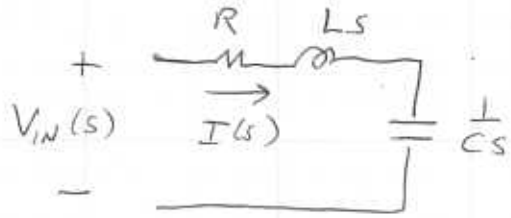
8.12a If $R = 20\Omega$, $L = 0.6H$, what C will make a series RLC circuit overdamped



$$I(s) = \frac{V_{IN}(s)}{R + Ls + \frac{1}{Cs}} = \frac{V_{IN}(s)\,Cs}{LCs^2 + RCs + 1} = \frac{V_{IN}(s)\,\frac{s}{L}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Therefore any circuit quantity (current or voltage) will have a transient governed by the quadratic term

$$s^2 + \frac{R}{L}s + \frac{1}{LC}$$

Matching with $s^2 + 2\alpha s + \omega_0^2$ gives

$$2\alpha = \frac{R}{L} \longrightarrow \alpha = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC} \longrightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

The condition for overdamped is $\alpha > \omega_0$ thus

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}} \quad \text{or} \quad \sqrt{LC} > \frac{2L}{R}$$

$$LC > \frac{4L^2}{R^2}$$

$$C > \frac{4L}{R^2}$$

Substituting $C > \frac{4(0.6)}{(20)^2}$ so $\boxed{C > 6mF}$

8.13 Following the solution to 8.12a

$$s^2 + \frac{R_{eq}}{L} s + \frac{1}{LC} = s^2 + 2\alpha s + \omega_0^2$$

so $\alpha = \frac{R_{eq}}{2L}$ $\quad \omega_0 = \frac{1}{\sqrt{LC}}$

To be critically damped $\alpha = \omega_0$ so

$$\frac{R_{eq}}{2L} = \frac{1}{\sqrt{LC}}$$

$$\frac{R_{eq}^2}{4L^2} = \frac{1}{LC}$$

$$R_{eq}^2 = \frac{4L^2}{LC} = \frac{4L}{C}$$

so

$$R_{eq} = \sqrt{\frac{4L}{C}} = 2\sqrt{\frac{L}{C}}$$

$$R_{eq} = 2\sqrt{\frac{4H}{.01F}} = 40\Omega$$

But $R_{eq} = 60 \parallel R = \frac{60R}{60+R} = 40$

$$60R = 2400 + 40R$$

$$20R = 2400$$

$$\boxed{R = 120\Omega}$$

8.15 For a series RLC circuit $i_C = i_R = i_L$

But $i_C = C\frac{dv_C}{dt} = i_L$ so

$$C\frac{dv_C}{dt} = C\frac{d}{dt}\left[30 - 10e^{-20t} + 30e^{-10t}\right]$$

$$C\frac{dv_C}{dt} = C\,200\,e^{-20t} - C\,300\,e^{-10t}$$

matching with i_L gives

$$C\,200e^{-20t} - C\,300e^{-10t} = .04\,e^{-20t} - .06e^{-10t}$$

so $200\,C = .04$ or $300C = .06$

giving $C = \underline{\underline{200\mu F}}$

Then the pole locations for the v_C & i_L expressions give $s = -20$ and $s = -10$ (from e^{-20t} & e^{-10t})

so

$$(s+20)(s+10) = s^2 + 30s + 200$$

matching with our quadratic for a series circuit

$$s^2 + \frac{R}{L}s + \frac{1}{LC} \quad \text{gives} \quad \frac{R}{L} = 30 \quad \frac{1}{LC} = 200$$

so $L = \frac{1}{200C} = \frac{1}{200(200\mu F)} = \underline{\underline{25H}}$

$R = 30L = \underline{\underline{750\Omega}}$