Problem Set #23

Chapter 17, Solution 1.

(a) This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega = \mathbf{2}$.

(b) y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c) Since sin A cos B = 0.5[sin(A + B) + sin(A – B)],

$$g(t) = \sin 3t \cos 4t = 0.5[\sin 7t + \sin(-t)] = -0.5 \sin t + 0.5 \sin 7t$$

which is harmonic or **periodic** with the fundamental frequency

$$\omega = 1 \text{ or } T = 2\pi/\omega = \mathbf{2\pi.}$$

(d) $h(t) = \cos^2 t = 0.5(1 + \cos 2t)$. Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = \mathbf{\pi}$.

Chapter 17, Solution 7.

$$T = 3, \quad \omega_o = 2\pi / T = 2\pi / 3$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{3}\left[\int_0^2 2\,dt + \int_2^3 (-1)dt\right] = \frac{1}{3}(4-1) = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\frac{2n\pi t}{3}\,dt = \frac{2}{3}\left[\int_0^2 2\cos\frac{2n\pi t}{3}\,dt + \int_2^3 (-1)\cos\frac{2n\pi t}{3}\,dt\right]$$

$$= \frac{2}{3}\left[2\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_0^2 - 1\frac{3}{2n\pi}\sin\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}\sin\frac{4n\pi}{3}$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\frac{2n\pi t}{3}\,dt = \frac{2}{3}\left[\int_0^2 2\sin\frac{2n\pi t}{3}\,dt + \int_2^3 (-1)\sin\frac{2n\pi t}{3}\,dt\right]$$

$$= \frac{2}{3}\left[-2x\frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_0^2 + \frac{3}{2n\pi}\cos\frac{2n\pi t}{3}\bigg|_2^3\right] = \frac{3}{n\pi}(1 - 2\cos\frac{4n\pi}{3})$$

$$= \frac{1}{n\pi}\left(2 - 3\cos\frac{4n\pi}{3} + 1\right) = \frac{3}{n\pi}\left(1 - \cos\frac{4n\pi}{3}\right)$$

Hence,

$$f(t) = 1 + \sum_{n=0}^{\infty}\left[\frac{3}{n\pi}\sin\frac{4n\pi}{3}\cos\frac{2n\pi t}{3} + \frac{3}{n\pi}\left(1-\cos\frac{4n\pi}{3}\right)\sin\frac{2n\pi t}{3}\right]$$

We can now use MATLAB to check our answer,

```
>> t=0:.01:3;
>> f=1*ones(size(t));
>> for n=1:1:99,
f=f+(3/(n*pi))*sin(4*n*pi/3)*cos(2*n*pi*t/3)+(3/(n*pi))*(1-
cos(4*n*pi/3))*sin(2*n*pi*t/3);
end
>> plot(t,f)
```