DC Motors

The purpose of this supplement is to present the basic material needed to understand the operation of simple DC motors. This is intended to be used as the reference material for lectures 15, 16 and 17 in EE301 as this material is not covered in the text.

Before introducing electric motors, it is helpful to define electric motors. In simplest term, they are machines which convert electrical energy into mechanical energy. A few examples of familiar electric motors include motors to raise and lower car windows, ceiling fans, and motors that spin CDs and DVDs.

A linear motor is a basic easy-to-understand electrical motor. Although the linear design is not useful for most practical electrical motors, the concepts, that are learned here, are useful in understanding rotating DC motors which are discussed in lessons 16 and 17. This section will introduce or review: some basic concepts of magnets and magnetic flux, the Lorentz force law, Faraday's law and then finally the simple DC linear motor.

15.1 Magnetic Flux

Before electrical machinery can be adequately discussed, it is first essential that we introduce magnets in a manner which explains how they are employed in electrical machinery. It is helpful to recap a few basic conclusions from physics.

> -Magnets may exist as permanently magnetized materials, logically called *permanent magnets*, or as a temporary magnet which exists only when electric current is flowing, also logically called *electromagnets*.
>
> -All magnets have two poles, north and south. These poles give rise to the familiar action between two magnets where opposite poles attract while similar poles repel.
>
> - Lines of magnetic flux can be imagined, which flow from the north pole to the south pole. These lines of flux, while they do not physically exist, help us visualize the direction and strength of the magnetic field between magnetic poles.

15.1.1 Permanent Bar Magnets

Figure 1.(a) below shows how the lines of magnetic flux are arranged for a permanent bar-shaped magnet. There are two important features of this diagram. First, lines of magnetic flux are continuous. That is, the lines of flux exist as unbroken lines flowing from the north pole and re-entering the south pole. The same number of lines leave the north pole as enter the south pole Second, the lines are more closely spaced close to the magnet than far away. A reasonable--and correct--conclusion is that since the lines of magnetic flux are denser close to the magnet, the magnet's magnetic field must be stronger close to the magnet. These two results will apply to all magnets.

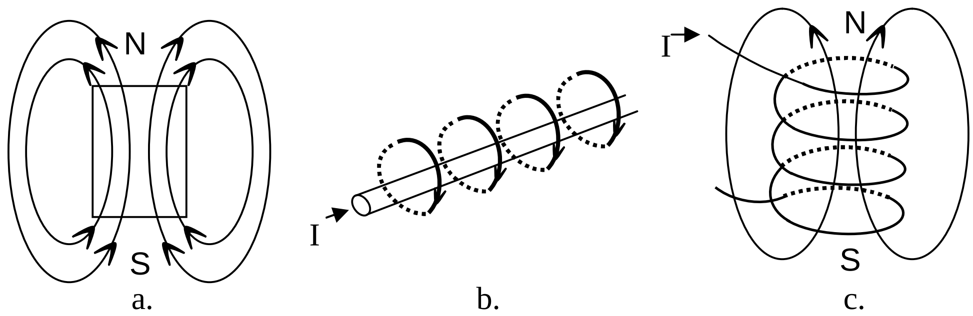

Figure 1: Lines of Magnetic Flux (Φ) for (a) A Permanent Magnet; (b) Current-Carrying Conductor; (c) Current-Carry Coil

15.1.2 Current Carrying Wire

As mentioned previously, a flowing current also produces a magnetic field, and therefore lines of magnetic flux. Figure 1(b) shows how lines of magnetic flux are arranged around a straight section of a current carrying wire. Notice that the lines of flux all form closed loops around the wire. Since they have no beginning or end we can not indicate where a north or south pole would exist, but the lines do have a defined direction. This direction is determined using the *right-hand rule* as follows. Point your right thumb in the direction of the current flow in the wire. Wrap your fingers around the wire. Your fingers indicate the direction of the lines of magnetic flux. It should be apparent that reversing the direction of current flow will also reverse the direction of the lines of magnetic flux. . Not as obvious is that the density of lines of flux is dependent on the current: higher current produces a higher density of lines of magnetic flux.

15.1.3 Current Carrying Coil

Next we will consider the lines of magnetic flux that result when a length of current-carrying wire is formed into a tightly wound coil. We will assume that the wire is insulated so that current must flow through the entire length of the wire, not short circuit between adjacent windings.

Figure 1(c) shows how the lines of magnetic flux are arranged around a coil. Notice first that a north and south pole are defined in this diagram even though they were not for a straight wire. Why this is so is easy to demonstrate graphically. Figure 2 shows two closely-spaced wires in cross section which can be imagined to be any two wires in the coil. Assuming the wire is wrapped around the coil in the same direction for both, the current direction will be the same in both wires. Therefore the lines of magnetic flux around each wire will be in the same direction. If the wires are closely spaced then the lines of flux will reinforce each other forming lines of flux which encompass the pair of wires.

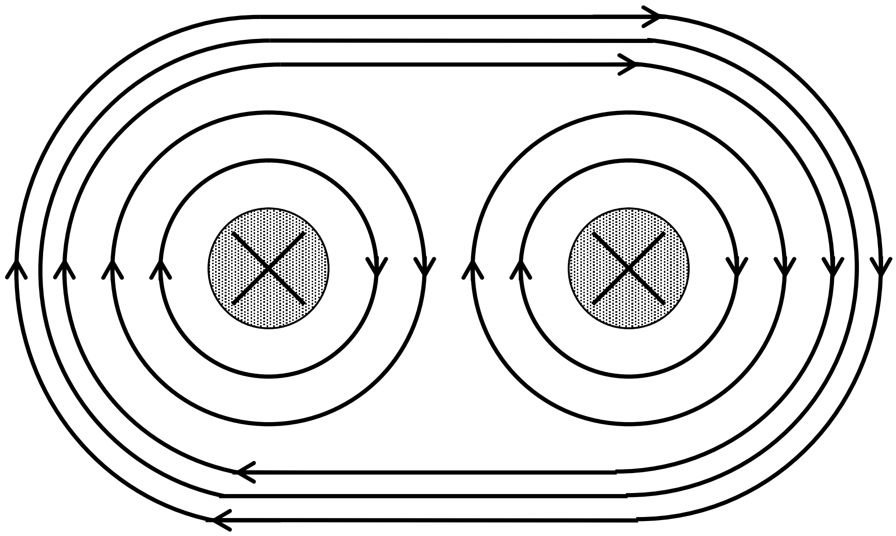

Figure2. Lines of Magnetic Flux for Two Current Carrying Wires

Now extend this simple example to a coil formed by many loops of wire carrying the same current. The net result then is similar to the bar magnet shown in figure 1(a). The coil has a defined north pole where the lines of flux leave and a south pole where they re-enter. It should be apparent that more loops of wire, or more *turns*, will produce more lines of magnetic flux.

It is useful to summarize the set of results regarding the lines of magnetic flux surrounding the electromagnet formed by a coil of current-carrying wire.

-The same number of lines leaves the north pole of the magnet as re-enter the south pole.
-These lines are denser close to the magnet, especially near the poles.
-The direction of the lines depends on the direction of the current through the coil.
-Changing current directions changes the poles of the magnet.
-The number of lines depends on the strength of the current. Higher current produces more lines of flux.

15.2 Magnetic Flux, Magnetic Flux Density, and Magnetic Field Intensity

It was alluded to in the previous section that lines of magnetic flux are only imagined. Nonetheless, they are useful in understanding how magnets interact. A slightly more rigorous treatment can be achieved by trying to count these lines of flux. Since these lines do not actually exist, we have to construct some model which allows us to count or measure them. In a very simple view, the amount of flux is described by the number of lines. *Magnetic flux* is given the symbol **Φ**. We measure magnetic flux in units of *Webers* (Wb), which have equivalent units of volts-seconds.

Now, we arrive at the concept of the magnetic field. One failing of using lines of magnetic flux is that this indicates that the magnetic field only exists where these lines occur, but numerous other undefined lines also exist. A better method would be to understand that the actual magnetic field from any magnet is not confined to a set of lines which occur along specific paths, but rather permeates all space. The direction of the magnetic field is the same as the lines of flux and the magnitude of the magnetic field is the flux **density**.

Thus, we are typically not concerned simply with the magnetic flux, but rather the magnetic flux density, $\vec{B}$, which is measured in *Tesla* (T), or Wb per square meter. The magnetic field is generally quantified by the magnetic flux density or simply B-field. Note that magnetic flux density is a vector quantity since the magnetic flux density has a direction and a magnitude.

The final magnetic field quantity to discuss is magnetic field intensity, which is given the symbol $\vec{H}$. The units of $\vec{H}$ are A/m. Magnetic field intensity is related to the magnetic flux density by

$$\vec{H} = \frac{1}{\mu}\vec{B} \quad \text{or} \quad \vec{B} = \mu\vec{H}$$

where μ is the permeability of a material. Permeability is a measure of how well a given material is able to conduct magnetic flux lines. Most materials have permeabilities roughly equal to that of free space (air). However, some materials, such as iron and iron-based compounds, have a much higher permeability, up to several thousand times higher, than free space. This explains why objects containing iron respond to magnetic fields even when not magnetized themselves.

We can summarize the key facts about magnetic fields as follows:

- A magnetic field surrounds any magnet.
- Lines of magnetic flux leave the north pole of the magnet and re-enter at the south pole of the magnet.
- The unit of flux, **Φ, is the Weber (Wb)**
- The magnetic field is generally quantified by the magnetic flux density, $\vec{B}$ or B-field and is measured in Teslas (T) or (Wb/m^2).
- Magnetic field intensity $\vec{H}$ varies by material, with most materials having the same permeability as free space. Iron and related materials have a much lower $\vec{H}$ for a given value of $\vec{B}$ than other materials. The units of $\vec{H}$ are A/m.

15.3 Magnetic Force

In section 15.1.2 we learned that a wire carrying current develops a magnetic field that surrounds it. As with any magnetic field, the magnetic field surrounding the wire will interact with any other magnetic field present. This interaction can result in a force applied to the current carrying wire, which can be used to move the wire. This force is described by the Lorentz Force Law.

Consider a straight length of current carrying wire. If the wire, of length L, is placed in a uniform magnetic field ($\vec{B}$) as shown in Figure 3, the wire will be subjected to a magnetic force given by

$$\vec{F} = I\vec{L} \times \vec{B}$$

The above equation treats the length as a vector with its direction given by the direction of the current. The equation states that the force is equal to the cross product of the current multiplied by the length vector and the magnetic field vector. The cross product implies that the force is proportional to the current and to the magnitude of the magnetic field vector and the angle between the vectors. The force is a maximum when the angle between the current carrying length and the magnetic field is 90 degrees (as in Fig. 3), and the force goes to zero when the current carrying length and magnetic field are parallel.

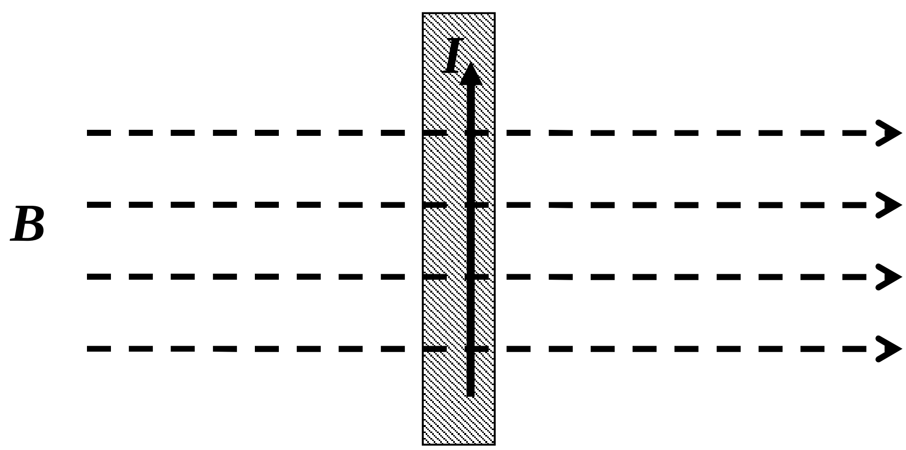

Figure 3. Current Carrying Wire in a Uniform Magnetic Field

The result of the Lorentz Force Law is that we can induce a force on a wire in a magnetic field by pushing current through the wire. That wire can then be used to perform mechanical work. This is the fundamental principle behind linear motors, as well as any other electric motors, and allows us to convert electrical energy into mechanical energy which can be used for mechanical work.

15.4 Linear Motors

We now have sufficient understanding to introduce the linear motor. As shown in Figure 4 a simple linear motor consists of a current source, a moveable wire and a magnetic field. Assuming that the current is flowing around the loop as indicated, and the magnetic field is shown going into the page, the effect of the force on the moveable wire will be to move the moveable wire to the right. This simple concept is how a linear motor operates—a current in moved through a wire where the direction of current flow is perpendicular to the magnetic field. The result is a force applied to the wire that makes it move. The force is dependent on the direction and magnitude of the current, the direction and magnitude of the magnetic field, and the angle between the current and magnetic field vectors.

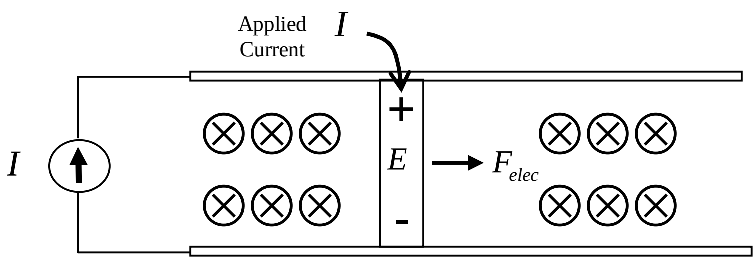

Figure 4: Linear Motor

15.4.1 Faraday's Law

From the description of a simple linear motor given above, we see how a force can be generated on a current carrying wire in a magnetic field. The equation in section 15.3 indicates that a constant current perpendicular to a uniform magnetic field will result in constant force on that wire. If the wire is free to move, the constant force would result in constant acceleration. Constant acceleration would eventually result in physically impossible speeds. It is clear, then, that another equation is needed to describe some opposition to this force.

It turns out that changing the magnetic **flux** flowing through the area defined by a closed loop of wire produces a current in the closed loop of wire. Faraday's law describes this result. It is the concept behind electric generators, which will be discussed later in the course. For now, we will simply accept that current is generated by changing the area of a closed loop of wire in a magnetic field. We needed to introduce Faraday's law in order to understand how this current works to counter the Lorenz law force.

For the linear motor in Fig. 4, Faraday's law states that changing the area of a conductor loop in a magnetic field results in a current. Hence an induced voltage is present which is given here by

$$E_{induced} = (\vec{v} \times \vec{B}) \cdot \vec{L}$$

where $\vec{L}$ represents the length and direction of the moveable wire, $\vec{v}$ represents the velocity of the moveable wire and , $\vec{B}$ represents the magnetic field,. The above equation indicates that the induced voltage is dependent on the velocity of the wire, the magnitude and direction of the magnetic field, and length and direction of the wire in the magnetic field. It is important to understand that the voltage induced by moving the wire supplies current which opposes the flow of current from the voltage source shown in Fig. 4. This opposition is described by Lenz' Law.

15.4.2 Linear Motor Operation

In order to describe a variation of the operation of a linear motor, let's apply a fixed voltage to the rails instead of the DC current. This would be in line with a linear motor used on a roller coaster, (not quite how it is done, but it gives the general idea.) Figure 5 shows a voltage source which will provide the current through the magnetic field, the uniform magnetic field pointing into the page, the free-moving bar and a resistor, R_{rail}, which represents the small but non-zero resistance of the wire loop. The direction of the initial current flow, the force developed, F_d, from the Lorentz law, the resulting velocity, and the induced voltage are all indicated on the diagram.

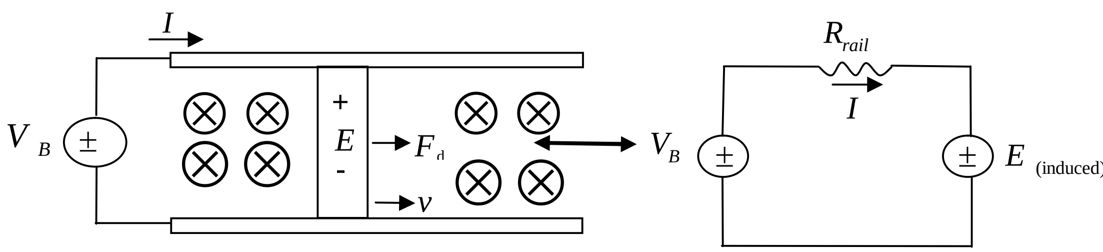

Figure 5: Linear Machine with Voltage Source Applied

Assuming the applied voltage is initially zero, the force on the wire will initially be zero. Similarly, if the wire in the magnetic field is initially at rest, the induced voltage across the wire is zero. Now assume at some time the voltage V_B is turned on and current begins to flow through the wire. The initial value for the current is $I = V_B / R_{rail}$. The initial current results in the Lorentz force being applied to the wire. Up to the time when the bar begins to move, the induced voltage, $E_{induced}$ given by the above equation is also zero. However, once the bar moves, the velocity vector $\vec{v}$ is no longer zero, so the induced voltage is no longer zero. A KVL equation for the circuit would be

$$V_B - IR - E_{\text{induced}} = 0$$

which can be rewritten to solve for current as

$$I = \frac{V_B - E_{\text{induced}}}{R}$$

The equations should make it obvious that as E_{induced} increases the current I through the bar decreases. As I decreases the Lorentz force decreases. Eventually E_{induced} will be equal and opposite to V_B, which means that no current will flow through the bar. Then the Lorentz force will go to zero and the bar will have no acceleration. In other words, it will be at a constant speed.

To use this linear machine as a motor it would be required to move some mechanical load such as a roller coaster cart with some friction that opposes movement. This force would be treated as a force that opposes the force developed, F_d, from the Lorentz law force. The net force on the bar would then be the difference between the developed force and the load force that could be frictional force.

$$F_{net} = F_d - F_{load}$$

Analysis of the operation of the linear machine can be conducted using these equations. For example, assume that a roller coaster is operating with a fixed applied load. Once steady state operation is achieved, the roller coaster is moving at a constant speed v and the net force on the roller coaster would be zero. Hence, a steady state load force would imply that

$$F_d = BLI = F_{load}$$

This in turn implies a steady state current of

$$I = \frac{F_{load}}{BL}$$

Substituting in the KVL expression above would then allow us to solve for the steady state speed.

$$v = \frac{V_{DC} - R_{rail} I}{BL} = \frac{V_{DC} - R_{rail} \frac{F_{load}}{BL}}{BL}$$

Certainly this analysis does not consider the acceleration of the roller coaster cart, also the algebra gets a bit harried if we make the load force dependent on speed. However, this result is useful in that it illustrates some key concepts that we will see again when we address DC machines. First, in a good design, the voltage drop across the resistance modeling the rails will be small so that the induced voltage will nearly balance the applied DC voltage. Therefore, if we want a high-speed coaster, we would want the induced voltage large and thus V_{DC} as large as practical. Further, we would want the frictional forces to be as small as practical. Second, the current that flows will be sufficient to create an electromagnetic force that balances the load force. The top speed and maximum force required will give us some sense of the power rating of the linear motor.

Example 1: Assume we wish to design a 20kW (output power) roller coaster that will reach speeds of 60mph. The maximum B-field that we can achieve is 0.8T. We have a 240V DC source available. If we desire a full-speed efficiency of 92%, what is the required rail resistance and source current and what is the effective length of the “bar”?

Solution. The desired output is telling us now much power is being delivered to the induced voltage source shown in Figure 5. Thus,

$$P_{OUT} = EI = 20kW$$

The input power is simply the product of the applied voltage and the current

$$P_{IN} = V_{DC} I$$

Since we desire an efficiency of 92%, this implies that

$$\eta = \frac{P_{OUT}}{P_{IN}} = 0.92$$
$$P_{IN} = \frac{P_{OUT}}{0.92} = \frac{20kW}{0.92} = 21.74kW$$

Substituting that we are applying 240 V gives

$$P_{IN} = V_{DC} I$$
$$I = \frac{P_{IN}}{V_{DC}} = \frac{21.74kW}{240V} = 90.58A$$

We can also find E now since

$$P_{OUT} = EI$$
$$E = \frac{P_{OUT}}{I} = \frac{20kW}{90.58A} = 220.8V$$

Next, we can use KVL to find the rail resistance from our equivalent circuit.

$$V_{DC} = IR_{rail} + E$$
$$240V = 90.58R_{rail} + 220.8V$$
$$R_{rail} = 0.212\Omega$$

Next, we solve for velocity in m/s

$$v = 60\frac{miles}{hr}\,x\,\frac{5280\,ft}{mile}\,x\,\frac{12in}{ft}\,x\,\frac{1m}{39.37in}\,x\,\frac{1hr}{60\,\text{min}}\,x\,\frac{1\,\text{min}}{60s} = 26.82m/s$$

To find the effective length of the “bar”:

$$E = Blv$$
$$l = \frac{E}{Bv} = \frac{220.8V}{(0.8T)(26.82m/s)} = 10.29meters$$

Now we would have to get practical in how we would implement this since for a mile-long track, it would not make much sense to have to drive current that far to develop the required forces. But this at least gives us a bit of “feel” to designing a linear motor with a constant voltage source applied.

Example 2: A 12V linear motor operates with a B-field of 0.4T. The effective length of the moveable “bar” is 0.2 meters. Assume the steady state mechanical load on the motor is 0.5N. Find the current flowing through the motor and the velocity of the bar if the rail resistance is $0.025\,\Omega$.

Solution

$F_d = F_{load}$ since $F_{net} = 0$ at steady state.

Hence

$$F_d = 0.5N$$

and

$$F_d = BLI$$
$$0.5 = (0.4)(0.2)I$$
$$I = 6.25A$$

Next, using KVL from our circuit model.

$$V_{DC} = IR_{rail} + E$$
$$12V = (6.25A)(0.025\Omega) + E$$
$$E = 11.84V$$

and

$$E = BLv$$

$$v = \frac{E}{BL} = \frac{11.84V}{(0.4T)(0.2m)} = 148m/\sec$$

OK, what have we learned? First, we need a magnetic field and a current-carrying conductor to create a force on the conductor. The force is maximum if the field and current are orthogonal. Second, a loop (or coil) experiencing a change of flux will exhibit an induced voltage. The induced voltage will attempt to push current in a direction to try to oppose the applied current and keep the flux constant. If the induced voltage is created by a conductor moving, then the factors that effect the magnitude of this voltage are the size of the magnetic field, the velocity of the conductor, and the length of conductor in the magnetic field. These two factors (in some form) are common to all electric machinery and will be key to us understanding the operation of a DC rotating motor next!