# EC262 Problem Set 1 (Solutions)

## Due: Friday 24 August 2012

Complete the following problems from the textbook *Digital Design with an Introduction to the Verilog HDL*, Mano and Ciletti, Fifth Edition.

### Chapter 1

Problems: 2, 4, 8, 9 (abe), 10.
Problems: 14 (2's complement only), 18.

### Additional Problems

**Exercise 1:** Compute the sum of 6-bit unsigned binary numbers.

a. 010100 + 101101
b. 011100 + 011010

**Exercise 2:** Show how the following 6-bit 2's complement numbers are stored in memory.

a. +15
b. -45
c. -1
d. -16

**Exercise 3:** Convert the following 6-bit 2's complement numbers to decimal.

a. 000101
b. 111111
c. 011111
d. 111001

**Chapter 1**

**<u>Problem 2</u>**: the exact numbers of bytes in a system that contains

a. 32 K bytes = 32 × 1K bytes = $2^5 \times 2^{10} = 2^{15}$ bytes = 32,768 bytes

b. 64 M bytes = 64 × 1M bytes = $2^6 \times 2^{20} = 2^{26}$ bytes = 67,108,864 bytes

c. 6.4 G bytes = 6.4 × 1G bytes = $6.4 \times 2^{30}$ = 6,871,947,674 bytes

**<u>Problem 4</u>**:

a. The largest 16-bit binary number (unsigned) = $1111\ 1111\ 1111\ 1111_2$

b. Equivalent decimal number = $2^{16}-1 = 65{,}535_{10}$

c. Equivalent hexadecimal number = $FFFF_{16}$

**<u>Problem 8</u>**: Convert the decimal number 431 to binary in 2 ways:

a. Convert directly to binary

| | | |
|---|---|---|
| 431 / 2 = 215 | rem 1 | _ _ _ _ _ _ _ _ 1 |
| 215 / 2 = 107 | rem 1 | _ _ _ _ _ _ _ 1 1 |
| 107 / 2 = 53 | rem 1 | _ _ _ _ _ _ 1 1 1 |
| 53 / 2 = 26 | rem 1 | _ _ _ _ _ 1 1 1 1 |
| 26 / 2 = 13 | rem 0 | _ _ _ _ 0 1 1 1 1 |
| 13 / 2 = 6 | rem 1 | _ _ _ 1 0 1 1 1 1 |
| 6 / 2 = 3 | rem 0 | _ _ 0 1 0 1 1 1 1 |
| 3 / 2 = 1 | rem 1 | _ 1 0 1 0 1 1 1 1 |
| 1 / 2 = 0 | rem 1 | 1 1 0 1 0 1 1 1 1 |
| | | 1 1 0 1 0 1 1 1 1 |

b. Convert to hex, then from hex to binary

| | | |
|---|---|---|
| 431 / 16 = 26 | rem F | _ _ F |
| 26 / 16 = 1 | rem A | _ A F |
| 1 / 16 = 0 | rem 1 | 1 A F |
| | | $1AF_{16}$ = 0001 1010 1111 (faster) |

**<u>Problem 9</u>**: Express the following numbers in decimal.

a. $10110.0101_2 = 2^4 + 2^2 + 2^1 + 2^{-2} + 2^{-4} = 16 + 4 + 2 + .25 + .0625 = 22.3125$

b. $16.5_{16} = 1*16^1 + 6*16^0 + 5*16^{-1} = 16 + 6 + 5*(.0615) = 22.3125$

e. $1010.1101_2 = 2^3 + 2^1 + 2^{-1} + 2^{-2} + 2^{-4} = 8 + 2 + .5 + .25 + .0625 = 10.8125$

**<u>Problem 10</u>**: Convert the following binary numbers to hexadecimal and to decimal.

a. $1.10010_2 = 0001.1001_2 = 1.9_{16} = 1 + 9*16^{-1} = 1.5625_{10}$

b. $110.010_2 = 0110.0100_2 = 6.4_{16} = 6 + 4*16^{-1} = 6.25_{10}$

Reason: $110.010_2$ is the same as $1.10010_2$ shifted to the left by two places (multiply by 4).

**<u>Problem 14</u>**: Obtain 2's complements of the following binary numbers.

| Binary | 2's complement |
|---|---|
| a. 00010000 | 11110000 |
| b. 00000000 | 00000000 |
| c. 11011010 | 00100110 |
| d. 10101010 | 01010110 |
| e. 10000101 | 01111011 |
| f. 11111111 | 00000001 |

**<u>Problem 18</u>**: Perform subtraction

a. 10011 – 10010 = 010011 – 010010

```
                 010011
2's comp:       +101110
Diff:            000001   (Positive)  Check:19-18 = +1
```

b. 100010 – 100110 = 0100010 – 0100110

```
                 0100010
2's comp:       +1011010
Diff:            1111100   sign bit indicates that the result is negative
2's comp         0000100   magnitude

Result: - 4
Check: 34 -38 = -4
```

c. 1001 – 110101 = 0001001 – 0110101

| | | |
|---|---|---|
| | 0001001 | |
| 2's comp: | +1001011 | |
| Diff: | 1010100 | sign bit indicates that the result is negative |
| 2's comp | 0101100 | magnitude |

Result: - 44
Check: 9 - 53 = -44

d. 101000 – 10101 = 0101000 – 0010101

| | | |
|---|---|---|
| | 0101000 | |
| 2's comp: | +1101011 | |
| Diff: | 0010011 | positive |

Result: 19
Check: 40 – 21 = 19

## Additional Problems

**Exercise 1:** Compute the sum of 6-bit unsigned binary numbers.

a. 010100 + 101101

| | | |
|---|---|---|
| | 010100 | $(20_{10})$ |
| + | 101101 | $(45_{10})$ |
| | 000001 | $(65_{10}) > (2^6-1)$ → overflow |

b. 011100 + 011010

| | | |
|---|---|---|
| | 011100 | $(28_{10})$ |
| + | 011010 | $(26_{10})$ |
| | 110110 | $(54_{10})$ |

**Exercise 2:** Show how the following 6-bit 2's complement numbers are stored in memory.

a. +15 = 0 0 1 1 1 1

b. -45 < -32 → out of range

c. -1
Start with the binary equivalent of the magnitude $1_{10}$ = $000001_2$
2's comp: $111111_2$

d. -16
Start with the binary equivalent of the magnitude $1_{10}$ = $010000_2$
2's comp: $110000_2$

**Exercise 3:** Convert the following 6-bit 2's complement numbers to decimal.

a. 000101

$000101_2 = 2^2 + 2^1 = 5_{10}$

b. 111111

111111 is a negative number
Find the magnitude of the number by finding the 2's complement: 000001 = $1_{10}$ (magnitude)

Result: $-1_{10}$

c. 011111

011111 = $2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 31_{10}$

d. 111001

111001 is a negative number
Find the magnitude of the number by finding the 2's complement: 000111 = $7_{10}$ (magnitude)

Result: $-7_{10}$