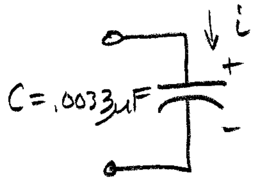
HW #24 Solutions EE301

Ch. 18 #'s 12, 13, 19, 25, 27

18.12)



$V_c = 170V \sin(377t + 40°)$

a. Phasor Form

$V_c = \frac{170V}{\sqrt{2}} \angle 40° = \boxed{120V \angle 40°}$

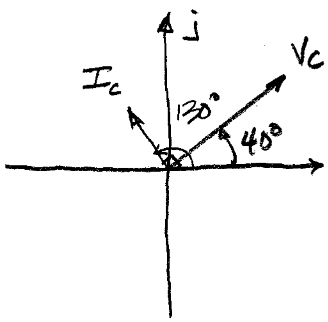
b/c. Sinusoidal waveforms and Phasor Diagram

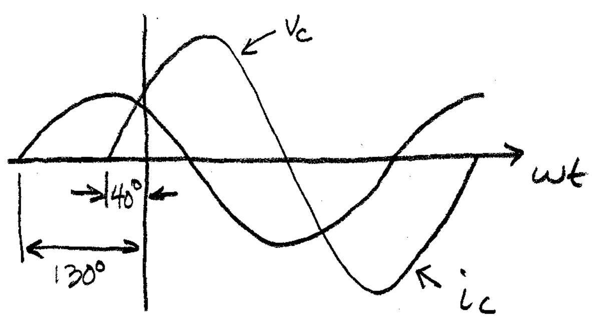
$Z_c = \frac{-j}{\omega C} = \frac{-j}{(377 \frac{rad}{s})(.0033 \times 10^{-6} F)} = -804k\Omega j = 804k\Omega \angle -90°$

$I_c = \frac{V_c}{X_c} = \frac{120V \angle 40°}{804k\Omega \angle -90°} = 149\mu A \angle 130°$

$I_m = \sqrt{2} I_{rms} = \sqrt{2}(149\mu A) = 211\mu A$

$i_c = 211\mu A \sin(377 \frac{rad}{s} t + 130°)$

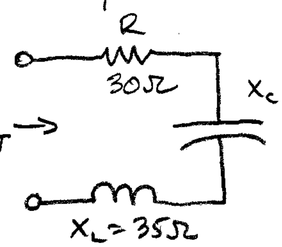




Note: Current leads voltage by 90°

18.13) Find Z_T

a.



$Z_R = 30\Omega$

$Z_c = -25\Omega j$

$Z_L = 35\Omega j$

$Z_T = Z_R + Z_c + Z_L$

$= 30\Omega - 25\Omega j + 35\Omega j$

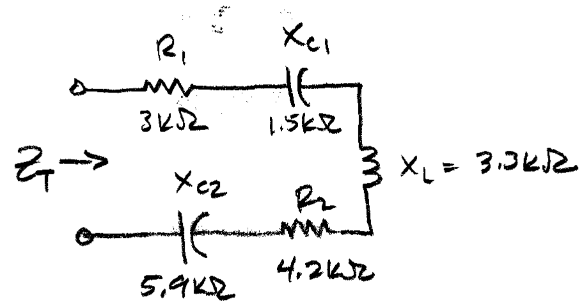
$\sqrt{(30\Omega)^2 + (10\Omega)^2} = 31.6\Omega$

$\theta = \tan^{-1} \frac{10\Omega}{30\Omega} = 18.4°$

$Z_T = 30\Omega + 10\Omega j$ rectangular

$Z_T = 31.6\Omega \angle 18.4°$ polar

18.13) b.



$Z_{R1} = 3k\Omega$, $Z_{C1} = -1.5k\Omega j$, $Z_L = 3.3k\Omega j$

$Z_{R2} = 4.2k\Omega$, $Z_{C2} = -5.9k\Omega j$

$$Z_T = Z_{R1} + Z_{C1} + Z_L + Z_{R2} + Z_{C2}$$

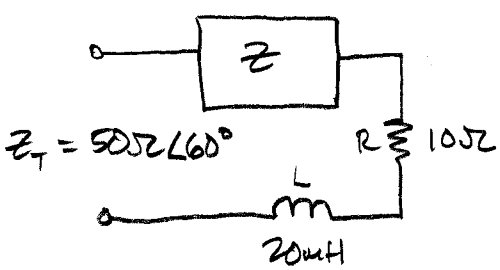
$$= 3k\Omega - 1.5k\Omega j + 3.3k\Omega j + 4.2k\Omega - 5.9k\Omega j$$

$$= 7.2k\Omega - 4.1k\Omega j$$

$$= \sqrt{(7.2k\Omega)^2 + (-4.1k\Omega)^2} \angle \tan^{-1}\left(\frac{-4.1k\Omega}{7.2k\Omega}\right)$$

$$\boxed{Z_T = 8.3k\Omega \angle -29.7^\circ}$$

18.19)



$f = 1kHz$

What RLC components are in Z to give Z_T?

$Z_R = 10\Omega$, $Z_L = \omega L j = \left(2\pi \frac{rad}{sec}\right)\left(1000 \frac{cycles}{sec}\right)\left(20\times10^{-3} H\right) j$

$Z_L = 126\Omega j$

$\omega = \left(2\pi \frac{rad}{s}\right)\left(1000 \frac{cycles}{sec}\right) = 6283 \, rad/s$

$$Z_T = Z + Z_R + Z_L = 50\Omega\angle 60^\circ$$

$$Z = 50\Omega\angle 60^\circ - Z_R - Z_L$$

$$= 50\Omega\cos 60^\circ + 50\Omega\sin 60^\circ j - 10\Omega - 126\Omega j$$

$$= 25\Omega + 43.3\Omega j - 10\Omega - 126\Omega j$$

$$Z = 15\Omega - 82.7\Omega j$$

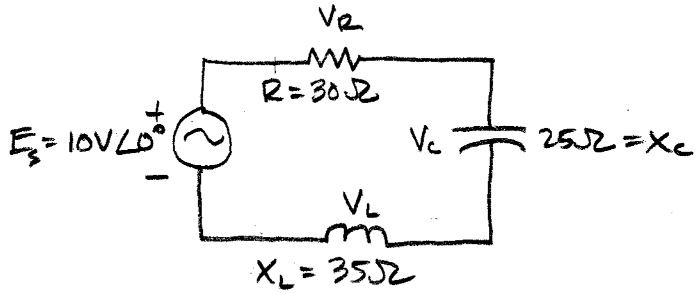
resistor Capacitor

$C = \frac{1}{\omega X_C} = \frac{1}{(6283\, rad/s)(82.7\Omega)}$

$C = 1.92\times10^{-6} F$

$\boxed{\text{Z is a } 15\Omega \text{ resistor in series with a } 1.92\mu F \text{ Capacitor}}$

18.25)



a. Use voltage divider rule to find V_R, V_C, V_L.

From 18.13, we already solved for $Z_T = 31.6\Omega\angle 18.4^\circ$

$$V_R = \frac{Z_R}{Z_T} E_s = \frac{30\Omega\angle 0^\circ}{31.6\Omega\angle 18.4^\circ}(10V\angle 0^\circ) = (0.95\angle -18.4^\circ)(10V\angle 0^\circ)$$

$$\boxed{V_R = 9.5V\angle -18.4^\circ}$$

$$V_C = \frac{Z_C}{Z_T} E = \frac{25\Omega\angle -90^\circ}{31.6\Omega\angle 18.4^\circ}(10V\angle 0^\circ) = (0.79\angle -108.4^\circ)(10V\angle 0^\circ)$$

$$\boxed{V_C = 7.9V\angle -108.4^\circ}$$

$$V_L = \frac{Z_L}{Z_T} E = \frac{35\Omega\angle 90^\circ}{31.6\Omega\angle 18.4^\circ}(10V\angle 0^\circ) = (1.11\angle 71.6^\circ)(10V\angle 0^\circ)$$

$$\boxed{V_L = 11.1V\angle 71.6^\circ}$$

b. $\Sigma E - \Sigma V = 0$ Kirchhoff's Voltage Law

$$E_s - V_R - V_C - V_L = 0$$

$$10V\angle 0^\circ - 9.5V\angle -18.4^\circ - 7.9V\angle -108.4^\circ - 11.1V\angle 71.6^\circ = 0$$

$$10V - [9.5V\cos(-18.4^\circ) + 9.5V\sin(-18.4^\circ)j] - [7.9V\cos(-108.4^\circ) + 7.9V\sin(-108.4^\circ)j]$$
$$- [11.1V\cos(71.6^\circ) + 11.1V\sin(71.6^\circ)j] = 0$$

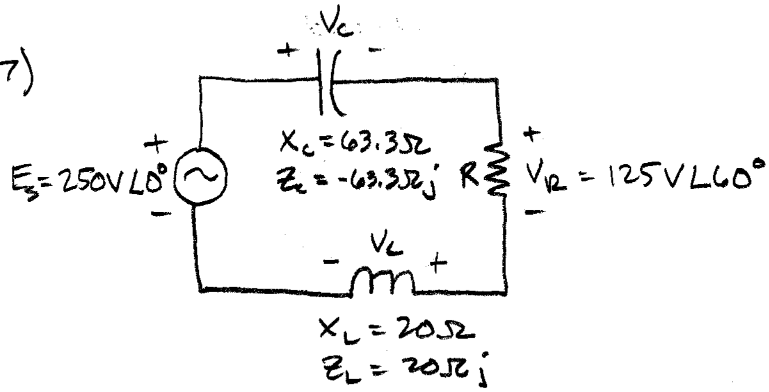
$$10V - 9.0V + 3.0Vj + 2.5V + 7.5Vj - 3.5V - 10.5Vj = 0$$

$$(10V - 9.0V + 2.5V - 3.5V) + (3.0V + 7.5V - 10.5V)j = 0$$

$$\boxed{0 + 0j \overset{\checkmark}{=} 0}$$

vPS

18.27)



a. Find V_c and V_L.

Using KVL,

$$E_s - V_c - V_R - V_L = 0$$

$$\begin{aligned} V_c + V_L &= E_s - V_R \\ &= 250V\angle 0^\circ - 125V\angle 60^\circ \\ &= 250V - 62.5V - 108.2Vj \\ &= 187.5V - 108.2Vj \\ V_c + V_L &= 216V\angle -30^\circ \end{aligned}$$

Then using the voltage divider rule,

$$V_c = \frac{Z_c}{Z_{c+L}} V_{c+L} = \frac{-63.3\Omega j}{-63.3\Omega j + 20\Omega j}(216V\angle -30^\circ)$$

$$= (1.46\Omega)(216V\angle -30^\circ)$$

$$\boxed{V_c = 316V\angle -30^\circ}$$

$$V_L = \frac{Z_L}{Z_{c+L}} V_{c+L} = \frac{20\Omega j}{-63.3\Omega j + 20\Omega j}(216V\angle -30^\circ)$$

$$= (-0.46\Omega)(216V\angle -30^\circ)$$

$$= -99.8V\angle -30^\circ$$

$$\boxed{V_L = 99.8V\angle 150^\circ}$$

b. Find R.

$$I_s = \frac{V_L}{Z_L} = \frac{99.8V\angle 150^\circ}{20\Omega\angle 90^\circ} = 5.0A\angle 60^\circ$$

$$Z_R = \frac{V_R}{I_s} = \frac{125V\angle 60^\circ}{5.0A\angle 60^\circ} = 25\Omega\angle 0^\circ$$

$$\boxed{R = 25\Omega}$$

18.27 can be solved another way by using Z_T and Ohm's Law to find I_s and V_R to solve for R. Then, V_C and V_L can be found using Ohm's Law

$$Z_T = Z_C + Z_R + Z_L = -63.3\Omega j + R + 20\Omega j = R - 43.3\Omega j$$

$$I_s = \frac{E_s}{Z_T} = \frac{250V\angle 0^\circ}{R-43.3\Omega j} = \frac{250V}{R-43.3\Omega j}$$

$$Z_R = \frac{V_R}{I_s} = \frac{125V\angle 60^\circ}{250V/R-43.3\Omega j} = \frac{1}{250V}\left[(125V\angle 60^\circ)(R-43.3\Omega j)\right]$$

$$R = Z_R = \frac{1}{250V}\left[(62.5V + 108.3V j)(R-43.3\Omega j)\right]$$

$$250V(R) = 62.5V(R) - 2706V\Omega j + 108.3V(R)j + 4689V\Omega$$
$$250V(R) = 62.5V(R) + 4689V\Omega + (108.3V(R) - 2706V\Omega)j$$

The real components on both sides of the equal sign are equal and the same applies to the imaginary components

Real: $250V(R) = 62.5V(R) + 4689V\Omega$

$$187.5V(R) = 4689V\Omega$$

$$R = \frac{4689V\Omega}{187.5V} = \boxed{25\Omega}$$

Imaginary: $108.3V(R) - 2706V\Omega = 0$

$$R = \frac{2706V\Omega}{108.3V} = 25\Omega \checkmark$$

$$\text{So, } I_s = \frac{250V\angle 0^\circ}{25\Omega - 43.3\Omega j} = \frac{250V\angle 0^\circ}{50\Omega\angle -60^\circ} = 5A\angle 60^\circ$$

$$\text{And, } V_C = Z_C I_S = (63.3\Omega\angle -90^\circ)(5A\angle 60^\circ)$$

$$\boxed{V_C = 317V\angle -30^\circ}$$

$$V_L = Z_L I_S = (20\Omega\angle 90^\circ)(5A\angle 60^\circ)$$

$$\boxed{V_L = 100V\angle 150^\circ}$$