Problem Set #24

Chapter 17, Solution 4.

$f(t) = 10 - 5t, \ 0 < t < 2, \ T = 2, \ \omega_0 = 2\pi/T = \pi$

$$a_0 = (1/T)\int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = 5$$

$$a_n = (2/T)\int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = 0$$

$$b_n = (2/2)\int_0^2 (10 - 5t)\sin(n\pi t)dt$$

$$= \int_0^2 (10)\sin(n\pi t)dt - \int_0^2 (5t)\sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\sin n\pi t\Big|_0^2 + \frac{5t}{n\pi}\cos n\pi t\Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = 10/(n\pi)$$

Hence $$f(t) = \underline{\mathbf{5 + \frac{10}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(n\pi t)}}$$

Chapter 17, Solution 18.

(a) $T = 2$ leads to $\omega_0 = 2\pi/T = \underline{\pi}$

$f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **<u>odd and half-wave symmetric</u>**.

(b) $T = 3$ leads to $\omega_0 = 2\pi/3$

$f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **<u>even</u>**.

(c) $T = 4$ leads to $\omega_0 = \pi/2$

$f_3(t)$ is **<u>even and half-wave symmetric</u>**.

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Wave-Form has even symmetry $\longrightarrow$ $b_n = 0$ and need to integrate over only half a period $(T=5)$

$$a_0 = \frac{2}{T}\int_0^{T/2} F(t)\,dt = \frac{2}{5}\left[\int_1^2 5\,dt + \int_2^{2.5} 10\,dt\right]$$

$$= \frac{2}{5}\left[5t\Big|_1^2 + 10t\Big|_2^{2.5}\right]$$

$$= \frac{2}{5}\left[(10-5) + (25-20)\right] = \underline{\underline{4}}$$

$$\omega_0 = \frac{2\pi}{T} = 2\pi/5$$

$$a_n = \frac{4}{T}\int_0^{T/2} F(t)\cos\left(n\frac{2\pi}{5}t\right)dt$$

$$= \frac{4}{5}\left[\int_1^2 5\cos\left(n\frac{2\pi}{5}t\right)dt + \int_2^{2.5} 10\cos\left(n\frac{2\pi}{5}t\right)dt\right]$$

$$= \frac{4}{5}\left[\frac{5\sin\left(n\frac{2\pi}{5}t\right)}{\frac{2\pi}{5}n}\Bigg|_1^2 + \frac{10\sin\left(n\frac{2\pi}{5}t\right)}{\frac{2\pi}{5}n}\Bigg|_2^{2.5}\right]$$

$$= \frac{4}{5}\frac{5}{2\pi n}\left[5\left[\sin\left(\frac{4\pi}{5}n\right) - \sin\left(\frac{2\pi}{5}n\right)\right]\right.$$

$$\left. + 10\left[\overset{0}{\cancel{\sin(\pi n)}} - \sin\left(\frac{4\pi}{5}n\right)\right]\right]$$

$$= \frac{2}{\pi n}\left[-5\sin\left(\frac{4\pi}{5}n\right) - 5\sin\left(\frac{2\pi}{5}n\right)\right]$$

$$= \frac{-10}{\pi n}\left[\sin\left(\frac{4\pi}{5}n\right) + \sin\left(\frac{2\pi}{5}n\right)\right]$$

if you integrate for the entire period

$$a_n = \frac{5}{\pi n}\left[-\sin\left(\frac{2\pi}{5}n\right) - \sin\left(\frac{4\pi}{5}n\right) + \sin\left(\frac{6\pi}{5}n\right) + \sin\left(\frac{8\pi}{5}n\right)\right]$$

$$\sin\left(\frac{6\pi}{5}n - \frac{10\pi}{5}\right) = \sin\left(-\frac{4\pi}{5}n\right) = -\sin\left(\frac{4\pi}{5}n\right)$$

$$\sin\left(\frac{8\pi}{5}n - \frac{10\pi}{5}\right) = \sin\left(-\frac{2\pi}{5}n\right) = -\sin\left(\frac{2\pi}{5}n\right)$$

and the two solutions converge!