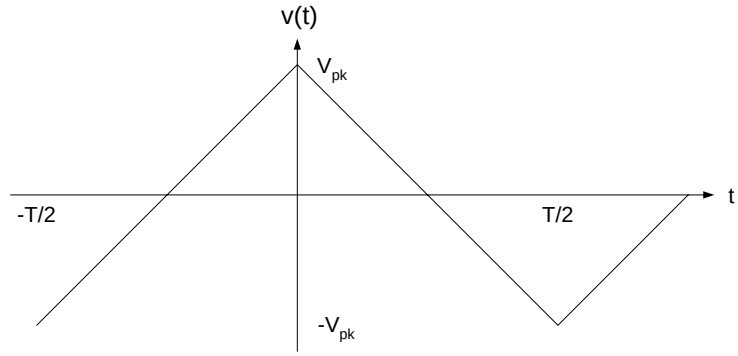
EE228 Spring 2007

LABORATORY EXPERIMENT #9

Today's laboratory exercise will reinforce our understanding of Fourier series. Specifically, we will practice determining the steady-state output of a filter circuit given a periodic waveform. A triangle signal



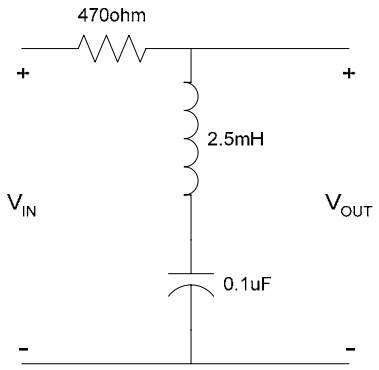
has the following Fourier Series

$$v(t) = \frac{8V_{pk}}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos\left[(2n-1)\omega_o t\right] \qquad \omega_o = \frac{2\pi}{T}$$

<u>Part 1:</u> Determine the amplitude (peak) and frequency (in terms of the variable ω_o) of the fundamental (first harmonic) and the first three harmonics present. Let $V_{pk} = 3V$.

<u>Frequency</u> <u>Amplitude (zero-to-peak)</u>

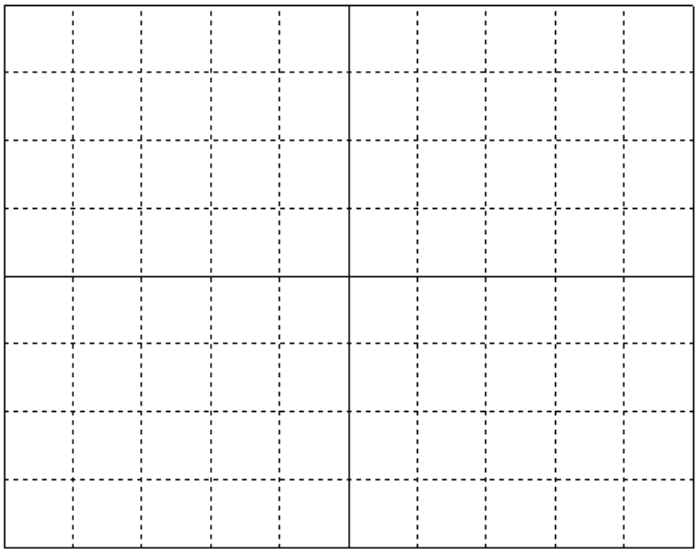
Part 2: Consider the following band-stop (BS) filter. Acquire the mounted components and measure their actual parameter values (record them on the schematic)



- Determine the center frequency (in Hz) and the Bandwidth (in Hz)

Part 3: Build the circuit on the quad-board. The input voltage should be the Function Generator (FG) set to the triangle signal with a zero-to-peak value of 3V 9confirm on the scope). Set the Function Generator frequency to the center frequency of the BS-filter. Monitor the input with CH1 of the scope and the output using CH2.

Part 4: Sketch ONE CYCLE of the input and output on the following axes (make sure to record volts/div and time/div and indicate where ground is for each channel)



Part 5: Next, we want to analytically predict our steady-state output (using Matlab) and compare against the previous plot. The following Matlab code has been emailed to you. It creates one period of a triangle wave given the frequency in Hertz that you provide. Our goal is to modify this code so that we can utilize Matlab to create the steady-state output for your circuit.

```
% Creates a triangle wave from its Fourier Series
% Input the frequency of the triangle wave in Hz
%   and calculate the angular frequency
wo = 2*pi*input('Gimme fo (Hz): ');
Vpk = 3.0;          % Triangle zero-to-peak
R = 470;
L = 2.5e-3;
C = 0.1e-6;

fund = 8*Vpk/(pi*pi);       % Fundamental amplitude
T = 2*pi/wo;                % Triangle period
t = linspace(0,T, 1000);    % Plot one period
vtri = 0.0;                 % Initialize to zero

% Loop through the first 20 non-zero series terms
for n = 1: 20
    vn = fund/((2*n-1)*(2*n-1));        % Find the peak amplitude
    vtri = vtri + vn*cos((2*n-1)*wo*t); % Add the term

end                                     % End the loop

plot(t,vtri);       % Plot the triangle
grid on
```

In order to solve this problem analytically, we need to break it up into a series of sinusoidal steady-state problems, then sum up the results in the time domain. As shown in the code, the frequency for a given term is given by $\omega = (2n-1)\omega_o$ where ω_o is the radian frequency of the triangle. Next consider the circuit, we can define two impedances (note that both will change as the frequency changes)

$$Z_1 = j\omega L - \frac{j}{\omega C} \qquad\qquad Z_{tot} = R + j\omega L - \frac{j}{\omega C}$$

The output voltage for each harmonic can then be expressed as

$$\tilde{V}_{out,h} = \tilde{V}_{in,h}\frac{Z_{1,h}}{Z_{tot,h}}$$

Or in terms of magnitudes and phase angles

$$\left|\tilde{V}_{out,h}\right| = \left|\tilde{V}_{in,h}\right|\frac{\left|Z_{1,h}\right|}{\left|Z_{tot,h}\right|} \qquad\qquad \angle\tilde{V}_{out,h} = 0^\circ + \angle Z_{1,h} - \angle Z_{tot,h}$$

We will assume that we will utilize peak values for the phasor magnitudes. Since our triangle waveform is an even function, the Fourier series only contains cosines with zero degrees as the phase angle for each term. Once we have the amplitude and angle for each harmonic, we return to the time domain via

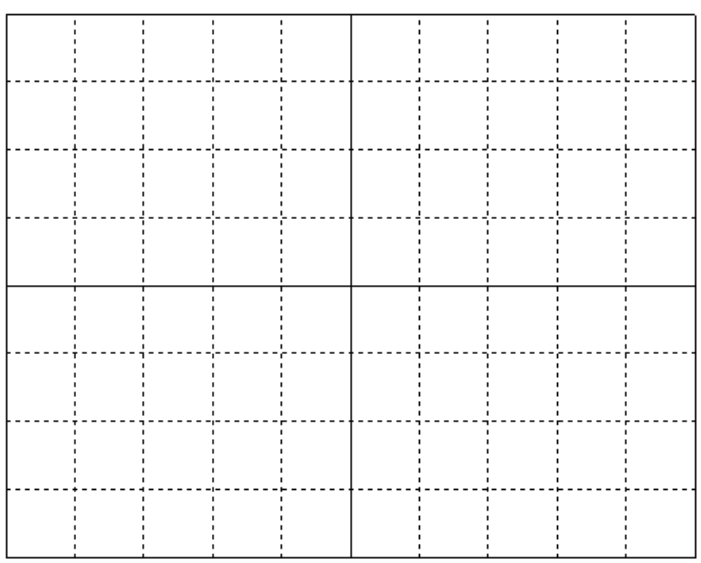
$$V_{out,h} = \left|\tilde{V}_{out,h}\right| \cos\left(\omega t + \angle \tilde{V}_{out,h}\right)$$

Implement this procedure in the code provided to you. Note, the bulk of the code must be placed in the for loop. Recall that the Matlab commands abs(x) finds the magnitude of a complex number and angle(x) finds the angle in radians. **Have the instructor verify your result.**

<u>Part 6:</u> Move the output (CH2) to across the capacitor. Adjust the frequency of the Function Generator. Over what *approximate* frequency range does the output look triangular?

Over what *approximate* frequency range does the output look sinusoidal?

<u>Part 7:</u> Set the FG frequency to 13kHz and sketch the low-pass filter input and output on the axes below (include the volts/div and time/div)



Carefully measure the zero-to-peak amplitude of the output and record it here

Part 8: Use the actual circuit values and the input fundamental and sinusoidal steady-state analysis to predict the theoretical output of the circuit at the fundamental frequency. Compare this analytical value to one measured in Step 7. What aspect of a "real" inductor have we forgotten to include in our analysis that may result in our calculation being too high?