AC Generators Revisited

5.1 Overall Power Consideration of a Three-phase AC Generator

A few lessons ago, three-phase AC generators were introduced and described. Now that you have learned about typical 3-phase circuit connections and calculated phase and line voltages, currents, and AC power, we return our attention to AC generators and look at them in more detail

To motivate our investigation, let's consider a shipboard application.

Example 1: Consider that we have a 3-phase, 1800 rpm, 450V synchronous generator rated to supply 3.75 MVA apparent power to a ship distribution system requiring a 0.8 lagging power factor. If this machine were operating at *rated conditions*, what would be the real and reactive power and the current being supplied? If the generator has an efficiency of 98%, what torque does the prime mover provide?

Solution. The real and apparent power are related to the power factor by

$$F_p = \frac{P_T}{S_T}$$

So we can compute that:

$$P_T = (0.8)\,(3.75 \text{ MVA}) = 3 \text{ MW}$$

This is the P_{out} of the generator and represents real work being performed on the ship: illuminating corridors, heating staterooms, and turning motors that spin pumps that pressurize the fire main. To get the total reactive power, Q_T, we use the power triangle result that:

$$\text{apparent power} = S_T = \sqrt{P_T^{\,2} + Q_T^{\,2}}$$

and find:

$$Q_T = \sqrt{(3.75 \text{ MVA})^2 - (3 \text{ MW})^2} = 2.25 \text{ MVAR}$$

In large measure, this quantifies the magnetic field requirements of the many shipboard (induction) motors. The *rated voltage* (here, 450 V) *is always a line voltage* (this is the voltage we can measure between any two cables in a 3-phase system), and the apparent power is related to the line voltage and line current by (from page 857 in the textbook):

$$S_T = \sqrt{3}\, V_L\, I_L$$

Thus, we calculate the magnitude of the line current as:

$$I_L = \frac{3.75\ MVA}{\sqrt{3}\ (450\,V)} = 4811\ A$$

In a shipboard application, the generator will normally be operated below "rated load" to provide margin for bringing on emergency equipment or equipment that may be shifted between generators due to a casualty (90% of rated load is a useful ceiling value for this margin). The lagging power factor will further tell us something about the angle between the **a**-phase voltage and current:

$$\theta = \cos^{-1}(F_p) = \angle V_a - \angle I_a$$

We will defer that calculation until we derive our per-phase equivalent circuit and review the difference between phase and line quantities. What can we say about the prime mover? Well, let's first determine the mechanical power delivered by the prime mover, which is P_{in} for the generator. We can calculate it by using the generator efficiency and the output power of the generator (i.e., P_T, which was calculated to be 3 MW):

$$\eta_{GEN} = \frac{P_{OUT}}{P_{IN}} = \frac{P_T}{P_{IN}}$$

So that:

$$P_{IN} = \frac{P_T}{\eta_{GEN}} = \frac{3\,MW}{.98} = 3.061\,MW$$

The prime mover output torque (which is T_{IN} for the generator) can then be calculated from:

$$T_{PM} = \frac{P_{IN}}{\omega_{mech}} = \frac{3.061\,MW}{1800\left(\dfrac{\pi}{30}\right)} = 16.24\,k\,N\cdot m$$

rpm × (π/30) → rad/sec

At this point it is useful to introduce the power conversion diagram for a generator, shown in Figure 1. Note that the power is mechanical at the input on the left, and is electrical at the output on the right, as we would expect for a generator.

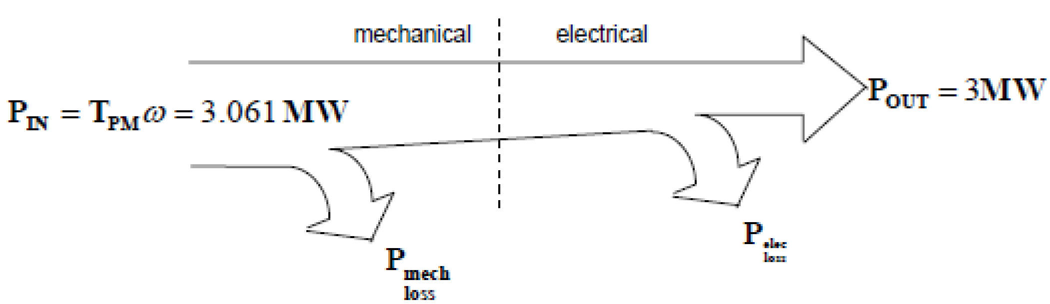


Figure 1: AC Generator Power Conversion Diagram

5.2 Per-Phase Equivalent Circuit

We can expand on the power conversion diagram by deriving a per-phase equivalent circuit representation of the three-phase synchronous machine. What is a per-phase circuit? Well, for three-phase systems that are balanced, we will find that the voltage between phases and the current between phases will only differ by an appropriate 120° shift. Therefore, not much additional information is captured by carrying along details of the voltage and current quantities of the two additional phases. Thus, a per-phase representation converts a three-phase circuit problem into a single-phase circuit problem. There are still some line and phase conversions that we would need to worry about, and there is some accounting to be done with relation to three-phase power, but per-phase analysis is a very convenient tool.

Let's first review phase and line quantities. Consider the three-phase, Y-connected generator shown in Figure 2. Note that **phase quantities** are associated with each generator coil. Thus the voltage directly across or the current directly through the turns of the coil are the phase quantities. In the case of the Y-connection, the ***phase voltage*** is a **line-to-neutral voltage** (the neutral point, N, being the common connection point for the coils of the three phases). The ***line voltage*** is the voltage between any two of the three lines that are distributed to the three-phase system. For a balanced system, the line voltage is √3 times larger than the phase voltage. We mainly refer to the line voltage because it is a quantity that we can always go into a system and measure (the neutral for some systems is not available and so the line-to-neutral voltage is not as readily accessible).

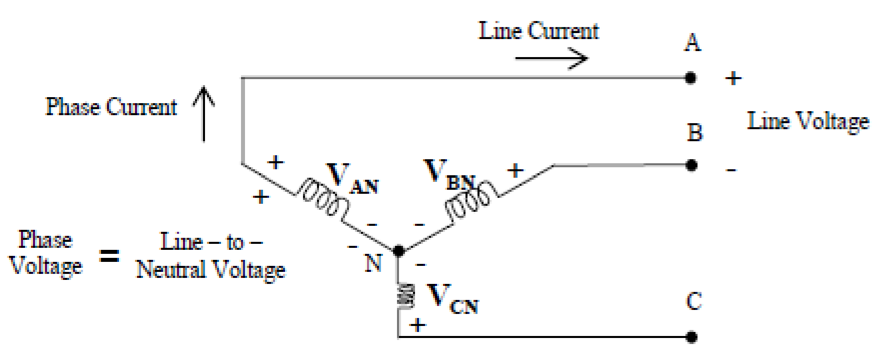


Figure 2: Three-Phase Synchronous Y-Connected Generator Stator

In considering a per-phase model of the three-phase synchronous machine, we will focus arbitrarily on the **a**-phase stator coil. Further, we will perform our derivation for a 2-pole machine and extrapolate our results to an arbitrary P-pole machine.

To derive the per-phase equivalent circuit for the **a**-phase stator coil, we take into account the effect of the magnetic fields present on the stator coil. We know, as Faraday's Law describes, that a voltage is induced in the **a**-phase stator coil when the prime mover rotates the field windings past the stator coil. If we could look into the terminals of the **a**-phase coil, we would see this induced voltage, $E_{induced}$. We would also see the resistance of the stator coil, R_s, and the voltage developed across the self-inductance of the stator coil (from Ohm's Law: voltage = reactance × current). From Kirchhoff's Voltage Law we can then write (see Figure 3):

$$\mathbf{E}_{\text{induced}} = \mathbf{j}\ \mathbf{X}_s\ \mathbf{I}_a + R_s\mathbf{I}_a + \mathbf{V}_{AN}$$

where V_{AN} is the terminal voltage of the **a**-phase coil. Note that this equation is stated in the phasor domain, so the voltage and current quantities have magnitudes and angles. We assigned the **synchronous reactance**, $X_s = \omega_e L_s$, to describe the self-inductance of the stator coil, where ω_e is the electrical frequency in rad/sec. $E_{induced}$ is the voltage generated as described by Faraday's Law. The amplitude of this AC voltage depends, of course, on the DC current flowing in the field winding, which provides the magnetic field that is rotated by the prime mover.

Our per-phase equivalent circuit can now be drawn as shown in Figure 3.

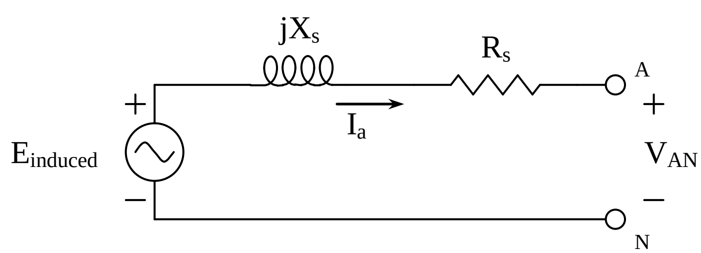


Figure 3: Per-Phase Equivalent Circuit for an AC Generator

<u>Example 2</u>: An Arleigh-Burke Class destroyer has a 3-phase, Y-connected, 4-pole, 60 Hz synchronous generator, rated to deliver 3.75 MVA with a 0.8 lagging power factor and a line voltage of 450 V. The machine stator resistance is negligible and the synchronous reactance is equal to 0.04 Ω. The actual system load on the machine draws 2 MW at a 0.8 F_p lagging. Assume that a voltage regulator has automatically adjusted the field current so that the terminal voltage, V_{AN}, is at its rated value.

a. What is the rated speed?

b. Determine the reactive and apparent power delivered by the generator.

c. Find the current drawn from the generator *using the terminal voltage, $\mathbf{V}_{AN}$, as the reference phasor*.

d. Determine the induced voltage, $\mathbf{E}_{\text{induced}}$.

<u>Solution</u>.

a. The speed of the machine in rpm is found once we know the number of poles, 4, and the system frequency, 60 Hz:

$$N = \frac{120\ f_e}{P} = \frac{120\,(60)}{4} = 1800\ rpm$$

b. The apparent power is found from our power factor relationship. Here we must use the *actual* power being delivered (2 MW), <u>not</u> the power we could derive from the *rated* 3.75 MVA value.

$$S_T = \frac{P_T}{F_p} = \frac{2\ MW}{0.8} = 2.5\ MVA$$

Next, the reactive power is found via the power triangle:

$$Q_T = \sqrt{{S_T}^2 - {P_T}^2} = \sqrt{(2.5\ MVA)^2 - (2\ MW)^2} = 1.5\ MVAR$$

c. We can find the line current of the machine a couple of different ways. Let's consider each.

<u>Method 1.</u>

First, we can use the apparent power expression from the textbook on page 857 to get the magnitude of the line current:

$$S_T = \sqrt{3}\, V_L\, I_L$$

The rated voltage is always the line voltage, so rearranging and substituting gives:

$$I_L = \frac{S_T}{\sqrt{3}\, V_L} = \frac{2.5\ MVA}{\sqrt{3}\,(450\ V)} = 3208\ A$$

To calculate the angle of the current phasor we use the load power factor and the fact that $\mathbf{V}_{AN}$ is our reference phasor (therefore the angle of $\mathbf{V}_{AN}$ is 0°):

$$\theta = \cos^{-1}(F_p) = \angle V_{AN} - \angle I_a$$

Substituting and solving for the angle of the current gives:

$$\angle I_a = \angle V_{AN} - \cos^{-1}(F_p) = 0° - \cos^{-1}(0.8) = -36.87°$$

Therefore the current phasor is:

$$\mathbf{I_a} = 3208\ \text{A} \angle -36.87°$$

Method 2.

Alternatively, we could have found the current using our three-phase power expression from page 857 in the textbook:

$$P_T = 3\, V_\phi\, I_\phi \cos\theta$$

[*Note that V_ϕ used in the textbook represents the same quantity that V_{AN} does in this supplement*.]

To use this expression, we need to correctly establish the magnitude of the phase voltage, V_ϕ, for the given line voltage, 450V. Since our machine is Y-connected, the phase voltage is a line-to-neutral voltage so we must convert the rated line voltage to its line-to-neutral value:

$$V_\phi = \frac{V_L}{\sqrt{3}} = \frac{450\,V}{\sqrt{3}} = 259.8\,V$$

Substituting into the total real power expression yields:

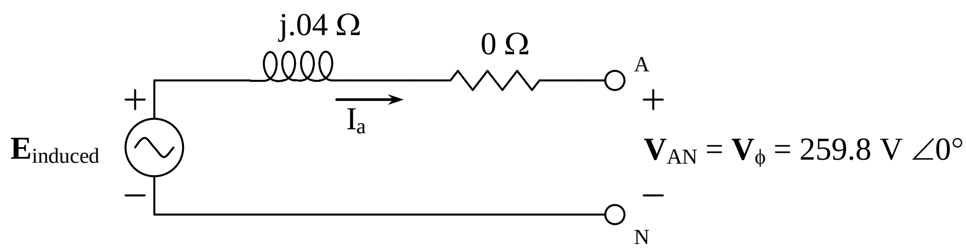
$$I_\phi = \frac{P_T}{3\,V_\phi \cos\theta} = \frac{2\ MW}{3\ (259.8)\ (0.8)} = 3208\ A$$

To retrieve the current phasor, we need to simply determine the current angle as shown in Method 1, above; therefore:

$$\mathbf{I}_a = 3208\ \mathbf{A}\ \angle{-36.87°}$$

as we found previously!

d. To determine the induced voltage, $\mathbf{E}_{induced}$, we apply the KVL equation for our equivalent circuit:



$$\mathbf{E}_{induced} = \mathbf{j\ X_s\ I_s} + R_s\ \mathbf{I_a} + \mathbf{V}_{AN}$$

Substituting, recognizing that the stator resistance was considered negligible, gives:

$$\mathbf{E}_{induced} = j\ (0.04\ \Omega)\ (3208\ A\angle{-36.87°}) + 259.8V\angle 0° = 352.1V\angle 16.94°$$

Note that the circuit contains no loss mechanism. Is this reasonable? Yes, for large machines the efficiency is typically greater than 98%. So we are saying that:

$$P_{IN} \approx P_T = P_{OUT}$$

where, P_T is the three-phase power at the terminals of the generator (i.e., with no loss, real power IN is equal to the real power OUT of the generator).