

Rotating DC Motors Part I

The previous lesson introduced the simple linear motor. Linear motors have some practical applications, but rotating DC motors are much more prolific. The principles which explain the operation of linear motors are the same as those which explain the operation of practical DC motors. The fundamental difference between linear motors and practical DC motors is that DC motors rotate rather than move in a straight line. The same forces that cause a linear motor to move “right or left” in a straight line cause the DC motor to rotate. This chapter will examine how the linear motor principles can be used to make a practical DC motor spin.

16.1 Electrical machinery

Before discussing the DC motor, this section will briefly introduce the parts of an electrical machine. But first, what is an electrical machine? An *electrical machine* is a term which collectively refers to motors and generators, both of which can be designed to operate using AC (Alternating Current) power or DC power. In this supplement we are only looking at DC motors, but these terms will also apply to the other electrical machines.

16.1.1 Physical parts of an electrical machine

It should be apparent that the purpose of an electrical motor is to convert electrical power into mechanical power. Practical DC motors do this by using direct current electrical power to make a shaft spin. The mechanical power available from the spinning shaft of the DC motor can be used to perform some useful work such as turn a fan, spin a CD, or raise a car window. The rotation of the DC motor is accomplished by the force which is developed on a current-carrying conductor in a magnetic field. The current-carrying conductor is connected to the shaft which is able to rotate relative to the stationary body of the DC motor. With this simple understanding, we can divide any motor into two physical parts; one part which rotates—called the *rotor*—and one part which doesn’t—called the *stator*. Figure 1 shows a simple diagram of an electrical machine showing the rotor and stator.

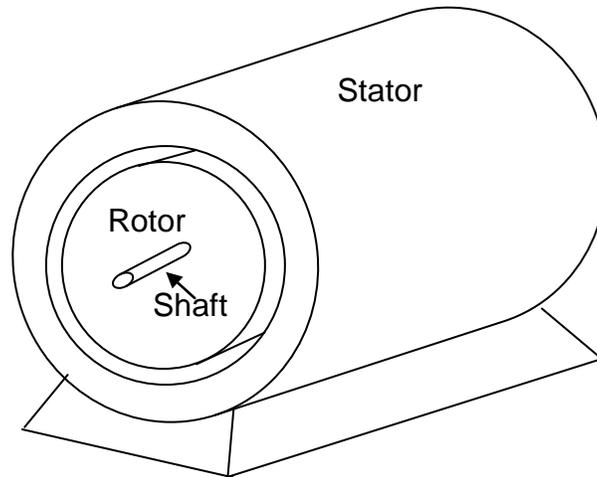


Figure 1: Layout of Typical Rotating Machine

DC motors use DC current and voltage to power the motor. For reasons which will be made clear below, it is necessary to change the direction of the DC current that is applied to the current-carrying conductor within the rotor. This is accomplished by utilizing a segmented metal ring, called a *commutator*. A commutator is directly connected to the current-carrying conductor, so it will rotate with the rotor. The commutator maintains electrical contact with its external DC electrical power source by using metal or hard carbon *brushes*. A more detailed description of commutation will be given below.

16.1.2 Functional parts of an electrical machine

As previously stated electrical machines are divided into two physical parts, rotor and stator. Electrical machines can also be divided into two *functional* parts. One functional part is the magnetic field, simply called the *field*, and the other functional part is the conductor, which is called the *armature*. In a given machine, one functional part is associated with one physical part, and the other functional part is then associated with the other physical part. So there are two possible configurations for electrical machines: 1) the field rotates with the rotor and the armature is on the stator, or 2) the armature rotates with the rotor while the field is on the stator. Any electrical machine can be designed in either configuration, although for practical reasons one design tends to dominate DC machines. The DC motor that we will study in EE301 will use a rotating armature inside a magnetic field, which is developed within the stator as shown in figure 2.

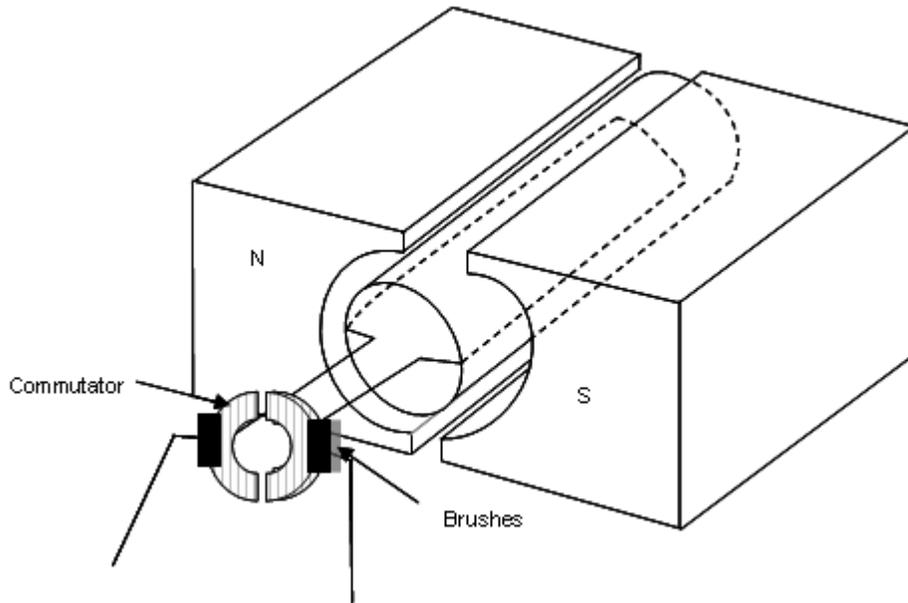


Figure 2. Rotating DC motor

16.2 DC Motor Operation

In simplest terms, a DC motor operates by using the force described by the Lorentz force law presented in Lesson 15. DC voltage is applied across the armature. The current-carrying armature is in the magnetic field generated in the stator. In Figure 2, the stator is represented by a permanent magnet with its North and South Pole as shown. The current-carrying wire in a magnetic field results in a force which will rotate the rotor.

Figure 3 is a simple diagram of a DC motor, showing the magnet poles as the stator and the direction of the current flowing through wire of the armature. Since the magnetic force ($\vec{F} = I\vec{L} \times \vec{B}$) is a cross product, we can see that the magnetic force acts to pull the top conductor (“a”) of the armature loop towards the left, and acts to pull the lower conductor (“b”) towards the right. These two forces rotate the armature that is attached to the rotor.

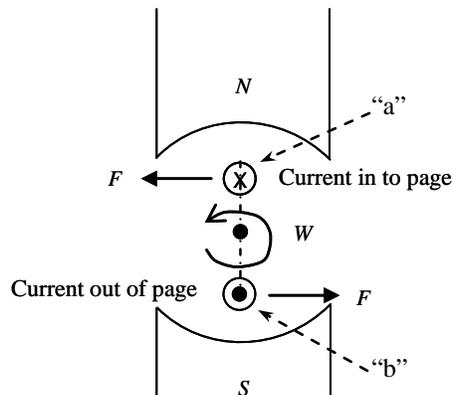


Figure 3.

You may be able to see that if the armature current is always in the same direction, the conductor (“a”) shown on the top in Figure 3 will always be pulled towards the left and the conductor (“b”) shown on the bottom in Figure 3 will always be pulled towards the right. At best, this motor would only rotate through one-half (180°) of a rotation and would stop when the “a” conductor is in the 9 o’clock position and the “b” conductor is in the 3 o’clock position. This is where the commutator comes into the picture. Recall that the function of the commutator is to reverse the direction of the current flowing through the armature in the rotor; this is precisely what happens. Again, the Lorentz force equation tells us that if the current direction is reversed, the sign of the cross product is also reversed. This means that the Lorentz force is now directed to pull the “a” conductor towards the right and the “b” conductor towards the left, allowing the motor to rotate through more than one-half of a rotation. By changing the current direction every half-rotation (when the conductors are in the 3 and 9 o’clock positions), the Lorentz force is always acting to keep the motor spinning 360° in one direction.

What about torque production? We apply a voltage to the brush terminals. This causes a current to flow into the top brush of the machine. Let’s call this current I_a (subscript “a” standing for “armature”) and consider it for the loop/commutator positions shown in Figure 4. Note, while the brushes contact both commutator segments (positions 1 and 3), the current is diverted from the loop and will flow directly from the top brush to the lower brush (this short-circuit condition will go away as we move to a more practical machine). In position 2, the current flows into the top brush and then must flow into coil-side **b** and back out coil-side **a**. The current then exits through the bottom brush. In position 4, the current enters the top brush but this time the commutator routes this current to coil-side **a**. The current traverses the loop and returns to the lower brush via coil-side **b**. Thus, the commutator ensures that the top conductor of the armature coil is always carrying current *INTO* the page while the bottom conductor of the armature coil is always carrying current *OUT OF* the page. The importance of this comes into play when we consider the production of force and recall our previous results regarding the Lorentz force:

$$\vec{F} = I\vec{L} \times \vec{B}$$

The equation used to determine torque is $\vec{T} = \vec{R} \times \vec{F}$, where \vec{R} is the vector which gives the armature position relative to the central axis of the motor and \vec{F} is the Lorentz force vector. If we apply the Lorentz force equation to the machine in position 2 (Figure 4), the top conductor will experience a force to the LEFT (I is directed IN, while \vec{B} is DOWN, so there is a natural 90 degrees between them) while the bottom conductor will experience a force to the RIGHT (I is OUT, while \vec{B} is DOWN, again 90 degrees between them). Since these forces are tangential to the rotor, the cross-product in the torque expression becomes multiplication and we arrive at the developed torque of

$$T_d = BLI_a R + BLI_a R = 2BLI_a R$$

Torque has a maximum value when \vec{R} is normal to \vec{F} , and its minimum value, 0, when \vec{R} and \vec{F} are parallel. This is shown in figure 4. In position 1, the armatures wires are located half-way between the two magnets, so \vec{R} is parallel to \vec{F} . No torque is developed by the motor until something makes the motor rotate slightly. Maximum torque, as shown in position 2 is developed when the armature wires are located at the top and bottom.

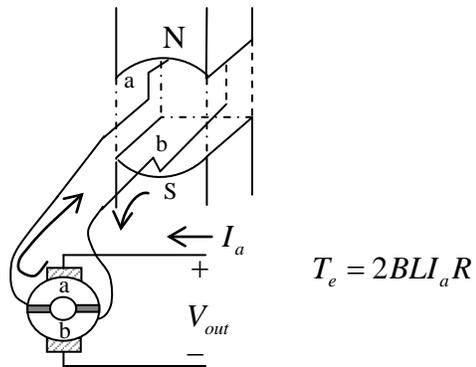
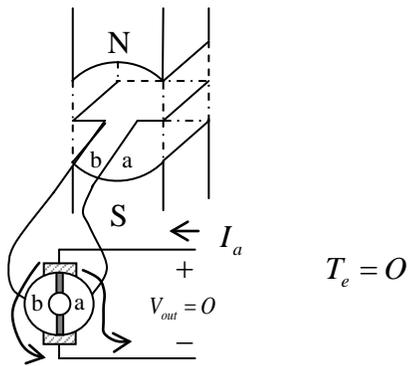
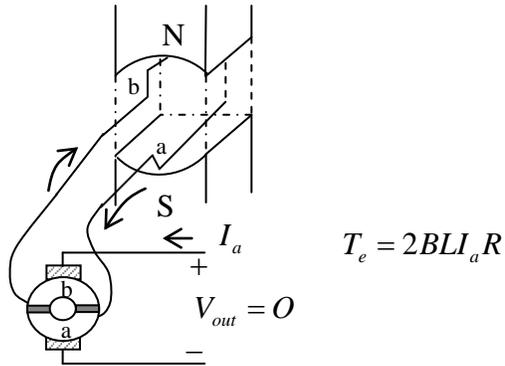
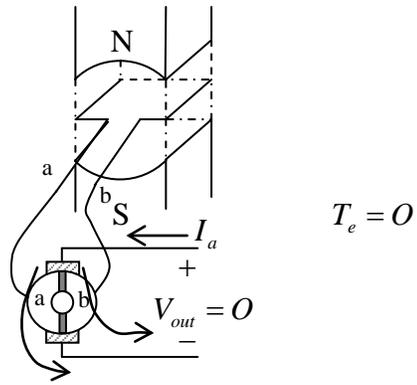


Figure 4: Current Flow and Torque Production in Rotating Loop

Again assuming a radially-directed B-field, as the loop and commutator turns, the developed forces will remain tangential. The resultant torque waveform is sketched in Figure 5. Note the large variation in torque when the brushes short-circuit the terminals of the loop.

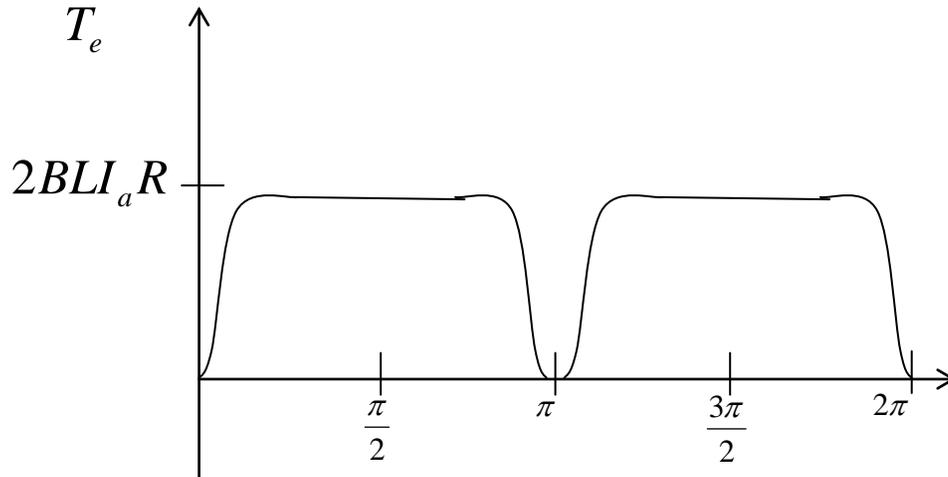


Figure 5: Developed Electromagnetic Torque for Single Loop Machine

To develop a more practical DC motor (less variation in the developed electromagnetic torque), we need to:

1. Add and spatially distribute more rotating loops
2. Increase the number of commutator segments

As noted in the section 16.1.2, the rotating loops are called the **armature** of our DC motor (hence the subscript “a” that we have been using).

Up to this point the armature shown in Figures 2, 3, and 4 has been simply a single loop of wire. If we calculated the maximum torque generated on such an armature using typical current and magnetic field values where $T = RF = 2RLIB$, the result would be a very small number. The maximum torque could conceivably be increased by using higher currents or stronger magnetic fields, but these are usually not practical solutions. A more practical solution is to construct the armatures using coils of wire rather than a single loop. This multiplies the maximum torque by the number of wire loops, N, (usually called the number of turns) in the armature. Then the equation for maximum torque becomes:

$$T = NRF = 2NRLIB.$$

The role of the commutator in multiple armature motors is also worth noting. Recall that the commutator in a single-loop armature motor simply changed the direction of the

current through the armature. In multiple-loop armature motors it still does this, but it also performs the additional function of delivering current only to the armature loop which is best positioned to develop maximum torque.

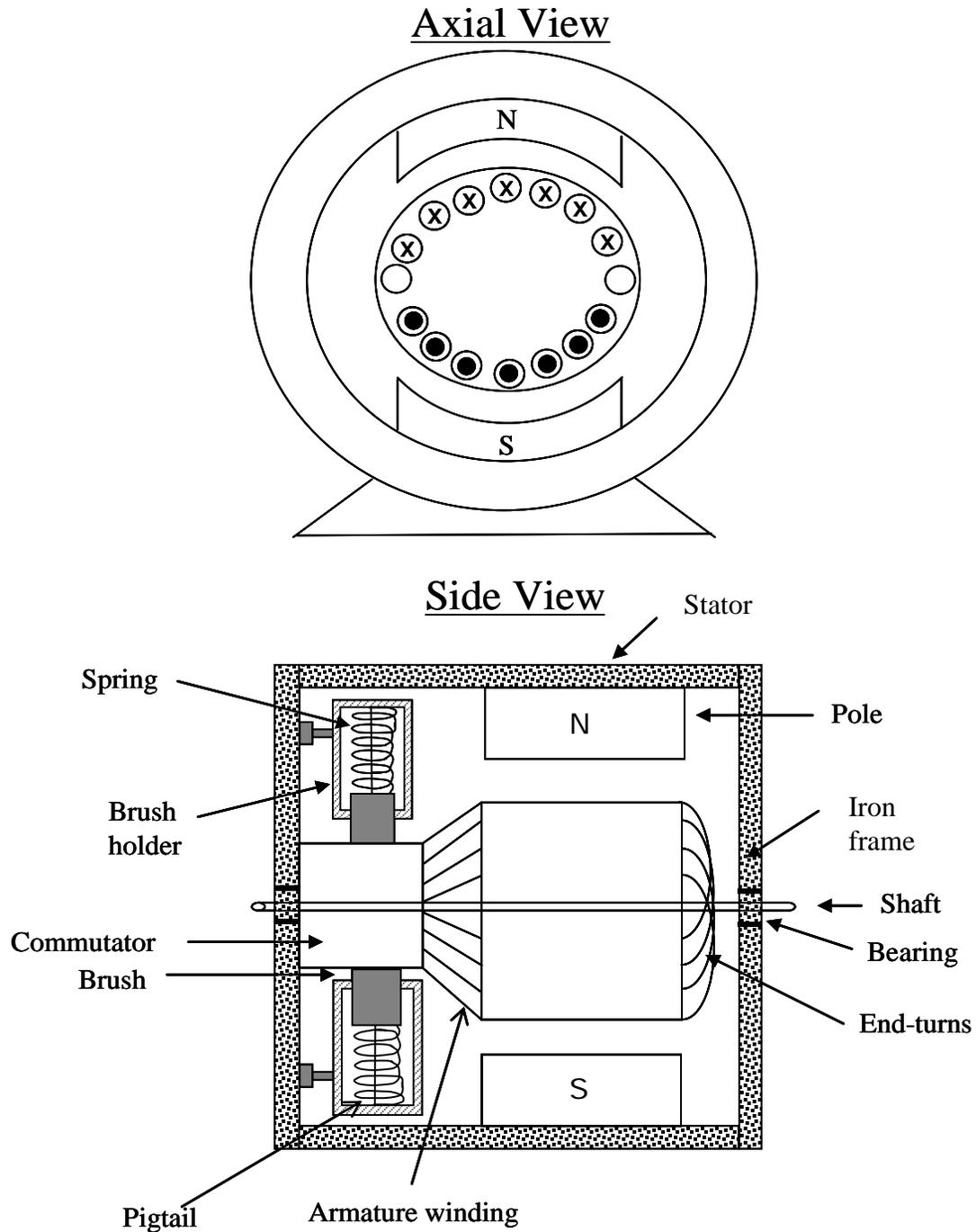


Figure 6: Layout for a Practical Permanent Magnet DC Machine

16.3 A Realistic DC Machine

A realistic DC machine is shown in Figure 6. In the axial (head-on) view, we observe that the armature conductors in the top half of the plane all carry current into the page, while the conductors on the bottom half of the plane all carry current out of the page. The spatial distribution of current, enforced by the action of the commutator, ensures that each of the top conductors will experience a force to the left while each of the lower conductors will experience a force to the right (hence the machine will rotate counter-clockwise). The north and south poles shown on the stationary part of the machine may be derived from permanent magnets or they could be generated from an electrically-excited stationary coil called the *field windings*. The side view illustrates some more practical machine issues. Note the actual length of the machine consists of the length of the armature, end-turns of the armature windings, the commutator, and the iron frame of the stator. The brushes are shown spring-loaded so they make adequate contact with the commutator. The shaft runs through the armature and commutator, and is supported in the stator iron frame by bearings. The “height” of the machine depends on the diameter of the armature, the poles of the field magnet, and the stator iron frame. Here, we consider a machine with two poles. A more compact and efficient design would utilize more poles.

16.4 Permanent Magnet Machines

In deriving an expression for the developed electromagnetic torque (T_d), we have said very little about the air-gap magnetic flux density, B . (The “air-gap” is the small space between the rotating armature and the stationary field poles.) We have assumed that it has been created somehow, perhaps with a coil wrapped about the stationary core. We have also been a bit remiss in not commenting on how the air-gap field is influenced by the magnetic field created by the armature windings. For now, we will assume that this “armature reaction” has been compensated for by our machine designer. The air-gap flux density can also be created by using permanent magnets attached to the stator core. The permanent magnet coupled with a fixed air-gap will establish a relatively constant value of B -field in the air gap.

To simplify our analysis and focus on one class of DC machine, we will only consider permanent-magnet machines (you are free to cheer at this point). If the B -field is fixed, then the term $N2BLR$ is simply a constant (recall, L is the axial length of the machine and R is the radius of the armature). We can replace this constant by a new parameter, K_v , so that:

$$T_d = (N2BLR)I_a = K_v I_a \quad (K_v \text{ is called the } DC \text{ motor constant, or the machine constant, and has units of } \textit{volt-seconds, V-s})$$

The mechanical dynamics of the DC motor are governed by Newton’s Laws. The developed electromagnetic torque is balanced by an inertial torque, a load torque, and a rotational loss torque:

$$T_d = K_v I_a = J \frac{d\omega}{dt} + T_{Load} + T_{Loss}$$

In the steady-state, the angular velocity ω is constant (thus $d\omega/dt = 0$) and therefore the electromagnetic torque must balance only the load and loss torques. What is the rationale for including T_{Loss} ? Even unloaded, the machine will have to produce torque to overcome friction and windage (air resistance) effects. Further, there are other secondary losses in the machine iron that are not accounted for in the armature resistance that we lump into this parameter. We can evaluate the no-load losses at various speeds. If this power loss characteristic is linear versus speed, then it is reasonable to model the extra loss component by a constant torque (since $P = T\omega$). If the no-load power varies more as a quadratic with speed, then we would replace T_{Loss} by a term like $C\omega$ where C is a constant.

It is interesting (to somebody) to observe that the mechanical power developed due to the electromagnetic torque is found to be

$$P_d = T_d \omega = K_v I_a \omega$$

If we ignore rotational losses so that $T_{Loss} = 0$, this developed power becomes the output mechanical power and we can evaluate the machinery efficiency as

$$\eta = \frac{P_{OUT}}{P_{IN}} 100 = \frac{P_d}{P_{in}} 100 \quad (T_{loss} = 0)$$

Example 1: A DC motor is used to raise and lower a window in an automobile. The motor draws 8.1A and runs at 800 rpm while raising the window. The machine constant is 0.095 V-s.

If the torque required to lower the window is 80% of that required to lift the window (gravity helps), find the current required to lower the window.

$$T_{d,up} = K_v I_a$$

$$T_{d,up} = (0.095 N - m / A)(8.1A)$$

$$T_{d,up} = 0.7697 N - m$$

$$T_{d,down} = 0.8 T_{d,up} = 0.8(0.7697 N - m)$$

$$T_{d,down} = 0.6158 N - m$$

Then:

$$I_{a,down} = \frac{T_{d,down}}{K_v} = \frac{0.6158 N - m}{0.095 N - m / A} = 6.48 A$$

16.5 Back EMF

In the previous sections the operation of rotating DC motors was introduced. It was shown that the force exerted on a current-carrying conductor in a magnetic field can be used to turn a motor – electrical energy is converted into a mechanical energy. In a later lesson, generators will be introduced. It will be shown that generators use mechanical energy to create electrical energy. They do this by forcing a conductor to move through a magnetic field. This forced motion of a wire through a magnetic field causes a voltage to be developed on the wire that causes a current to flow. This developed voltage was called “ $E_{induced}$ ” in the lesson on linear motors. This generation of an induced voltage exists anytime a conductor is moved through a magnetic field, including such motion in a motor. In other words, the intended rotation of the armature in a motor also creates an unintended generation of voltage. This is the induced voltage described by Faraday’s Law in the sections on linear motors.

We will use the symbol “ E_a ” from now on to represent the induced voltage. This generated voltage on the armature is referred to as *induced electromotive force*, or, more commonly, the *back EMF*. As the label “back EMF” implies, the polarity of the back EMF is opposite of the polarity of the applied voltage which causes the armature current to flow, just as we saw for the linear motor. The equation used to calculate back EMF is:

$$E_a = K_v \omega$$

where K_v is the *DC motor constant*, which depends on the construction of the motor, and ω represents the angular velocity of the armature (in *radians per second*). Greater back EMF can be developed by turning the rotor faster.

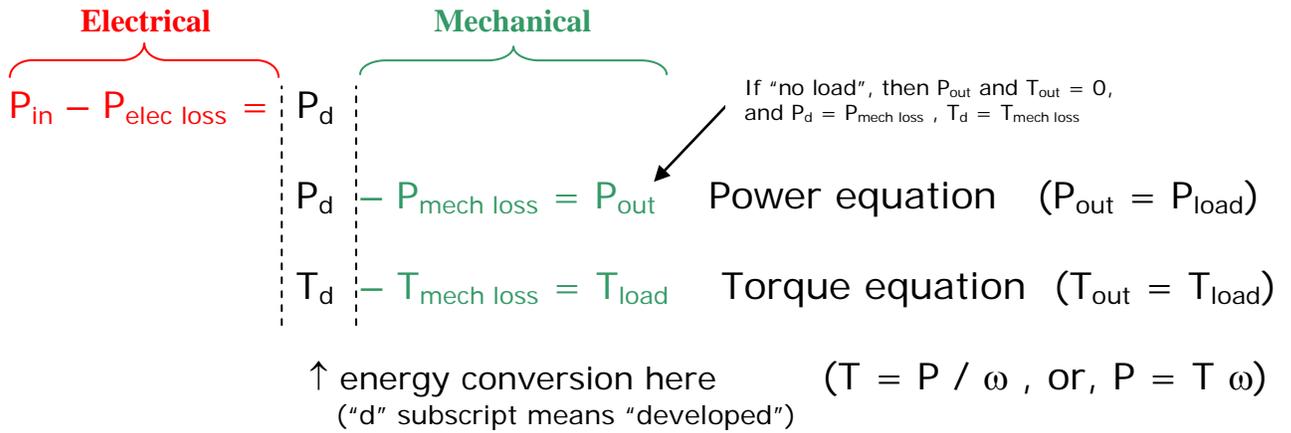
As mentioned previously, the equation for torque using the DC motor constant is:

$$T = K_v I_a .$$

It should be clear from this equation that a greater torque can be generated by increasing the armature current.

Here is a summary of the rotating motor equations:

Rotating Motor Equations



$$P_{in} = V_{DC} I_a$$

$$P_{elec\ loss} = P_a = I_a^2 R_a$$

$$P_d = E_a * I_a = T_d \omega = K_v I_a \omega$$

$$E_a = K_v \omega$$

$$T_d = K_v I_a$$

$$\eta = P_{out} / P_{in}$$