Appendix C

LOGARITHMS

A. COMMON LOGARITHMS

The common, or base 10, logarithm of a number is the power to which 10 must be raised to obtain the number. For example, the common logarithm of 100 (written as **?**log 100") is 2, because raising 10 to the second power gives 100. In general,

$$\text{if } x = 10^y, \text{ then } \log x = y$$

The common logarithms of numbers which are integral powers of 10 are the whole number exponents:

$$\log 100 \text{ or } \log 10^2 = 2$$
$$\log 10 \text{ or } \log 10^1 = 1$$
$$\log 1 \text{ or } \log 10^0 = 0$$
$$\log 0.1 \text{ or } \log 10^{-1} = -1$$
$$\log 0.01 \text{ or } \log 10^{-2} = -2$$
$$\log 0.001 \text{ or } \log 10^{-3} = -3$$

The log values for numbers between 1 and 10 are decimal fractions. For example, the log of 3 is 0.4771. The log of 3 is between the log of 1.0 which is 0 and the log of 10 which is 1. In general:

$$\log (c \times 10^n) = n + \log c$$

where $1 < c < 10$. The log c is a positive decimal fraction called the mantissa and n is a positive or negative whole number called the characteristic.

B. OPERATIONS INVOLVING LOGARITHMS

Because logs are exponents, the rules for calculations with logs are the same as those for exponents. They are summarized below.

	Exponential Notation	Logarithms
Multiplication	$10^x \cdot 10^y = 10^{(x+y)}$	$\log (xy) = \log x + \log y$
	$10^2 \cdot 10^4 = 10^{(2+4)} = 10^6$	$\log (10^2 \cdot 10^4) = \log 10^2 + \log 10^4 = 2 + 4 = 6$
Division	$10^x / 10^y = 10^{(x-y)}$	$\log (x/y) = \log x - \log y$
	$10^2 / 10^4 = 10^{(2-4)} = 10^{-2}$	$\log (10^2 / 10^{-4}) = \log 10^2 - \log 10^4 = 2 - 4 = -2$
Raising to a power	$(10^x)^n = 10^{(x \cdot n)}$	$\log x^n = n \log x$
	$(10^2)^3 = 10^{(2 \cdot 3)} = 10^6$	$\log [(10^2)^3] = 3 \log 10^2 = 3 \cdot 2 = 6$
Extracting a root	$(10^x)^{1/n} = 10^{x/n}$	$\log x^{1/n} = (1/n) \log x$
	$(10^6)^{1/2} = 10^{6/2} = 10^3$	$\log [(10^6)^{1/2}] = ½ \log 10^6 = ½ \cdot 6 = 3$

C. NATURAL LOGARITHMS

Up to this point, common, or base-10, logarithms have been described. However, several of the equations used in chemistry are expressed in terms of natural logarithms (written as “ln”), or logarithms to the base *e*, where *e* = 2.7183. As with common logs, where the log value is the power to which 10 is raised, the natural logarithm is the power to which *e* is raised. For example, ln 10 = 2.303 because $e^{2.303} = 10$.

The relationship between base-10 logarithms and natural logarithms is as follows:

$$\ln x = 2.303 \log x$$

D. ANTILOGARITHMS

The process of obtaining the number for which a log value corresponds is called finding an antilogarithm or antilog. For example, continuing with the example used above, the log of 3 is 0.4771 and the antilog of 0.4771 is 3. The process of obtaining an antilog is the same as raising 10 or *e* to the power of the log value. Antilog $0.4771 = 10^{0.4771} = 3$. The natural log, or ln, of 3 is 1.099; the natural antilog of 1.099 or $e^{1.099}$ is 3.

E. USING A CALCULATOR TO FIND LOGARITHMS AND ANTILOGARITHMS

On most calculators, the logarithm of a number (base 10 or base *e*) is obtained by using the LOG or LN key. The logarithm of the number will appear in the display.

Finding the antilogarithm of a number also can be done on an electronic calculator. This operation is generally carried out either with a 10^x or e^x key or with the sequence of keys INV LOG or INV LN.