Complex Power Calculations

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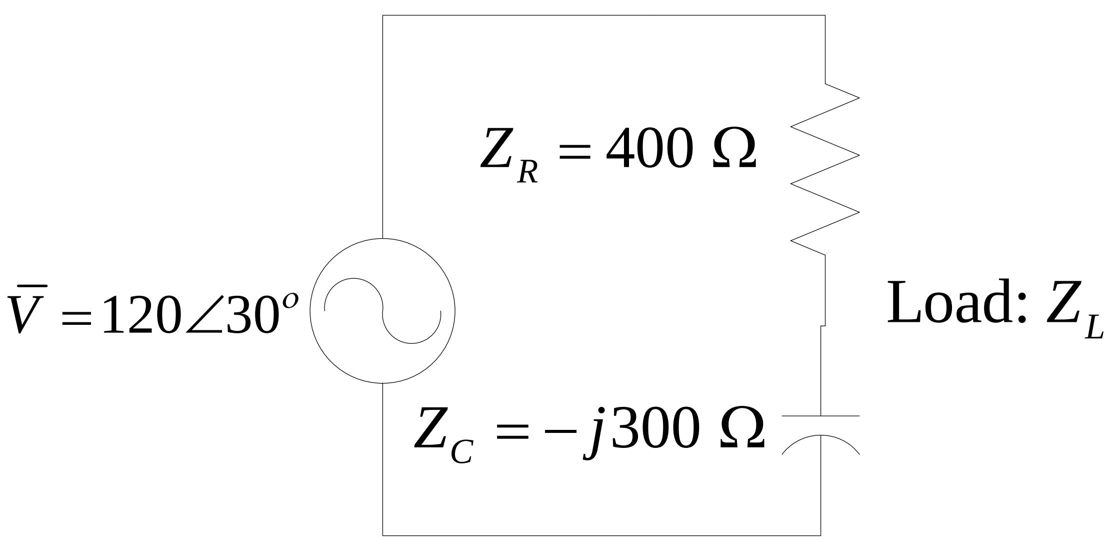
There are several techniques for calculating the real, reactive, and apparent powers of arbitrary impedances. Some common methods potentially require a large number of steps; however, in some cases, it may be desirable to use an alternate approach. Specifically, the approach presented here is widely applicable, and often requires fewer (*or less tedious*) calculations.

Now, let's agree on the notation we're going to use. In these problems, we'll be making use of voltage and current RMS phasors, $\bar{V}$ and $\bar{I}$. The bar over the letters indicates that V and I are **phasored quantities**; i.e. they are expressed as complex numbers, typically in polar form. For example, the sinusoidal voltage, $v(t)=120\sqrt{2}\sin(120\pi t+60^{\circ})$, has an RMS voltage phasor of $\bar{V}=120\angle 60^{\circ}$. (*Note, the* $\sqrt{2}$ *went away when converting from peak to RMS*) Next, we need to review the complex conjugate operator, "*****". For example, the complex conjugate of $\bar{I}$ is written $\bar{I}^{*}$. Mathematically, if $\bar{I}=15\angle 30^{o}$, then $\bar{I}^{*}=15\angle -30^{o}$ (*just flip the sign on the angle*). Additionally, if the value is expressed in rectangular form, as is often the case for impedances, it's just as easy. Suppose that $Z=3+j4$, then $Z^{*}=3-j4$ (*just flip the sign of the imaginary part*). Now, let's move to the actual calculations. Suppose that you have the following circuit (*you'll need to convert the R's, L's and C's to Z's and sources to phasors first*):



Now the question is, "*What are the* ***real, reactive, and apparent powers*** *in the load?*" Answering this question is simple with the alternative approach.

The equation you'll be using is:

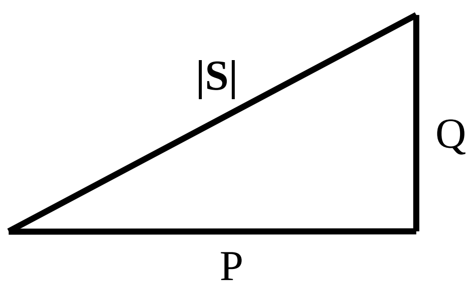
$$S = P + jQ = \bar{V}\ \bar{I}^* = \frac{\bar{V}\bar{V}^*}{Z^*} = \bar{I}\ \bar{I}^* Z \tag{1.1}$$

So, in the circuit above, since we know the voltage across the entire load, and we know the impedance of the load, we have that:

$$S = \frac{\bar{V}\bar{V}^*}{Z^*} = \frac{(120\angle 30^o)(120\angle 30^o)^*}{(400 - j300)^*} = \frac{(120\angle 30^o)(120^o\angle -30^o)}{(400 + j300)}$$

$$= \frac{120^2}{400 + j300} \approx (23.0 - j17.3)\ \text{ VA}$$

This means that the **real power if 23.0 W** and the **reactive power is 17.3 VAR** capacitive. Since the apparent power is the hypotenuse of the power triangle: (*remember that **S is a complex number**, so its **magnitude** is the length of the hypotenuse*)



Power Triangle:
S = P + jQ

If we convert S into polar form using the calculator, we'll get that:

$$S = (23.0 - j17.3) = 28.8\angle 36.9^o\ \text{ VA}$$

This means that the **apparent power is 28.8 VA**.

*Although the complex power S can be expressed as a polar number, it **IS NOT** a phasor. Remember, phasors come from sinusoids in the time domain, and that's not what's going on here.*

That's it. We're done.