EE 302 PS 08 - SOLUTIONS

Chapter 3

Questions: 22, 23, 27
Problems: 13
Critical Thinking: None

Additional Problems: 1

Question 22

Assuming 100% modulation ($m = 1$), 66.67% of the AM signal power is in the carrier; 16.67% is in one sideband, and 33.33% is in both sidebands.

Question 23

No. All information is contained in each sideband.

Question 27

Advantages of SSB transmission over conventional AM transmission

(1) SSB signals occupy half the spectrum space that conventional AM signals do.
(2) SSB transmission is more efficient (more power is devoted to the signal and less to the carrier than in conventional AM signals). This allows for smaller, lighter transmitters; reduced power consumption; and a stronger signal with better range.
(3) SSB signals are less susceptible to noise by virtue of their reduced bandwidth.
(4) SSB signals are less susceptible to selective fading than conventional AM signals.

Problem 13

From the relationship

$$I_T = I_c\sqrt{1 + \frac{m^2}{2}}$$

m can be determined to be

$$m = \sqrt{2\left[\left(\frac{I_T}{I_c}\right)^2 - 1\right]} = \sqrt{2\left[\left(\frac{2.7\text{ A}}{2.4\text{ A}}\right)^2 - 1\right]} = 0.7289$$

This is 72.89% modulation.

Additional Problem 1

1.a. $T = 1$ ms.

1.b. For a square wave in general, the Fourier expansion is:

$$f(t) = \frac{4V}{\pi}\left[\sin(2\pi f t) + \frac{1}{3}\sin(2\pi 3 f t) + \ldots + \frac{1}{n}\sin(2\pi n f t) + \ldots\right]$$

where $f = 1/T$ is the fundamental frequency and V is the amplitude of the square wave. In this problem, $f = 1000$ Hz and $V = 5$ V. Plugging these into our equation, and using only the first three odd harmonics, we get:

$$\begin{aligned} f(t) &= \frac{4(5\text{ V})}{\pi}\left[\sin(2\pi(1000\text{ Hz})t) + \frac{1}{3}\sin(2\pi(3000\text{ Hz})t) + \frac{1}{5}\sin(2\pi(5000\text{ Hz})t)\right] \\ &= 6.37\text{ V}\sin(2\pi(1000\text{ Hz})t) + 2.12\text{ V}\sin(2\pi(3000\text{ Hz})t) + 1.27\text{ V}\sin(2\pi(5000\text{ Hz})t) \end{aligned}$$

1.c. See Figure 1.

1.d. The carrier is a 50-kHz ($f_c = 50$ kHz) sine wave with amplitude $V_c = 10$ V. Thus, in the frequency domain, the AM signal includes the carrier sine wave with its original amplitude along with the modulating signal (message) which is reduced in amplitude by a factor of one half, translated up in frequency by $\Delta f = f_c$ and reflected about the carrier. See Figure 2 for the frequency-domain plot.

1.e. Given a sinusoid as the modulating signal (message), recall that power is:

$$P_T = P_c\left(1 + \frac{m^2}{2}\right) \quad (1)$$

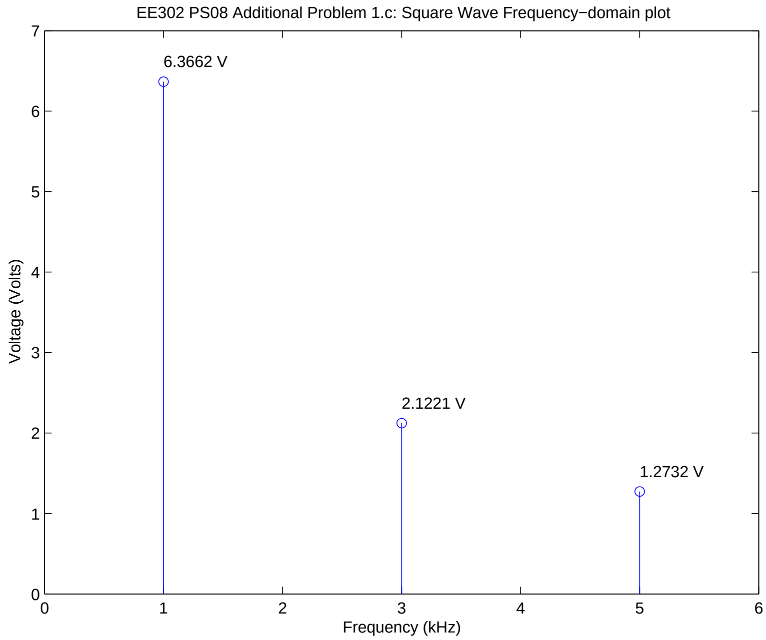

FIGURE 1. Baseband signal for additional problem 1.c.

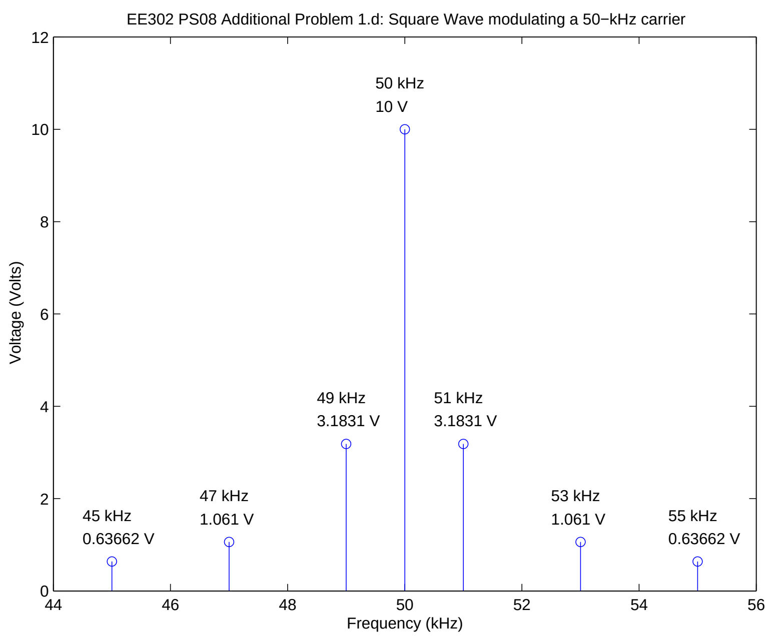

FIGURE 2. AM signal for additional problem 1.d.

Consider the truncated square wave, which is a weighted sum of three sinusoids. We can apply Equation (1) to three separate AM waves (one for each sinusoidal component of the square wave). The only considerations are that:

(1) All three AM waves share one carrier so that there is only one term (not three) to account for the carrier in our equation for P_T.
(2) Each of the three components of the truncated square wave has a different modulation index:

$$m_1 = \frac{6.37 \text{ V}}{10 \text{ V}} = 0.637$$

$$m_2 = \frac{2.12 \text{ V}}{10 \text{ V}} = 0.212$$

$$m_3 = \frac{1.27 \text{ V}}{10 \text{ V}} = 0.127$$

Thus, carrier power is:

$$P_c = \frac{V_{c,rms}^2}{R} = \frac{V_{c,peak}}{2R} = \frac{(10 \text{ V})^2}{2 \times 75 \ \Omega} = 0.6667 \text{ W}$$

and total power is:

$$\begin{aligned} P_T &= P_c \left(1 + \frac{m_1^2}{2} + \frac{m_2^2}{2} + \frac{m_3^2}{2}\right) \\ &= (0.6667 \text{ W}) \left(1 + \frac{0.637^2}{2} + \frac{0.212^2}{2} + \frac{0.127^2}{2}\right) \\ &= 0.8222 \text{ W} = 822.2 \text{ mW} \end{aligned}$$

We've already determined that carrier power is $P_c = 666.7$ mW. Thus, the combined power in the sidebands is $P_{\text{USB,LSB}} = P_T - P_c = 155.5$ mW. The power in the upper sideband is the same as that of the lower sideband: $P_{\text{USB}} = P_{\text{LSB}} = \frac{1}{2} P_{\text{USB,LSB}} = 77.7$ mW.

Now we present an alternate solution. Recall that for a sinusoidal voltage with RMS value V_{rms} over a resistance R, the power delivered to the resistance is

$$P = \frac{V_{rms}^2}{R} \qquad (2)$$

Equivalently, we can use a power equation that uses amplitude V_{pk} in lieu of V_{rms}. Remember that $V_{rms} = V_{pk}/\sqrt{2}$:

$$P = \frac{V_{rms}^2}{R} = \frac{(V_{pk}/\sqrt{2})^2}{R} = \frac{1}{2}\frac{V_{pk}^2}{R} \tag{3}$$

The amplitudes of the various sinusoids plotted in the frequency domain are given. We can either calculate the power of each one by first converting it to RMS and using Equation (2) or use the amplitudes directly with Equation (3). To get the power of the lower sideband, sum the powers of all sinusoids in the lower sideband. The same can be done for the upper sideband, and for the entire AM signal. The calculations are tabulated in Table 1.

Table 1. Additional Problem 1.e: Power in various components of the AM signal.

Lower Sideband		**Carrier**		**Upper Sideband**	
f (kHz)	$P = \frac{1}{2}\frac{V_{pk}}{R}$ (mW)	f (kHz)	$P = \frac{1}{2}\frac{V_{pk}}{R}$ (mW)	f (kHz)	$P = \frac{1}{2}\frac{V_{pk}}{R}$ (mW)
45	2.70	50	666.6	51	67.5
47	7.50			53	7.50
49	67.5			55	2.70
LSB Total	$P_{LSB} = 77.7$			USB Total	$P_{USB} = 77.7$
Total Power: $P_T = P_{LSB} + P_C + P_{USB} = 822.2$ mW					