

$$\begin{aligned}
 (1) \quad X &= \overline{(A B) + C} + B C \\
 &= (\overline{A B}) \bar{C} + B C \\
 &= (\bar{A} + \bar{B}) \bar{C} + B C \\
 &= \bar{A} \bar{C} + \bar{B} \bar{C} + B C \quad \rightarrow \text{K-MAP} \\
 &\quad \downarrow \\
 &\quad \text{Boolean} \\
 &= \bar{C} (\bar{A} + \bar{B} + B) \\
 &= \bar{C} (\bar{A} + (\bar{B} + B)) \\
 &= \bar{C} (\bar{A} + 1) \\
 &= \bar{C} (1)
 \end{aligned}$$

$$X = \bar{C}$$

	\bar{C}	C
$\bar{A} \bar{B}$	0	0
$\bar{A} B$	1	0
$A \bar{B}$	1	0
$A B$	1	0

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$\begin{aligned}
 (2) \quad X &= \overline{A B + (\bar{A} + C)} \\
 &= (\overline{A B}) \overline{(\bar{A} + C)} \\
 &= (\bar{A} + \bar{B}) (\bar{\bar{A}} \bar{C}) \\
 &= (\bar{A} + \bar{B}) (A \bar{C}) \\
 &= (\bar{A} A \bar{C}) + A \bar{B} \bar{C} \\
 &= (0 \bar{C}) + A \bar{B} \bar{C} \\
 &= 0 + A \bar{B} \bar{C} \\
 &= A \bar{B} \bar{C}
 \end{aligned}$$

$$X = A \bar{B} \bar{C}$$

$$\begin{aligned}
 (3) \quad X &= \overline{A + \bar{B}} \\
 &= \bar{A} B
 \end{aligned}$$

$$X = \bar{A} B$$

$$\begin{aligned} \textcircled{4.} \quad X &= \overline{\overline{A} \overline{B}} \\ &= \overline{\overline{A}} + \overline{\overline{B}} \end{aligned}$$

$$\boxed{X = A + B}$$

$$\begin{aligned} \textcircled{5.} \quad X &= \overline{\overline{\overline{A} \overline{B} C} D} \\ &= \overline{\overline{\overline{A} \overline{B} C}} + \overline{D} \\ &= \overline{\overline{A} \overline{B} C} + \overline{D} \\ &= (\overline{A} + \overline{B}) C + \overline{D} \end{aligned}$$

$$\boxed{X = \overline{A} C + \overline{B} C + \overline{D}}$$

$$\begin{aligned} \textcircled{6.} \quad X &= \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} C + A B \overline{C} \\ &= \overline{A} \overline{B} C + B C (\overline{A} + A) + A \overline{B} (\overline{C} + C) \\ &= \overline{A} \overline{B} C + B C (1) + A \overline{B} (1) \\ &= \overline{A} \overline{B} C + B C + A \overline{B} \\ &= \overline{A} \overline{B} C + \overline{A} \overline{B} + B C \\ &= \overline{B} (\overline{A} \overline{C} + A) + B C \\ &\quad \downarrow \\ &\quad A + \overline{A} \overline{C} \\ &\quad \downarrow \\ &\quad A + \overline{A} B = A + B \\ \text{SO, } A + \overline{A} \overline{C} &= A + \overline{C} \\ &= \overline{B} (A + \overline{C}) + B C \end{aligned}$$

$$\boxed{X = A \overline{B} + \overline{B} \overline{C} + B C}$$

$$\begin{aligned}
 \textcircled{7.} \quad y &= \overline{C+D} + \overline{A}C\overline{D} + A\overline{D}\overline{E} + \overline{A}\overline{B}C\overline{D} + A\overline{C}\overline{D} \\
 &= \overline{C}\overline{D} + (\overline{A}\overline{A})C\overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= \overline{C}\overline{D} + (1)C\overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= \overline{C}\overline{D} + C\overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= (\overline{C}+C)\overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= (1)\overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= \overline{D} + A\overline{B}\overline{C} + \overline{A}\overline{B}C\overline{D} \\
 &= \overline{D} + \overline{A}\overline{B}C\overline{D} + A\overline{B}\overline{C} \\
 &\quad \downarrow \quad \downarrow \quad \downarrow \\
 &\overline{D} + \overline{D}\overline{A}\overline{B}C \\
 &\quad \downarrow \quad \downarrow \quad \downarrow \\
 &\overline{A} + \overline{A}\overline{B} = \overline{A}\overline{B}
 \end{aligned}$$

$$\text{SO, } \overline{D} + \overline{D}\overline{A}\overline{B}C = \overline{D} + \overline{A}\overline{B}C$$

$$= \overline{D} + \overline{A}\overline{B}C + A\overline{B}\overline{C} \quad \text{*K-map produces the same expression!}$$

$$\begin{aligned}
 \textcircled{8.} \quad z &= C(\overline{A}B + \overline{A}C) + (D + \overline{C})\overline{A} \\
 &= \overline{A}BC + \overline{A}CC + \overline{A}D + \overline{A}\overline{C} \\
 &= \overline{A}BC + \overline{A}C + \overline{A}D + \overline{A}\overline{C} \\
 &= \overline{A}(BC + C + D + \overline{C}) \\
 &= \overline{A}(BC + D + (C + \overline{C})) \\
 &= \overline{A}(BC + D + 1) \\
 &= \overline{A}(1) \\
 &= \overline{A}
 \end{aligned}$$

$$\boxed{z = \overline{A}}$$