Rotating DC Motors Part II

II.1 DC Motor Equivalent Circuit

The next step in our consideration of DC motors is to develop an equivalent circuit which can be used to better understand motor operation. The armatures in real motors usually consist of many windings of relatively thin wire. Recall that thin wires have larger resistance than thick wires. The equivalent circuit then must include a resistor R_a which accounts for the total resistance of the armature winding. Figure 1 shows the equivalent circuit for a DC motor. V_{DC} represents the applied voltage which causes the armature current to flow, R_a is the resistance of the armature, and E_a is the generated or induced "back EMF" in the armature.

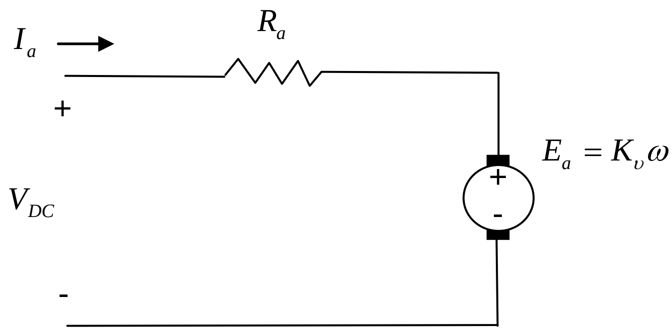


Figure 1: Permanent Magnet DC Motor Equivalent Circuit

Figure 1 shows that armature current, I_a, flows from the positive terminal of the voltage source, V_{DC}, and the value of I_a is determined by $I_a = (V_{DC} - E_a)/R_a$. The back EMF acts to limit the flow of armature current, but E_g must always be less than V_{DC} in order for I_a to be positive. The equivalent circuit for the DC motor can be analyzed using any circuit technique learned earlier in this course.

II.2 Power in DC Motors

The power input to a DC motor is simply the input current multiplied by the applied voltage:

$$P_{IN} = I_a V_{DC}.$$

Due to electrical and mechanical losses in the motor, the mechanical power out of the motor must be less than the electrical power in. The most obvious electrical loss is due to the armature resistance, $P_{elec\ loss} = I_a^2 R_a$. As discussed in previous sections, power losses also occur due to:

- friction between parts of the machine

- magnetic inefficiencies of the material used
- air resistance (windage) of the rotating armature
- small resistance across the brushes used to couple current onto the commutator

These losses can be approximated collectively by specifying an overall value for mechanical power lost, $P_{mech\ loss}$.

As with any other system which does not store energy, the electrical power input to a DC motor must equal the sum of the mechanical power output and the various loss mechanisms:

$$P_{IN} = P_{OUT} + P_{elec\ loss} + P_{mech\ loss}$$

The output power can be calculated from the power balance equation above, or from the product of output torque and angular velocity:

$$P_{OUT} = T_{load}\,\omega$$

Mechanical power is often expressed in units of horsepower, *hp*. This is a non-SI unit of power equal to 746 watts. If torque is expressed in N·m and angular velocity is expressed in units of *radians per second*, then the units out of $P_{OUT} = T_{load}\ \omega$ will be expressed in watts. To convert to horsepower, simply divide the answer in watts by 746.

A power conversion diagram is a useful tool to document the power flow in a motor. It clearly shows that the input power is electrical and that the output power is mechanical. The power developed, P_d, denotes the change from electrical to mechanical power.

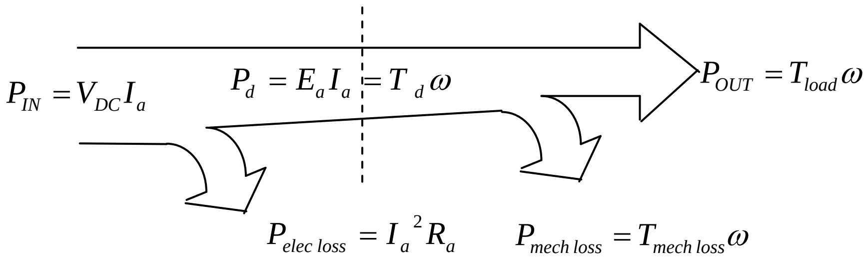


Figure 2 : Power Conversion Diagram

II.3 Efficiency

Efficiency of a system can be generally defined as the ratio of power output relative to power input. In DC motors this becomes the mechanical power out relative to the electrical power in:

$$\eta = P_{OUT} / P_{IN}$$

The expression for power out, $P_{OUT} = T_{load}\omega$, can be recast to provide a simple equation for efficiency as follows. Earlier in this chapter, developed torque was given as $T_d = K_v I_a$. Multiplying both sides of this equation by the angular velocity, ω, gives:

$$T_d\ \omega = K_v\ I_a\ \omega$$

But $T_d\omega$ equals the power developed, P_d. Thus:

$$P_d = K_v I_a \omega$$

Back EMF was also defined earlier as $E_a = K_v\omega$. Hence, we can arrive at a new equation for mechanical power developed as shown at the dashed line in Figure 2 above:

$$P_d = T_d\omega = K_v I_a \omega = E_a I_a$$

In many cases the friction losses will be small compared to the armature loss. ***If we assume that mechanical losses can be ignored***, the efficiency of a DC motor can be expressed simply as the ratio of back EMF to applied voltage because:

$$T_d = T_{load} + T_{mech\ loss}$$

and if: $$T_{mech\ loss} = 0$$

then: $$T_d = T_{load}$$

hence:

$$P_{OUT} = T_{load}\omega = T_d\omega = K_v I_a \omega = E_a I_a$$

and

$$\eta = \frac{P_{OUT}}{P_{IN}}(100\%) = \frac{E_a I_a}{V_{DC} I_a}(100\%) = \frac{E_a}{V_{DC}}(100\%)$$

Again, *this final equation assumes all mechanical losses are ignored*.

Here is a summary of the rotating motor equations:

Rotating Motor Equations

$P_{in} - P_{elec\ loss} = P_d$ ↓ mechanical ↓

If "no load", then P_{out} and $T_{out} = 0$, and $P_d = P_{mech\ loss}$, $T_d = T_{mech\ loss}$

↑ electrical ↑ $P_d - P_{mech\ loss} = P_{out}$ Power equation ($P_{out} = P_{load}$)

$T_d - T_{mech\ loss} = T_{out}$ Torque equation ($T_{out} = T_{load}$)

↑ energy conversion here ($T = P / \omega$, or, $P = T\,\omega$)

("d" subscript means "developed")

$$P_{in} = V_{DC}\ I_a$$

$$P_{elec\ loss} = I_a^2\ R_a$$

$$P_d = E_a * I_a = T_d\,\omega = K_v\,I_a\,\omega$$

$$E_a = K_v\,\omega$$

$$T_d = K_v\,I_a$$

$$\eta = P_{out} / P_{in}$$

Example II-1: A permanent magnet DC motor is rated for 25 *V, 2 A* and *1300 rpm.* If the machine is 90% efficient at rated conditions find R_a and K_v if $T_{mech\ loss} = 0.0334\ N \cdot m$.

SOLUTION: We use the given rated electrical information to determine the power into the motor.

$$P_{IN} = V_{DC} I_a = (25V)(2\,A) = 50\,W$$

The efficiency value then enables us to calculate the output power at rated conditions.

$$P_{OUT} = (0.90)(50\,W) = 45\,W$$

We can calculate the power loss at rated conditions by

$$\begin{aligned} P_{mech\ loss} &= T_{mech\ loss}\,\omega \\ &= (0.0334\ N \cdot m)\left(\frac{1300\,rev}{\min}\right)\left(\frac{1\min}{60\ s}\right)\left(\frac{2\pi\ rad}{1rev}\right) \\ &= 4.55\,W \end{aligned}$$

The power developed must be

$$P_d = P_{mech\ loss} + P_{OUT}$$

$$P_d = 4.55\,W + 45\,W = 49.55\,W$$

Now, we can solve for K_v.

$$P_d = T_d\,\omega = K_v I_a \omega$$

$$49.55\,W = K_v (2\,A)\,1300\,rpm\left(\frac{\pi}{30}\right)$$

$$K_v = 0.182\,V \cdot s$$

To find R_a:

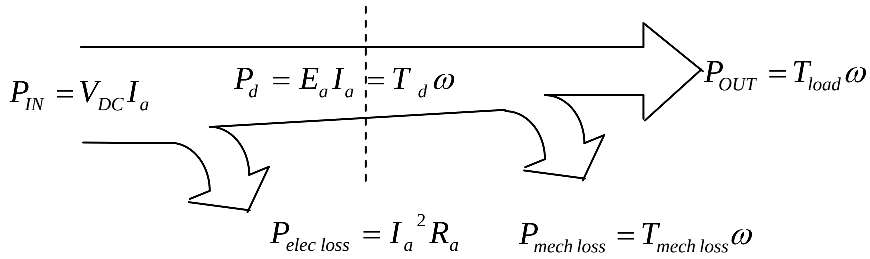
$$V_{DC} = R_a I_a + E_a$$

$$V_{DC} = R_a I_a + K_v \omega$$

$$25 = R_a (2\,A) + (0.182\,V \cdot s)(1300)\left(\frac{\pi}{30}\right)$$

$$R_a = 0.11\,\Omega$$

A power conversion diagram shows another approach to this problem if we consider electrical power loss in the armature.



From the power conversion diagram.

$$P_{IN} = P_{elec\ loss} + P_{mech\ loss} + P_{OUT}$$
$$50\,W = P_{elec\ loss} + 4.55\,W + 45\,W$$
$$P_{elec\ loss} = 0.45\,W$$
$$P_{elec\ loss} = I_a^2 R_a$$
$$0.45\,W = (2A)^2 R_a$$
$$R_a = 0.11\,\Omega$$