using a Chow-Type Theorem and Desingularization

Researchers: Associate Professor Caroline Grant Melles and Pierre Milman (University of Toronto)

We construct complete Kähler metrics on the nonsingular set of a subvariety X of a compact Kähler manifold. To that end, we develop (i) a constructive method for replacing a sequence of blow-ups along smooth centers, with a single blow-up along a product of coherent ideals corresponding to the centers and (ii) an explicit local formula for a Chern form associated to this 'singular' blow-up. Our metrics have a particularly simple local formula of a sum of the original metric and of the pull back of the classical Poincaré metric on the punctured disc by a 'size-function' S_I of a coherent ideal I used to resolve the singularities of X by a 'singular' blow-up, where $(S_I)^2 := \sum_{j=1}^{r} \left|f_j\right|^2$ and the f_j's are the local generators of the ideal I. Our proof of (i) makes use of our generalization of Chow's theorem for coherent ideals. We prove Saper type growth for our metric near the singular set and local boundedness of the gradient of a local generating function for our metric, motivated by results of

Donnelly-Fefferman, Ohsawa, and Gromov on vanishing of certain L_2-cohomology groups. In an appendix we give a simple constructive proof of a valuation criterion due to M. Lejeune and B. Teissier.

This 68-page joint paper has been accepted for publication in the *Annales de la Faculté des Sciences de Toulouse* **.**

Investigations into Sudoku

Researchers: Associate Professor T. S. Michael and Professor Mark D. Meyerson

This research in progress investigates extensions of a su doku problem that recently appeared in the *American Mathematical Monthly.* We have constructed a set of four su doku squares, each of which is orthogonal to the other three - and to its mirror-image about the main diagonal. The notion of orthogonality considered here is in the combinatorial sense of Latin squares.

Maple Animation of Space Filling Curves

Researcher: Professor Mark D. Meyerson

Dr. Meyerson is writing Maple programs to help with the visualization of classical Space Filling Curves. By varying the usual definition, one can produce families of curves with fractional (between 1 and 2) dimensions (fractals).

Viewing Möbius Transformations with Maple Animations

Researcher: Professor Mark D. Meyerson

We use Maple's ability to easily draw and animate to examine all possible Möbius transformations. A grid on the complex sphere stereographically embedded in $\mathbb{R}^3$ is distorted conformally as it continuously varies to the image under any Möbius transformation. There are two different cases: one or two fixed points (parabolic and loxodromic).

Wrestling Tournaments: How to Select First-Round Byes

Researcher: Associate Professor T. S. Michael

The Eastern Intercollegiate Wrestling Association (EIWA) encounters a scheduling problem at the season-ending conference tournament: How should the first-round byes be distributed among the wrestlers in each weight class? For many years a random selection was used, but this caused controversies when some teams received many more byes than others. I discovered a new mathematical method that is fair to individuals and teams. The new method was implemented at the 2006 EIWA tournament, which is the regional qualifier for the NCAA championship meet. The method performed well, and will be adopted at future EIWA tournaments. Because Navy is a member of the athletic conference, the new method has a direct impact on midshipmen in athletic competition.

Tiling with Three Squares

Researchers: Associate Professor T.S. Michael and Associate Professor Courtney Moen

This research in progress investigates a particular tiling problem in the spirit of the Frobenius coin-exchange problem. We are given three squares with pairwise relatively prime edge lengths. We seek the largest integer square that cannot be tiled with the three given squares.

Nonlinear Interaction of Light with Liquid Crystals

Researcher: Assistant Professor Aurelia Minut

In this research, we propose a new model of Partial Differential Equations (PDE's) in the field of nonlinear optics of liquid crystals to model the evolution of nematic liquid crystals in the presence of an electromagnetic field. The current research in the field of liquid crystals uses the Oseen-Frank model, the Ericksen model or the Ericksen-Leslie theory and no electromagnetic fields are present. There are indications from this research that the Oseen-

Frank theory needs modifications in order to fit real physical phenomena. We provide modifications by coupling this equation with Maxwell's Equations. One particular application we have in mind is to use this theory to model the propagation of light through liquid crystals and compare it with the propagation of light through optical fibers. We are currently working on the model for the system of differential equations and will study the existence of solutions.

Competition in Two-Dimensional Heterogeneous Environments

Researchers: Dr. Daniel Kern (University of Nevada - Las Vegas)
and Assistant Professor Aurelia Minut

Competition in a heterogeneous environment is studied in two spatial dimensions. We examine the case of two competing populations (one native, one invasive) expanding in a random manner through a domain alternating between two patch types. We focus on a small population of the invasive species being introduced in a limited portion of the environment. We developed the model of differential equations and among the main goals for these equations are:

Examining boundaries, especially between patch types,
Condition for successful invasion -- stability analysis, and
Traveling waves and wave propagation speed.

On a New Notion of Ellipticity

Researchers: Professor George Nakos
and Associate Professor Christopher W. Brown (USNA Computer Science Department)

In this work we generalize the notion of the Douglis-Nirenberg ellipticity (DN-ellipticity) for square systems of partial differential equations. DN-ellipticity was introduced in 1955 for the following reason: under the new notion ellipticity can be preserved when a higher order elliptic equation or system reduces to a lower order one. Unfortunately, DN-ellipticity involves the choice of a weight system that makes the definition noninvariant under singular transformations. This lack of good functoriality properties of DN-ellipticity is a serious obstacle in developing properties of solutions for such systems. In this research we propose a new notion of ellipticity that has all the desired properties and includes all DN-elliptic systems both square and rectangular and does not involve any choice of weights.

Mathematical Foundations for Statistical Learning

Researchers: Associate Professor John F. Pierce and Associate Professor Gary O. Fowler

We are investigating how to develop measures to assess the reliability of empirical estimators for the currents, salinity and temperature gradients in the Chesapeake Bay, and to assess bounds on the level of confidence of decisions based upon them about accepting or rejecting mathematical models for the dynamics in the bay based upon the numerically approximated systems of partial differential equations that arise from hydrodynamic and thermodynamic principles.

The works of S. Smale, R. Devore and others that link results from the theory of functional analysis to problems of statistical learning provide the rational foundation for this form of empirical reasoning. Their tools specify more subtle bounds on rates of convergence for statistical estimators that, in principle, can be applied to the empirical datasets of finite size being collected in the Chesapeake Bay and to the simulations developed from the mathematical models of the bay. In principle, these estimators can be used to ascertain: (1) Are the empirical data being collected even relevant for discerning the validity of the mathematical models? (2) If not, does the source of the discordance lie in the nature of the mathematical models, in the assumptions that lead to the numerical approximations and simulations, or in the experimental or sampling design which produces the empirical data?

A prototypical problem that demonstrates how to engage these functional analytical tools with empirical data was fashioned at Cornell University by T. Joachims when he formulated a rational model for assessing designs for creating learning machines capable of distinguishing documents according to subject. The endeavor for the fall semester involves understanding how Joachim's design of the estimators for the elements in the a-priori bounds in the functional analytical tools alter the design of the sampling of the texts and the design of the vector of features erected from them. The endeavor for the spring semester will be to adapt this paradigm to the context of the Chesapeake Bay.

A Dynamical Systems Approach to Cardiac Arrhythmias

Researchers: Assistant Professor Irina Popovici, Professor B. Mitchell Baker and Professor Mark Kidwell

This project concerns the properties of an iterated system proposed by B. M. Baker and R. Kline to model the behavior of the heart, based on the connection between membrane current kinetics, action potential duration and cardiac rhythm. The starting point is a dynamical system incorporating one or two kinetic parameters which control repolarization, and hence action potential duration. A simply formulated family of even one-dimensional dynamical systems is mathematically capable of producing restitution curves, bifurcation schemes, rhythmic response and chaotic behavior mimicked by cardiac preparations. The family of one- and two-dimensional maps in which the kinetic parameters are modeled follow three basic assumptions: 1) exponential increase of the parameter until a threshold for repolarization is reached, 2) exponential decay of this parameter immediately after threshold is reached, 3) a periodic constraint. The parameter sets we studied consisted of points where the dynamical system transitions from nearly-periodic behavior to chaos for the models that include a refractory period and for the two-dimensional (two loader) model.

Locating Edges and Removing Ringing Artifacts in JPEG Images by Frequency-Domain Analysis

Researchers: Assistant Professor Irina Popovici and Professor W. Douglas Withers

We present a method of locating edges in JPEG-coded images which operates in frequency space on the DCT coefficients. Applied to the quantized DCT coefficients of a block containing a sharp, straight edge, the method an equation for the edge in a fraction of the operations needed to dequantize and transform the coefficients to pixel values. As a sample application of this method, we present a technique for alleviating ringing artifacts in JPEG-coded images.

A Dynamical Systems Approach to Membrane Phenomena Underlying Cardiac Arrhythmias

Researchers: Assistant Professor Irina Popovici, Professor B. Mitchell Baker and Professor Mark Kidwell

We study a model of the human heart cells as a family of dynamical systems parameterized by the concentrations of potassium and magnesium. The two-dimensional maps follow three basic assumptions: 1) increase of the parameters until a threshold for re-polarization is reached, 2) exponential decay of these parameters immediately after threshold is reached, 3) a periodic constraint. The principal results give the existence and stability properties of orbits analogous to the so called escalator orbits of the one-dimensional family, the co-existence of stable orbits for fixed values of the period parameter, existence of bunny-ears orbits. We consider the latter results our most important, owing to conjectures that such period parameter values could produce arrhythmic behavior in a potentially spatially disorganized fashion.

Stochastic Generalized Assignment Problem

Researcher: Commander David Spoerl, USN

Solving a stochastic version of the Elastic Generalized Assignment Problem (EGAP), which incorporates independent, identically distributed resource-consumption coefficients. The Stochastic Elastic GAP (SEGAP) is a two-stage stochastic integer program with simple recourse. We construct a deterministic equivalent and solve using the branch-and-price algorithm. Currently I am completing the model and the math program to solve the SEGAP.

Differential Operators on Orbifolds

Researcher: Assistant Professor William N. Traves

We study differential operators on orbifolds, the quotient of a vector space by a finite group action. Generators and relations for the ring of invariant differential operators are presented. These are also used to compute the ring of differential operators on the orbifold. The techniques used involve Grobner bases and a careful study of the map from the ring of invariant differential operators to the differential operators on the orbifold.

The Fundamental Theorem of Invariant Theory and Differential Operators on Grassmannians

Researcher: Assistant Professor William N. Traves

A presentation for the ring of invariant differential operators on an affine cone over a Grassmannian variety is presented. Generators can be obtained directly from the fundamental theorem of invariant theory, applied to an associated graded ring; more care is needed to lift the relations to the ring of operators. A valid lifting of the relations is presented and it is shown that these, together with the commutator relations, generate the entire ideal of relations. Finally, the map g from the invariant operators to the ring of invariants is used to obtain a presentation of the ring of differential operators on a Grassmannian variety. This is done explicitly for G(2,4), the variety parameterizing two-planes in four-space (projective lines in projective 3-space). Finally a conjecture is presented that describes the kernel of the map g.

Enumerative Algebraic Geometry: Counting Conics

Researchers: Ensign Andrew C. Bashelor, USNR (Class of 2005 Trident Scholar)
Assistant Professor Amy E. Ksir and Associate Professor William N. Traves

This project began as Midshipman Bashelor's Trident Scholar project in 2004-2005. The problems studied were generalizations of Steiner's Problem, which asks, "How many conics are tangent to five given plane conics?" Bashelor solved all of the twenty-one relevant generalizations of this problem, and wrote up these results in his 2005 Trident Report.

Algebraic geometry has a reputation for being very difficult to learn; most introductory books and articles on the subject are aimed at second year graduate students and start by introducing a lot of heavy mathematical "machinery." We realized that Steiner's problem and Bashelor's solution of it would make a very nice expository article, introducing some of the key ideas and tools of algebraic geometry to an undergraduate audience. Over the course of this year, we wrote this expository article and submitted it to a leading mathematics journal aimed at a broad audience. While the mathematical content of this article is largely the same as that of Bashelor's Trident report, the focus and explanation are entirely new.

Quadratic Forms and Cohomology

Researcher: Associate Professor JoAnn S. Turisco

The relatively new methods of "motivic" homotopy, developed by V. Voevodsky, (for which he received the Fields Medal in 2002) have lead to many advances in the algebraic theory of quadratic forms. This theory in algebraic geometry is similar to homotopy theory in algebraic topology. The motivic homotopy category provides a motivic cohomology, K-theory, and algebraic cobordism.

My work involves the application of motivic methods, along with the more established methods using Galois cohomolgy, to compute some of the several invariants associated to quadratic forms. Many new results in quadratic form theory have been obtained using motivic methods. In particular, a Hopf condition for the composition of quadratic forms over fields of characteristic greater than 2 has been proved fairly recently using motivic cohomology. This problem has interested me for years and I am currently working to extend the previous results.

Menu Program for using Stats/List Editor on Voyage 200

Researcher: Professor John C. Turner

This work is a direct outcome of my work for SM230 (and SM219 previously). The Voyage 200 and TI92 include a powerful statistical package. However, the default interface has several serious deficiencies. The biggest is that after a problem has been worked, it is not possible to return to previous calculations. This lack of a "paper trail" inhibits the student from comparing results from previous problems. It also prevents a student from learning from mistakes, say, on an exam. When the exam is returned, it is not possible to verify what the student actually entered into the calculator. Thus, it is not possible for the student to see how to correct this work in the future.

I used the Custom Menu feature of the calculator to create an interface to the Stats/List programs. This approach leaves the inputs and outputs from previous problems on the home screen. This makes it easier for the student to see what was done wrong. It also allows the instructor to see exactly what was typed in for a given problem, rather than what the student thought or meant to type in. Also, the ability to save the home screen (and all the student inputs) provides a way to verify the work on an exam.

A Fast Method for Determining Taylor Series Weights for Finite Difference Approximations of Derivatives on Uniform Grids

Researcher: Commander Vincent Van Joolen, USN

Finite difference approximations for derivatives provide an important means for numerically solving systems modeled with ordinary and partial differential equations. Taylor series polynomials are a major conveyance by which finite difference expressions are generated. Though basic expressions for the lower derivatives are well known, easily derived, or readily available in mathematical tables, expressions involving higher order derivatives or that sample a greater number of grid points are more difficult to come by. Here a scheme is proposed that will allow the reader to theoretically calculate finite difference weights to approximate any derivative using any combination of grid points.

Matrices over Commutative Rings

Researcher: Professor William P. Wardlaw

For years I have been interested in matrices over commutative rings. I have continued my research into such matrices with special attention given to (1) the various definitions of the rank of a matrix over a commutative ring and (2) the characterization of the classical adjoint (adjugate) of a matrix over a commutative ring. This has led to a short proof that row rank equals column rank which was published in *Mathematics Magazine*, as well as a paper with Associate Professor Jody Lockhart on determinants of matrices over the integers modulo m, which has been accepted by *Mathematics Magazine*.

Various Mathematical Problems

Researcher: Professor William P. Wardlaw

Three problems with solutions have been prepared for submission to mathematical journals. These were done in collaboration with Associate Professor Courtney Moen, Assistant Professor Amy Ksir, and Lieutenant Joshua Wood, USN.

A Modified Hill Cipher

Researcher: Professor William P. Wardlaw

Several modifications to the cipher invented in 1931 by Lester S. Hill are suggested to make the cipher more secure. This paper has been accepted for publication in *Cryptologia* upon revision suggested by the referees.

Paving Small Matrices

Researchers: Assistant Professor Vrej A. Zarikian and Gary Weiss (University of Cincinnati)

Anderson's "Paving Conjecture" states that for every $\varepsilon > 0$, there exists a positive integer k such that every norm-one matrix A with zero diagonal admits a k-paving of norm less than or equal to $1 - \varepsilon$. The truth of the Paving Conjecture is equivalent to an affirmative answer to the "Kadison-Singer Problem", a famous question in operator algebra theory, open since 1959. Owing to recent work of Casazza et al. showing that the Kadison-Singer Problem is equivalent to a number of important questions in math and engineering, there is renewed interest in the Kadison-Singer Problem, and therefore the Paving Conjecture. Together with Gary Weiss, we have investigated whether every matrix is 3-paveable (it is known that not every matrix is 2-paveable). Using a combination of operator theory, graph theory, and computer experimentation, we have produced examples of 7-by-7 circulant matrices whose 3-paving norm is 0.8231. Before our work, the highest known 3-paving norm was 0.6667, and it was only attained for very large matrices. We are hopeful that we can produce a "small" matrix whose 3-paving norm is 1.000, thereby disproving the Paving Conjecture for the case $k = 3$ (we believe that the Paving Conjecture is false).

Alternating-Projection Algorithms for Operator-Theoretic Calculations

Researcher: Assistant Professor Vrej A. Zarikian

We show how alternating-projection algorithms can be used to solve a variety of operator-theoretic problems, including deciding complete positivity, computing completely bounded norms, computing norms of Schur multipliers, and matrix completion/approximation problems.

Alternating-Projection Algorithms for Operator-Theoretic Calculations

Researcher: Assistant Professor Vrej A. Zarikian

In the recently developed theory of operator spaces, the important linear transformations are not the bounded ones, but rather the completely bounded ones. Often, problems in operator space theory come down to computing the norm (or size) of such a linear transformation. Unfortunately, such calculations are usually impossibly difficult, and one must be content with a reasonable estimate. In finite-dimensions, however, one would hope that a numerical algorithm could be devised to compute completely bounded norms. I succeeded in doing just that. The key realization was that computing completely bounded norms reduces to deciding whether certain convex sets intersect, a problem which can be solved using von Neumann's alternating-projection algorithm from 1933.

One-Sided M-Ideals and Multipliers of Operator Spaces

Researcher: Assistant Professor Vrej A. Zarikian

The M-ideal theory of Alfsen and Effros, which dates to 1972, is an important tool in the study of Banach spaces. In my dissertation under Effros, I generalized M-ideals to operator spaces, introducing the notion of a one-sided M-ideal. One-sided M-ideals are best studied using Blecher's theory of one-sided multipliers. Together with Blecher, I have advanced the theory of one-sided M-ideals and multipliers considerably (see Attachment #4). Consequences include streamlined proofs of characterization results for operator algebras and operator modules. The current project attempts to resolve some of the many remaining questions. For example, the proximinality question: Does every element of an operator space have a best approximation from the one-sided M-ideal?

Midshipman Research Course Projects

Justice in Liver Transplant Allocation, an Integer Programming Approach

Researcher: Midshipman 1/C Peter Barkley, USN
Adviser: Assistant Professor Sommer Gentry

The existing system for distributing donated livers is geographically inequitable. As livers are given to the neediest person within the region, people in great need of a liver do not receive one in some regions, while people with much less need receive a transplant in other regions. We show how to make the system more just, by redistricting the regions so that the difference in the boundary need level at which people receive organs is constrained to be small. Organs have a limited maximum shipping distance, and political constrains have frequently hindered attempts to modify the regional system. Furthermore, the time between removal and transplation (Cold Ischemia Time) is directly correlated with the subsequent success of the transplant. Our model, therefore, maintains the current number of regions at 11, and minimizes the size of the regions while constraining the justice of the distribution. Using methods similar to those used in school redistricting problems, we have designed an integer program to minimize the distance between a "seed" geographical unit for each region and all the other geographical units contained in that region. This minimization is constrained by a limit on the difference between the levels of neediness at which each region runs out of donors to provide organs. The completed research constituted Mr. Barkley's honors project, which won the math department's Military Applications Award for a thesis in operations analysis.

Linear Feedback Shift Registers and Cyclic Codes in Sage

Researcher: Midshipman 1/C Timothy B. Brock, USN
Adviser: Professor W. David Joyner

Honors Project 2006: This paper discusses the history of linear feedback shift registers (LFSR) in cryptographic applications and will attempt to implement an algorithm in SAGE and Python (www.python.org) to create a linear feedback shift register sequence (LFSR sequence) in cryptography. Also, this paper will attempt to implement the Berlekamp Iterative Algorithm in SAGE and Python. This algorithm will be able to use the Linear Feedback Shift Register sequence generated by the first algorithm to find the sequence's connection polynomial.

The research attempted to show that the connection polynomial of a given LFSR sequence is the reverse of a generator polynomial of the cyclic code of length *p,* where *p* is also the period of the LFSR sequence. This will provide a connection between cyclic error-correcting codes and LFSR sequences.

Long Quadratic Residue Codes

Researcher: Midshipman 1/C Greg Coy, USN
Adviser: Professor W. David Joyner

A long standing problem has been to develop "good" binary linear block codes to be used for error correction. The goal of this project was to investigate a family of codes called Quasi-Quadratic Residue (QQR) codes. The numerical and theoretical investigation of these codes yielded little in finding a "good" family to be used in error-correction. However, an interesting family of codes was discovered called Long Quadratic Residue (LQR) codes. These codes are based off the construction of the QQR codes. The LQR codes have "good" parameters, essentially meaning that their information rate and relative minimum distance are bounded away from zero. Further, I have shown that this family of curves is incompatible with the "fake Goppa conjecture," since this family of codes beats the Gilbert-Varshamov bound for binary codes. Most importantly, I have used LQR codes to develop the "small content conjecture." This essentially shows that the Goppa conjecture is incompatible with a conjecture on hyperelliptic curves over finite fields.

Nearly-Involutive Matrices for the Keyspace of the Hill Cipher

Researcher: Midshipman 1/C Kindle Clarke, USN
Advisers: Professor George Nakos and Assistant Professor Alexis A. Alevras

The Hill Cipher was an important cryptosystem of classical cryptology in the 1930's (Hill 2; 3). The basic idea of the Hill Cipher is you first pick a modulus m, where m is a positive integer m>1. A message is converted into an n-vector **x** with components in Z_m. Let *A* be in GL(n, Z_m), so *A* is an invertible matrix modulo m with entries in Z_m. The encryption of **x** is **y**=*A*x and therefore the decryption is **x**=A^{-1}**y**. In 1931, Hill suggested that the use of involutive matrices, i.e. matrices satisfying the identity: $A^2 = I$

This project studied of the number of solutions of the closely related matrix equation: $X^2 = cI$ where $c \in Z_p$, *p* being a prime and *I* is the identity matrix. Results were obtained for 2x2 and 3x3 matrices.

Singular Analysis of an N-dimensional Cosmic String

Researcher: Midshipman 1/C Gregory Dietzen, USN
Advisers: Professor Deborah A. Konkowski, Professor Mitch Baker and Assistant Professor Alexis A. Alveras

Cosmic strings are theoretical vestiges of our early universe which may help us understand its formation. Their study is also intriguing because of their possible link to M-Theory. In this research Midshipman Dietzen strove to understand the singularities, both classical and quantum mechanical, of an n-dimensional cosmic string.

The study began with a study of the literature for 2- and 4-dimensional cosmic strings and then an analysis of the next most difficult case, a 5-dimensional cosmic string. Classical singularities were studied using MAPLE for a curvature singularity check and, finding no curvature singularities, a topological analysis was completed which showed the existence of a quasiregular singularity. Classical scalar waves were then studied. This involved writing and solving the general relativistic Klein-Gordon equation. With the classical analysis in hand quantum singularities were next studied. This involved Midshipmen Dietzen learning some of the basics of operator theory in functional analysis and the application of this knowledge to this particular case. Quantum singularities were found for some, but not all, cases. Midshipman Dietzen then tackled the n-dimensional cosmic string and found similar classical and quantum singularity structure.

In conclusion, Midshipman Dietzen's analysis showed that a cosmic string has a classically singular spacetime. It is cone-shaped causing a topological defect which made it impossible to analyze using the usual curvature analysis of Einstein's relativity. However, basic quantum theory could be applied showing the n-dimensional string may or may not be quantum mechanically singular. He successfully found the range of parameters when it is, values that must be determined from the study of each wave mode and each individual string.

Dynamic Systems: Spring, Cycloid and Pendulum Systems

Researcher: Midshipman 1/C John H. Doherty, USN
Adviser: Professor Mark Kidwell

The simplest of the dynamic systems that was examined in this project is the motion of a mass attached to a spring under the influence of an outside force such as an electromagnet. This produced a simple linear ordinary differential equation where the position x is dependent upon a single variable t. The particular solution was found with simplistic methods learned in an introductory course on differential equations.

The second system that was studied is the cycloid. This system can be physically represented by tracing the path of a specific point of a bicycle wheel while the wheel moves along a surface. Although it is not intuitively obvious, the upside down arc of the cycloid also plots the motion of the quickest possible path of a bead sliding along a string acting solely under the influence of its own gravity traveling between two points as determined by Johann Bernoulli in the Brachistochrone Problem. For the purposes of this study, the motion of a marble along a frictionless and dragless Brachistochrone was considered. It was revealed that the motion of this marble had numerous interesting properties under these circumstances including that it maintains constant period for any initial displacement.

Another conceptually similar but more complicated dynamical system is a pendulum with an external torque acting upon it. For this problem, the external torque acting upon the pendulum bob was either zero or a constant value. However; the torque due to the weight of the bob changed as theta changes. The perpendicular distance from the point of rotation served as the effective torque arm. This distance was equal to the product of the length of the pendulum arm and the sine of the angle theta between the pendulum arm and a vector normal to the ground. Therefore, the mass times the second derivative of theta with respect to time equaled the sum of the moment due to the torque and the moment due to the weight. This can be expressed mathematically as follows:

$$m * L * \frac{d^2(\theta(t))}{d(t)^2} = -m * L * \sin(\theta(t)) + T_0$$

It was revealed that these three unique systems have numerous interrelated properties and provide a fascinating application of differential equations.

Higher Dimensional Linear Regression as a Mathematical Foundation for Data Classification

Researcher: Midshipman 1/C Robert B. Irving, USN
Adviser: Associate Professor Gary O. Fowler

With the ultimate goal of constructing a classification machine which uses only the information contained within a training sample in order to classify as-yet-unseen data points, this project built upon Shawe-Taylor and Christianini's mathematical analysis and analyzed the geometrical features of their perspective (Shawe-Taylor, 25). The author begin by creating a geometric model for the more familiar regression problem and then applied similar techniques to the classification problem. More specifically, it was shown how higher-dimensional linear regression and classification can be captured by embedding the sample data into a higher-dimensional space of features, analyzing this embedded data with simple geometric tools, and then projecting the model back into our original data space to obtain intricate fits (regression) or separations (classification).

Mathematics Honors Project Summary: Low Density Parity Check Codes

Researcher: Midshipman 1/C Gordon R. McDonald, USN
Adviser: Professor W. David Joyner

This honors project investigated the subject of Low Density Parity Check codes (LDPC codes). In it the author attempted to define a general set of conditions that ‘good' Low Density Parity Check codes each contained two iterative methods were also investigated which theoretically decoded LDPC codes. Those algorithms are called Bit-Flipping and Gallager Hard Decision decoding. The algorithms were illustrated by means of detailed examples and implemented in GAP, an open source computer algebra system. Furthermore, the functions that were written in GAP are now part of the newest GAP update. Overall, the author was highly unsuccessful in generating LDPC codes which were of any use. This lead to some difficulty in implementing the few ‘good' LDPC codes which appeared in two decoding algorithms. In conclusion, it was found that the decoding algorithms depend upon very specific constructions of LDPC codes which are oftentimes too idealistic to obtain using the method of construction attempted. There is, however, an alternative method to construct LDPC codes which could possibly make the decoding algorithms much more successful. This could easily be investigated with further research.

Dynamical Systems and Irrational Angle Construction by Paper-Folding

Researcher: Midshipman 1/C Cayanne McFarlane, USN
Adviser: Professor W. Douglas Withers

Hilton and Pedersen have previously shown how to construct any rational angle to arbitrary precision by a periodic sequence of folding a strip of paper. We show how this method can be generalized to construct any angle, rational or irrational, by using a connection between the method and a simple dynamical system to generate the appropriate aperiodic folding sequence.

God’s Algorithm on the Edges and Corners of the Rubik’s Cube

Researcher: Midshipman 1/C Daniel C. Ryan, USN
Adviser: Assistant Professor Amy E. Ksir

Rubik’s cube is a puzzle that can be represented using group theory. “God’s Algorithm” is the unknown answer to the following question: Given a Rubik’s cube in its maximal state of disorder, what is the smallest possible number of moves necessary to solve it? In this project, Midn Ryan tackled the two smaller, related problems: finding God’s Algorithm for just the edges of the Rubik’s cube, and finding God’s Algorithm for just the corners of the Rubik’s cube. Midn Ryan used group theory and his knowledge of the Rubik’s cube to tackle the first part of the problem, which was to understand the structure of the Rubik’s cube group. For the second part of the project, he was able to adapt some existing computer programs towards finding the solution. He was able to confirm intermediate results, and only lack of computer power stopped him from obtaining a final result.

Profession A Probability City Patrol

Researcher: Midshipman 1/C William Sumpton, USN
Advisers: Lieutenant Commander Kyle Kliewer, USN, and Professor Emeritus Charles Mylander

The United States Marine Corps currently engages in city patrols in hostile areas, most noticeably cities like Baghdad and Fallujah. A searcher’s ability to visually cover an area from its perimeter depends on how far the lines of sight, propagating from the perimeter inward, can travel before becoming obstructed. The probability of finding a target in the city block is then proportional to the area covered and time spent by the search. A battlefield commander could use information as to what areas of a city can be covered by an observer on the ground to better implement his assets.

The search of a block of structures in a city is modeled using a combination of matrices and line of sight from a variety of observation positions based on searching around the parameter. From the data, two specific observations are made: what area can the observer see and not see, as well as how many times a location is spotted from a parameter position. The probabilities of finding a target for the entire block as well as specific locations in the interior are computed. These probabilities will be affected by different parameters such as a diminishing line of sight with distance, a randomly moving target, as well as a target that moves between predetermined points. Safer ingress routes into the block can then be realized by examining the results of the computations.

Publications

Journal (Refereed) Manuscripts

ALEVRAS, Alexis A., Assistant Professor, and PRICE, Geoffrey, Professor, (co-authors), "Cocycles for One-parameter Flows of B(H)", *Journal of Functional Analysis*, 230 (2006), pp. 1-64.

> The set of local cocycles is a natural invariant for an E_0-semigroup. It has a multiplicative structure, as well as a partial order structure among its positive elements. In particular, the unitary local cocycles form a topological group which may be appropriately viewed as the automorphism group of the E_0-semigroup, while the set of positive contractive local cocycles is order isomorphic to the set of flows of completely positive maps dominated by the semigroup. The local cocycles have been computed for the standard, type I examples of the CAR/CCR flows by W. Arveson and R. Bhat. In this paper we compute for the first time the local cocycles for a class of type II E_0-semigroups of B(H) with index zero, and describe the order structure as well as the multiplication in terms of the boundary weight associated with such a semigroup.

ALEVRAS, Alexis A., Assistant Professor, "The Gauge Group of an E_0-semigroup", *Contemporary Mathematics*, in press.

> We discuss the role of the group of unitary local cocycles in the study of E_0-semigroups of B(H) and describe the computation of that group for a class of E_0-semigroups of type II and index zero.

BUCHANAN, J.L., Professor, Gilbert, R.P., and Xu, Y., "Ultrasound as a Diagnostic Tool to Determine Osteoporosis", *Advances in Analysis*, pp. 345-354, World Scientific.

> Cancellous bone is known to be poroelastic in structure. Ultrasonic wave propagation in cancellous bone can be formulated by using Biot's equations. In this paper we present some results in our ongoing research on the reflection and transmission of ultrasonic waves in cancellous bone. We investigate the relations among reflected waves, transmitted waves and the Biot coefficients. We present an algorithm for the determination of the porosity of cancellous bone.

CRAWFORD, Carol G., Professor, "6 Steps to Successful, Interactive Math WEB Design - An Innovative Real World Calculus Laboratory Designed with Java for the United States Naval Academy", *Proceedings of the Joint Meeting of the 3rd International Conference on Education and Information Systems, Technologies and Applications (EISTA 2005) and the International Conference on Cybernetics and Information Technologies, Systems and Applications (CITSA 2005).* Volume I, pages 223-228.

> The author presents "6 Steps" to help in the design of an effective, interactive mathematics resource for higher education. These steps reflect her work with Professor Mark Meyerson in the creation of an innovative, online Real World Calculus Laboratory for the 3- semester Calculus sequence taken by all midshipmen at The United States Naval Academy. The Curriculum Development Program at the Academy funded the research for this two-year design project.

GAGLIONE, Anthony M., Professor (co-author), "Notions of Discrimination," *Communications in Algebra*, September 2005, in press.

> As an outgrowth of the study of algebraic geometry over groups, discriminating groups were introduced. Many important universal type groups such as Higman's universal group and Thompson's group F were shown to be discriminating. Squarelike groups were then introduced to better capture axiomatic properties of discrimination. In the present article squarelike groups are reinterpreted in terms of discrimination of quasi-varieties, and the relationship with an older version of discrimination, termed varietal discrimination here, is studied.

GAGLIONE, Anthony M., Professor (co-author), "An Embedding Theorem for Groups Universally Equivalent to Free Nilpotent Groups", *Groups St. Andrews 2005***,** November 2005, in press.

> Let F be a finitely generated nonabelian free group. Kharlampovich and Myasnikov proved that any finitely generated group H containing a distinguished copy of F is universally equivalent to F in the language of F if and only if there is an embedding of H into Lyndon's free exponential group $F^{Z[t\}}$ which is the identity on F. Myasnikov posed the question as to whether or not a similar result holds for finitely generated free nilpotent groups with Lyndon's group replaced by Philip Hall's completion with respect to a suitable binomial ring. We answer his question here in the affirmative.

GAGLIONE, Anthony M., Professor, (co-author), "Reflections on Discriminating Groups", *Journal of Group Theory*, February 2006, in press.

> Here we continue the study of discriminating groups as introduced by Baumslag, Myasnikov and Remeslennikov in [BMR2]. First we give examples of finitely generated groups which are discriminating but not trivially discriminating in the sense they embed their direct squares and then show how to generalize these examples. In the opposite direction we show that if F is a nonabelian free group and R is normal in F such that F/R is torsion free, then F/R′, where R′=[R,R] is the commutator subgroup of R, is nondiscriminating.

GAGLIONE, Anthony M., Professor (co-author), "The Search for Origins of the Commutator Calculus", *Proceedings in Honor of Gehard Rosenberger*, April 2006, in press.

> The origins of the commutator calculus and the relationships between some of the key investigators are discussed. Also their subsequent work in the Age of the Enigma,1930-1950,is presented. Several questions about the history are proposed.

GAGLIONE, Anthony M., Professor (co-author), "A Note on Nondiscrimination of Nilpotent Groups and Mal'cev Completions," *Proceedings in Honor of Gehard Rosenberger*, April 2006, in press.

> In this note we prove that if a finitely generated nilpotent group is discriminating then so is its Mal'cev completion. From this, we recover that a finitely generated nilpotent group can be discriminating only if it is torsion free abelian. This was proved in a very different way by Myasnikov and Shumyatsky but the method of this paper should shed additional light on nondiscrimination in an alternate manner.

GAGLIONE, Anthony M., Professor, (co-authored), "Finitely Presented Infinite Torsion Groups and a Question of V.H. Dyson," *Proceedings in Honor of Gehard Rosenberger*, April 2006, in press.

> Let L be a first-order language appropriate for group theory. The universal theory of a class of groups is the set of all universal sentences of L true in every group in this class. In this note, it is shown that if the universal theory of the torsion groups coincides with the universal theory of the finite groups, then there cannot exist a finitely presented infinite group of finite exponent.

GAGLIONE, Anthony M., Professor (co-author), "Unions of Varieties and Quasivarieties," *Proceedings in Honor of Gehard Rosenberger*, April 2006, in press.

> In this paper we characterize unions and direct unions of varieties and quasivarieties of operator algebras in terms of closure properties. In terms of groups these apply to many important classes such as nilpotent groups, solvable groups and equationally Neotherian groups.

GENTRY, Sommer, Assistant Professor (co-author), "Kidney Paired Donation: Optimizing the Use of Live Donor Organs", *Journal of the American Medical Association,* vol. 293, pp.1883-1890, 2005. This paper was awarded the 2005 Vanguard Prize from the American Society for Transplant Surgeons.

> Blood type and crossmatch incompatibility will exclude at least one third of patients in need from receiving a live donor kidney transplant. Kidney paired donation (KPD) offers incompatible donor/recipient pairs the

opportunity to match for compatible transplants. We designed a mathematically verifiable optimized matching algorithm and compared it with the scheme currently used in some centers and regions. We simulated patients from the general community with characteristics drawn from distributions describing end-stage renal disease patients eligible for renal transplantation and their willing and eligible live donors. Even if only 7% of patients awaiting kidney transplantation participated in an optimized national KPD program, the health care system could save as much as $750 million. Optimized matching affords patients the flexibility of customizing their matching priorities and the security of knowing that the greatest number of high-quality matches will be found and distributed equitably.

GENTRY, Sommer, Assistant Professor (co-author), "A Comparison of Populations Served by Kidney Paired Donation and List Donation", *American Journal of Transplantation*, 2005, August, 5(8), pp.1914-1921.

Options for utilizing live donor kidneys from those who are blood type incompatible or crossmatch positive with their intended recipients include kidney paired donation (KPD), list paired donation (LPD) and desensitization. We simulated patients and their potential donors to determine which recipients could receive a kidney through KPD and LPD. At population sizes predicted to be achieved by a national paired donation system, the role of LPD became minimal, with only 3.9% of pairs unmatched through KPD eligible for LPD. Considerable overlap was seen between the pairs unmatchable by KPD and those ineligible for LPD, namely less-demanded donors and hard-to-match recipients. For this population, the best option may be desensitization.

GENTRY, Sommer, Assistant Professor (co-author), "Characterization of Waiting Times in a Simulation of Kidney Paired Donation", *American Journal of Transplantation*, 2005, October 5(10), pp.2448-2455.

A national kidney paired donation (KPD) program will substantially increase transplant opportunities for recipients with blood type incompatible or cross-match positive donors. It seems likely that donor-recipient pairs with certain blood types, races or restrictions will wait longer than others for a match, although no data exist to confirm this assumption. We simulated patients and characterized the predicted waiting times for different blood type sub-groups, as well as the effects of patient-imposed restrictions on waiting time. We also compared waiting times of different racial sub-groups. These data provide the first waiting time predictions that can aid patients with incompatible donors in choosing between KPD and desensitization, and can facilitate planning for a national paired kidney donation program.

GENTRY, Sommer, Assistant Professor (co-author), "Multiplying the Benefit of Live Non-directed Donors through Domino Kidney Paired Donation", *Lancet*, in press.

The allocation of living nondirected donor (LNDD) organs has been at the discretion of individual transplant programs, including the model that governs recipient selection. In all cases, the LNDD's gift has resulted in a single transplant. We demonstrate using optimized allocation on a simulated patient databases, that an LNDD donation can be multiplied by allocating the kidney to a recipient with an incompatible donor who is willing to donate their kidney to a patient on the deceased donor waiting list, producing a domino effect. We also present a case report of an LNDD kidney that was allocated to a domino paired donation (DPD) and, as a result, two transplants were accomplished. This allocation strategy is then discussed in the context of the ethical arguments that guide current models of allocation for LNDD.

GENTRY, Sommer, Assistant Professor (co-author), "Relative Roles for List Paired Exchange, Live Donor Paired Exchange, and Desensitization", *American Journal of Transplantation*, 2006, February, 6(2), p. 437.

This letter is an invited response to Morrissey P., in support of list paired exchange, *American Journal of Transplantation* February, 6(2): 2006. We describe our data showing that list paired exchange will become less important as living donor paired exchange becomes more widely available. Our simulated matching results also suggest certain subgroups of patients are unlikely to be served well by either of these two modalities and might benefit most from desensitization.

HOFFMAN, Michael E., Professor, "Symmetric Functions, Quasi-symmetric Functions and Rooted Trees," *Oberwolfach Reports,* in press.

> The Hopf algebra of symmetric functions is a subalgebra of the larger Hopf algebra of quasi-symmetreic functions. Similarly, there is an injective map of Hopf algebras from the Grossman-Larson Hopf algebra of rooted trees to L. Foissy's Hopf algebra of planar rooted trees. These four Hopf algebras fit into a commutative diagram, and several results can be obtained by consideration of the images of objects under the maps forming this diagram.

JACKSON, R.K., Assistant Professor, Pinto, D.J., and Wayne, C.E., "Existence and Stability of Traveling Pulses in a Continuous Neuronal Network", *SIAM J. Appl.Dyn. Syst.,* 4 (2005), pp. 954-984.

> In this work, we examined the existence and stability of traveling pulse solutions of a set of integro-differential equations that describe activity in a spatially extended population of synaptically connected neurons. Our results were guided by the local behavior of individual neurons. When neurons have a single stable state, we demonstrated the existence of two traveling pulse solutions in a connected network. For bistable neurons, we demonstrated the existence of a stationary pulse solution and, in some cases, a single traveling pulse solution. Additionally, we performed a linear stability analysis of all these pulse solutions by constructing an Evans function. Numerical evaluation of these traveling pulses suggests that for the network of monostable neurons, fast pulses are stable and slow pulses are unstable. For the bistable system, all pulses are unstable. The system simplifies considerably in the case of stationary pulses and we present an alternate simpler analysis of their existence and instability.

JOYNER, W.D., Professor and KSIR, A.E. Assistant Professor, "Decomposing Representations of Finite Groups on Riemann-Roch Spaces," *Proceedings of the American Mathematical Society*, in press.

> If G is a finite subgroup of the automorphism group of a projective curve X and D is a divisor on X stabilized by G, then under the assumption that D is nonspecial, we compute a simplified formula for the trace of the natural representation of G on the Riemann-Roch space L(D).

JOYNER, W.D. Professor and KSIR, A.E. Assistant Professor, "Automorphism Groups of Some AG Codes", *IEEE Transactions on Information Theory*, July 2006, in press.

> We show that, in many cases the automorphism group of a curve and the permutation automorphism group of a corresponding AG code are the same. This generalizes a result of Wesemeyer beyond the case of planar curves.

JOYNER, W.D., Professor and KSIR, A.E. Assistant Professor, "Modular Representations on Some Riemann-Roch Spaces of Modular Curves X(N)," *Computational Aspects of Algebraic Curves*, Lecture Notes Ser. Comput., 13, World Sci. Publ., Hackensack, NJ, 2005, pp.163-205.

> We compute the PSL(2,N)-module structure of the Riemann-Roch space L(D), where D is an invariant non-special divisor on the modular curve X(N), with N > 5 prime. This depends on a computation of the ramification module, which we give explicitly. These results hold for characteristic p if X(N) has good reduction mod p and p does not divide the order of PSL(2,N). We give as examples the cases N=7, 11, which were also computed using GAP. Applications to AG codes associated to this curve are considered, and specific examples are computed using GAP and MAGMA.

KONKOWSKI, Deborah A., Professor, (co-author), "Mining Metrics for Buried Treasure," *Proceedings of Malcolm@60 Festschrift Conference*, *General Relativity and Gravitation*, to appear. It is currently available on the Web :"On Line" Link: http://www.springerlink.com/(jneqzy45hfdwbz45gzuky355)/app/home/contribution.asp?referrer=parent&backto=issue,9,25;journal,1,391;linkingpublicationresults,1:101151,1 ArXiv Link: http://arxiv.org/abs/gr-qc/0412137

> The same but different: That might describe two metrics. On the surface CLASSI may show two metrics are locally equivalent, but buried beneath one may be a wealth of further structure. This was beautifully

described in a paper by M.A.H. MacCallum in 1998. Here I will illustrate the effect with two flat metrics -- one describing ordinary Minkowski spacetime and the other describing a three-parameter family of Gal'tsov-Letelier-Tod spacetimes. I will dig out the beautiful hidden classical singularity structure of the latter (a structure first noticed by Tod in 1994) and then show how quantum considerations can illuminate the riches. I will then discuss how quantum structure can help us understand classical singularities and metric parameters in a variety of exact solutions mined from the Exact Solutions book.

KONKOWSKI, Deborah A., Professor, (co-author), "Classical and Quantum Singularity of Levi-Civita Spacetimes with and without Cosmological Constant," appears in the Web Proceedings of Workshop on Dynamics and Thermodynamics of Black Holes and Naked Singularities (Milan, Italy).(14 pages): Publication Link: http://www.mate.polimi.it/bh/ ArXiv Link: http://arxiv.org/abs/gr-qc/0410114

Levi-Civita spacetimes have classical naked singularities. They also have quantum singularities. Quantum singularities in general relativistic spacetimes are determined by the behavior of quantum test particles. A static spacetime is said to be quantum mechanically singular if the spatial portion of the wave operator is not essentially self-adjoint on a C_0^∞ domain in L^2, a Hilbert space of square integrable functions. Here we summarize how Weyl's limit point-limit circle criterion can be used to determine whether a wave operator is essentially self-adjoint and how this test can then be applied to scalar wave packets in Levi-Civita spacetimes with and without a cosmological constant to help elucidate the physical properties of these spacetimes.

KSIR, A.E., Assistant Professor, (co-author), "*FoxTrot* Brings Mathematics to the Comics Page," *Math Horizons,* November 2005, pp. 18-20.

Fox Trot is a syndicated comic strip appearing in newspapers across the country and around the globe. Its creator, Bill Amend, was a physics major in college and often includes mathematical references in the strips. We examine some of these references and conduct an interview with Amend.

LIAKOS, A., Assistant Professor, (co-author), "Time Dependent Flow Across a Step with Slip with Friction Boundary Condition", *International Journal for Numerical Methods in Fluids*, to appear. (Manuscript number FLD-05-0058.R1, July 2005.

This paper studies numerically the slip with friction boundary condition in the time-dependent incompressible Navier-Stokes equations. Numerical tests on two-and three-dimensional channel flows across a step using this boundary condition on the bottom wall are performed. The influence of the friction parameter on the flow field is studied and the results are explained according to the physics of the flow. Due to the stretching and tilting of vortices, the three-dimensional results differ in many respects from the two-dimensional ones.

LIAKOS, A., Assistant Professor, (co-author), "A 3-dimensional Simulation of Barrier Properties of Nanocomposite Films", *Journal of Membrane Science*, Volume 263, Issues 1-2, 15 October 2005, Pages 47-56.

Monte Carlo simulations were conducted in two and three dimensions to compute the diffusion coefficients for membranes containing oriented platelets. The effects of platelet aspect ratio, relative separation, loading and spatial dimension were studied. The model of Aris (1986) was found to accurately predict these effects in two dimensions. However, in 3D this model over-predicts the effects of the filler by 50% when compared to simulations. The results have particular value for nanocomposite film currently under development for use in food packaging applications.

LOCKHART, Jody, Associate Professor, and WARDLAW, William P., Professor, "Determinants of Matrices over the Integers Modulo m", *Mathematics Magazine,* in press.

In this paper, the number of matrices over the integers modulo m with determinant k is computed. The problem is first reduced to the case when m and k are both prime powers. This is done by showing that the number of such matrices is multiplicative in m and that the number with determinant a constant times a power of a prime p is equal to the number with determinant the power of p if the constant is not divisible by

p. Next, a recursive formula giving the number of matrices with determinant 0 over the integers modulo m where m is a prime power is found. The proof of the formula involves a very careful counting argument. Finally, the number of matrices over the integers modulo m with determinant k where m and k are both prime powers is expressed in terms of the number of matrices with determinant 0.

LOCKHART, Jody, Associate Professor, and WARDLAW, William P., Professor, “A Modified Hill Cipher”, *Cryptologia*, in press.

Several modifications of the cipher invented in 1931 by Lester S. Hill are suggested to make the cipher more secure.

LOCKHART, Jody, Associate Professor, with WARDLAW, William P., Professor, Problem 1727, *Mathematics Magazine*, October 2005.

LUCAS, Marc D., Lieutenant Commander, USN, (co-author), “Drilling for $4,100 a Day”, *U.S. Naval Reserve “Reservist” Magazine 2005.*

This entry provides an actuarial perspective of the present value of a drill or active duty day as a portion of the remaining commitment to obtain a reserve retirement benefit.

LUCAS, Marc D., Lieutenant Commander, USN, “Drilling for $4,200 a Day”, *The Reserve Officer Association “Officer” Magazine 2006.*

This entry provides an actuarial perspective of the present value of a drill or active duty day as a portion of the remaining commitment to obtain a reserve retirement benefit updated for 2006.

MARUSZEWSKI, Richard, Professor, and CAUDLE, Kyle, Lieutenant Commander USNR, “Approximating Integrals Using Probability”, *MACE Journal*, volume 39, #2, pp 143-147, 2005.

Monte Carlo techniques and expectations from probability are used to approximate the value of a definite integral. The concepts are implemented via algorithms written in Visual Basic.

MARUSZEWSKI, Richard, Professor, "Do Differential Equations Swing?”, *MACE Journal,* in press.

A look at forcing functions and resonance using both analytic techniques and MAPLE approximations. The ideas are illustrated by the example of the playground swing.

MELLES, Caroline, Associate Professor, (co-author), “Classical Poincaré Metric Planted off Singularities using a Chow-Type Theorem and Desingularization” *Annales de la Faculté des Sciences de Toulous*, in press.

We construct complete Kähler metrics on the nonsingular set of a subvariety X of a compact Kähler manifold. To that end, we develop (i) a constructive method for replacing a sequence of blow-ups along smooth centers, with a single blow-up along a product of coherent ideals corresponding to the centers and (ii) an explicit local formula for a Chern form associated to this ‘singular’ blow-up. Our metrics have a particularly simple local formula of a sum of the original metric and of the pull back of the classical Poincaré metric on the punctured disc by a ‘size-function’ S_I of a coherent ideal I used to resolve the singularities of X by a ‘singular’ blow-up, where $(S_I)^2 := \sum_{j=1}^{r} |f_j|^2$ and the f_j's are the local generators of the ideal I. Our proof of (i) makes use of our generalization of Chow's theorem for coherent ideals. We prove Saper type growth for our metric near the singular set and local boundedness of the gradient of a local generating function for our metric, motivated by results of Donnelly-Fefferman, Ohsawa, and Gromov on vanishing of certain L_2-cohomology groups. In an appendix we give a simple constructive proof of a valuation criterion due to M. Lejeune and B. Teissier.

MICHAEL, T. S., Associate Professor (co-author), "Packing Boxes with Bricks", *Mathematics Magazine***,** 79 (2006), pp.14-30.

We provide an expository account of arithmetic and geometric conditions that are necessary and sufficient for a d-dimensional rectangular box to be tiled (packed) by translates of two given rectangular bricks. Our work unifies a number of results in the tiling literature.

MICHAEL, T. S., Associate Professor (co-author), "Optimal Strategies for Node Selection Games: Skew Matrices and Symmetric Games", *Linear Algebra and Its Applications,* 412 (2006), pp.77-92.

Each of two players simultaneously selects a node in a directed graph with the payoff determined by the direction of any arc between the selected nodes. We investigate the optimal strategies for the two players in this node selection game. We show that when the graph is a non-trivial bipartite graph, the optimal strategy is never unique.

MICHAEL, T. S., Associate Professor, "Ryser's Embedding Problem for Hadamard Matrices", *Journal of Combinatorial Designs,* 14 (2006), pp.41-51.

We obtain better bounds for the minimum order of a Hadamard matrix that contains a rectangular all 1's submatrix of a given size.

MICHAEL, T. S., Associate Professor (co-author), "Guarding the Guards in Art Galleries", *Math Horizons,* 14 (2006), 22-23, 25.

We give an expository account of some recent research in computational geometry. Let P be a polygon (the art gallery) with n sides. We wish to select a set G of points (the guards) so that (a) every point in the polygon is visible to some guard; and (b) every guard is visible to at least one other guard. What is the minimum number of guards $g(n)$ that is guaranteed to suffice for all polygons with n sides? We prove that g(n) = floor[(3n-1)/7] for n at least 5. Also, if we restrict to rectilinear galleries the function is floor(n/3) for n at least 6.

POPOVICI, Irina, Assistant Professor, and WITHERS, W. Douglas, Professor, "Custom-Built Moments for Edge Location", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 28 (2006), no. 4, pp.637-642.

We present a general construction of functions whose moments serve to locate and parameterize step edges within an image. A characteristic of the moment approach to edge location is that the result is specified parametrically rather than as a set of pixels; thus our method performs best on sharp, straight edges. Previous use of moments to locate edges was limited to functions supported on a circular region, but our method allows the use of ``custom-designed" functions supported on circles, rectangles, or any desired shape, and with graphs whose shape may be chosen with great freedom. We present analyses of the sensitivity of our method to pixelization errors or discrepancy between the image and an idealized edge model. The parametric edge description provided by our method makes it especially suitable as a component of wedgelet image coding.

POPOVICI, Irina, Assistant Professor, and WITHERS, W. Douglas, Professor, "Locating Thin Lines and Roof Edges by Custom-Built Moments", *Proceedings of the 2006 International Conference on Image Processing*, in press.

The method of custom-built moments allows location of step edges in an image in parametric form, as an equation $ax+by=c$, using a customizable mask of almost any desired shape, or moment functions adapted to calculation in a particular basis. We adapt this method to the problem of locating both thin lines and roof edges in parametric form.

WARDLAW, William P., Professor, "Row Rank Equals Column Rank", *Mathematics Magazine*, Vol. 78, No. 4, (October 2005) 316-318.

This note gives a short (perhaps the shortest) proof that row rank is equal to column rank and continues to derive some properties of matrix rank.

WARDLAW, William P., Professor, problem 811, *College Mathematics Journal*, Vol.36, No. 5, (November 2005), 413.

This problem is to find the characteristic polynomial of the classical adjoint (adjugate) of a matrix, given the characteristic polynomial of the matrix.

ZARIKIAN, Vrej A., Assistant Professor (co-author), "The Calculus of One-Sided M-Ideals and Multipliers in Operator Spaces", *Memoirs of the American Mathematical Society*, Vol. 179, No. 842, viii+85 pp., January 2006.

The theory of one-sided M-ideals and multipliers of operator spaces is simultaneously a generalization of classical M-ideals, ideals in operator algebras, and aspects of the theory of Hilbert C*-modules and their maps. Here we give a systematic exposition of this theory. The main part of this memoir consists of a "calculus" for one-sided M-ideals and multipliers, i.e. a collection of the properties of one-sided M-ideals and multipliers with respect to the basic constructions met in functional analysis. This is intended to be a reference tool for "noncommutative functional analysts" who may encounter a one-sided M-ideal or multiplier in their work.

Technical Reports

BUCHANAN, J.L., Professor, "An Assessment of the Biot-Stoll Model of a Poroelastic Seabed", Naval Research Laboratory Memorandum Report NRL/MR/7140-05-8885, 83 pages, August 5, 2005, Washington DC.

Discussed in this report are:
The derivation of the equations of the Biot-Stoll model, including later extensions of Biot's original formulation.
The determination of the Biot-Stoll parameters.
The predictions of the Biot-Stoll model for wave speed, attenuation, reflection and transmission.
Difficulties with and controversies about the Biot-Stoll model.
The incorporation of frequency-dependent viscoelastic effects into the model.

GENTRY, Sommer E. Assistant Professor (co-author), "Consensus Statement of the Mathematics Subgroup of the United Network for Organ Sharing Kidney Paired Donation Working Group", 2 pages, April 2006.

This is an extremely short consensus statement on the mathematics of optimizing kidney paired donation addressed from the primary academic researchers in the field to the federally-funded agency which is planning to create and run a national kidney paired donation registry in the United States. The length of the statement is inversely proportional to the difficulty we experienced in coming to the consensus that it presents.

MINUT, Aurelia, Assistant Professor, "Modeling of MEMS Gyroscopes", *Institute for Scientific Research*, Technical Report.

The MEMS (microelectromechanical systems) gyroscopes can be used in navigation systems in cases where the GPS is denied. The MEMS gyroscopes are small devices that can be mounted on a wide range of platforms, such as UAVs, guided missiles.

We developed a model for the equations of motion and solved the equations numerically. Then we tried coupling two, four, nine and twelve gyroscopes in different ways and compared the results. This research attempts to improve the performance of current gyroscopes by weakly coupling an array of MEMS gyroscopes.

Presentations at Professional Meetings and Conferences

ALEVRAS, Alexis A., Assistant Professor, “The Gauge Group of an E_0-semigroup”, Third East Coast Operator Algebras Symposium, University of Pennsylvania, 2 October 2006. (invited talk)

GAGLIONE, Anthony M. Professor, “An Embedding Theorem for Groups Universally Equivalent to Free Nilpotent Groups,” Groups St. Andrews, St. Andrews, Scotland, 30 July - 6 August 2005.

GAGLIONE, Anthony M. Professor, “Notions of Discrimination,” Special Session of AMS, Bard College, New York, 7-9 October 2005.

GAGLIONE, Anthony M. Professor, “On a Question of Verena Huber Dyson,” Conference in Honor of Ol’shanski, Vanderbilt University, 6-10 May 2006.

GENTRY, Sommer E., Assistant Professor, “Maximum Matching for Kidney Paired Donation”, Computational Research in Boston seminar, Massachusetts Institute of Technology, 2 September 2005.

GENTRY, Sommer E. Assistant Professor, “Optimizing Kidney Paired Donation”, Economics Department seminar, U.S. Naval Academy, 13 September 13, 2005.

GENTRY, Sommer E., Assistant Professor, “Mathematical Optimization for Transplant Professionals”, University of Cincinnati Institute for the Study of Health, 26 September 2005. (invited talk)

GENTRY, Sommer E., Assistant Professor, “Matching Inequalities in Kidney Paired Donation”, Institute for Operations Research and Management Science (INFORMS) Conference on Optimization and Healthcare, San Antonio, TX, 4 February 2006.

GENTRY, Sommer E., Assistant Professor, “Maximum Matchings on Graphs for Kidney Paired Donation”, The Johns Hopkins University Applied Mathematics and Statistics department seminar, 6 April 2006.

GENTRY, Sommer E., Assistant Professor, “Neat Teaching Ideas: Index Card Essays.” MAA Section NeXT meeting, 8 April 2006.

GENTRY, Sommer E., Assistant Professor. “Graph Theory and Abstractions”, extended interview taped 5 May 2006, as part of the *Discovery Productions* series entitled Discovering Math. Discovering Math is an educational video series being developed for use in high school mathematics classrooms.

HOFFMAN, Michael E., Professor, “Updown Categories and Algebraic Structures,” AMS Special Section on Algebraic and Enumerative Combinatorics, Joint Mathematics Meetings, San Antonio, TX, 13 January 2006.

HOFFMAN, Michael E., Professor, “Quasi-symmetric Functions and Applications,” Combinatorics seminar, George Washington University, Washington, D.C., 15 February 2006.

HOFFMAN, Michael E., Professor, “Some Hopf Algebras of Physical Interest,” Renormalization and Galois Theories, Centre International de Rencontres Mathématiques, Marseille, France, 13 March 2006. (invited talk)

HOFFMAN, Michael E., Professor “Some Generating Functions for Permutations,” U.S. Naval Academy Pure Mathematics Colloquium, 29 March 2006.

HOFFMAN, Michael E., Professor “The Algebra of Multiple Zeta Values and Euler Sums,” Mini-Conference on Zeta Functions, Index and Twisted K-Theory: Interactions with Physics, Mathematisches Forschungsinstitut, Oberwolfach, Germany, 2 May 2006.

HOFFMAN, Michael E., Professor, “Rooted Trees and Symmetric Functions,” Mini-Conference on Zeta Functions, Index and Twisted K-Theory: Interactions with Physics, Mathematisches Forschungsinstitut, Oberwolfach, Germany, 4 May 2006.

KONKOWSKI, Deborah A., Professor, “Quantum Singularities in Generalized Spacetimes,” 13th Meeting of the European Physical Society, Bern, Switzerland, 11-15 July 2005.

KONKOWSKI, Deborah A., Professor, “Quantum Singularities in General Relativistic Spacetimes,” Albert Einstein International Centenary Conference, 17-23 July 2005.

KONKOWSKI, Deborah A., Professor, “Black Holes, Big Bangs and Cosmic Strings,” World Year of Physics lecture, Hood College, Fredrick, MD, 3 November 2005.

KONKOWSKI, Deborah A., Professor, “Einstein and Quantum Mechanics,” World Year of Physics lecture, University of Maryland, Baltimore County, MD, 1 March 2006.

KONKOWSKI, Deborah A., Professor, “Quantum Mechanical Healing of Classical Singularities,” American Physical Society April Meeting, Dallas, TX, 22-25 April 2006.

KSIR, Amy E., Assistant Professor, “Finite Group Actions on Riemann Roch Spaces and Automorphisms of Algebraic Geometry Codes.” American Mathematical Society Summer Research Institute for Algebraic Geometry, University of Washington, Seattle, WA, 28 July 2005.

KSIR, Amy E., Assistant Professor. “Error-Correcting Codes and Algebraic Curves”, Gettysburg College Mathematics Department Colloquium, Gettysburg, PA, 17 November 2004.

KSIR, Amy E., Assistant Professor, “Group Representations on Some Riemann-Roch Spaces of Hurwitz Curves”, AMS Eastern Section meeting, 22 April 2006.

LIAKOS, A., Assistant Professor, “Slip with Friction Boundary Condition for the Navier-Stokes Equations – Numerical Studies for Time Dependent Laminar Flows”, 27 October 2005. (Sigma Xi invited talk)

LOCKHART, Jody, Associate Professor, “Determinants of Matrices over the Integers Modulo m”, MD-VA-DC Section Spring MAA meeting, Loyola College, Baltimore, MD, 8 April 2006.

LUCAS, Marc D., Lieutenant Commander, USN, “Midshipman to Millionaire - Practical Applications of Mathematics to Personal Financial Security”. Math Open House U.S. Naval Academy, Annapolis, MD, 21 February 2006.

LUCAS, Marc D., Lieutenant Commander, USN, “An Overview of Actuary Science”, SM280, Topics in Mathematics, U.S. Naval Academy Annapolis, MD, January 2006.

MCCOY, Peter A., Professor, “On the Propagation of Electromagnetic Waves,” CP 22 Part III, SIAM Annual Meeting, New Orleans, LA, 15 July 2005.

MCCOY, Peter A., Professor, “An Overview of Shannon’s Sampling Theorem,” U.S. Naval Academy Applied Math Seminar (Chesapeake Bay Group), Annapolis, MD, 2 November 2005.

MCCOY, Peter A., Professor, “On the Normal Mode Expansion of Solutions to the Paraxial Wave Equation,” Session on PDE’s I, AMS-MAA annual joint meetings, San Antonio, TX, 13 January 2006.

MICHAEL, T. S., Associate Professor, “Art Gallery Theorems”, University of Delaware, Combinatorics Seminar, Newark, DE, October 2005.

MICHAEL, T. S., Associate Professor, “Bridge-Ratings, Wrestling Tournaments, and Combinatorics” George Washington University, Combinatorics Seminar, Washington, D.C., April 2006.

PENN, Howard L., Professor “Which Ballparks are Homer Friendly?”, Mathematical Association of America Mathfest, Albuquerque, NM, 12 August 2005.

PENN, Howard L., Professor “Which Ballparks are Homer Friendly, Part II? Mathematical Association of America Sectional Meeting, Baltimore, MD, 8 April 2006.

POPOVICI, Irina, Assistant Professor, “A Two-dimensional Dynamical System Underlying Cardiac Arrhythmias”, AMS Special Session on Dynamic Equations with Applications, 2005 Joint Meeting of the Mathematical Societies, San Antonio, TX, January 2006.

POPOVICI, Irina, Assistant Professor, “A Dynamical Systems Approach To Membrane Phenomena Underlying Cardiac Arrhythmias”, International Conference on Chaos and Nonlinear Dynamics, Bethesda MD, January 2006.

POPOVICI, Irina, Assistant Professor, “A Two-dimensional Dynamical System Underlying Cardiac Arrhythmias”, MAA MD/DC/VA Regional Fall Meeting, Montgomery College, MD, November 2005.

SANDERS, Thomas J., Professor, “Core Mathematics at the United States Naval Academy,” Academic External Review Group Meeting, Institute of Defense Analysis, Arlington, VA, 4 October 2005.

TURNER, John, Professor, “The Flat Earth Guide to Probability and Statistics”, Joint Statistical Meetings, Minneapolis, MN, 10 August 2005.

WARDLAW, William P., Professor, “A Modified Hill Cipher", U.S. Naval Academy Colloquium Talk, 8 February 2006.

WARDLAW, William P., Professor, “Row Rank Equals Column Rank”, Spring meeting of MD-VA-DC Section of Mathematical Association of America(MAA), Loyola College, Baltimore, MD, 8 April 2006.

ZARIKIAN, Vrej A., Assistant Professor, “Paving Small Matrices”, Special Session on Recent Progress in Operator Algebras, American Mathematical Society (AMS) Central Section Meeting, Lincoln, NE, 22-23 October 2005.

ZARIKIAN, Vrej A., Assistant Professor, “Alternating-Projection Algorithms for Operator-Theoretic Calculations”, Special Session on Geometry of Banach Spaces and Connections with Other Areas, American Mathematical Society (AMS) Southeastern Section Meeting, Miami, FL, 1-2 April 2006.