

Lab #6 Magnetic Equivalent Circuits

Background: In this lab exercise, we will be building magnetic circuits and measuring current, flux, and induced voltage. In order to compare the measured values with our theory, let's analyze the circuit shown in Figure 1a. Note this circuit contains two coils, each wrapped around a laminated ferromagnetic core. If we consider applying a voltage source to coil #1, current will flow and we will establish lines of flux as shown in Figure 1b. Some will couple to the second coil (ϕ_{mag}) and some will leak out into the air (ϕ_{leak}). The sum of those two components must be what couples through coil #1 (ϕ_1).

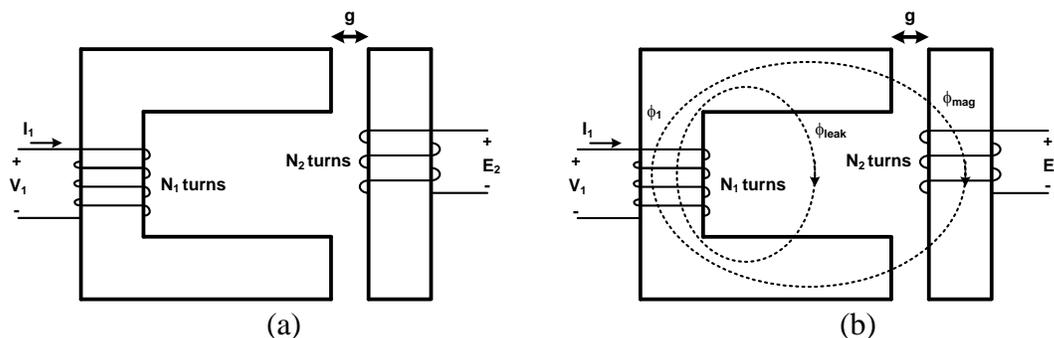


Figure 1: The target (a) two-coil circuit and (b) proposed flux paths

This assumption about the relevant flux paths enables us to imagine the magnetic equivalent circuit shown in Figure 2a. Since the steel segments (R_{S1} and R_{S2}) are in series as are the two air gaps (R_g), we can combine them to arrive at the reduced circuit shown in Figure 2b where

$$R_{mag} = R_{S1} + R_{S2} + 2R_g \quad (1.1)$$

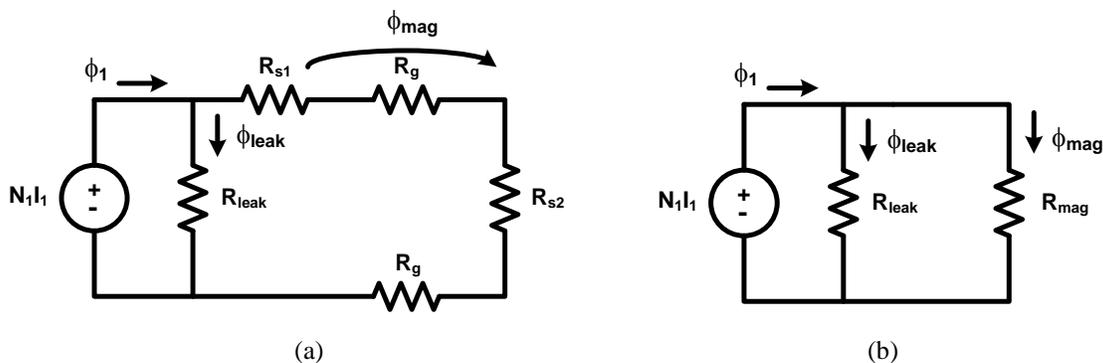


Figure 2: (a) Magnetic Equivalent Circuit and (b) Reduced Version

The flux which flows from coil 1, through the air gap, and links coil2 is then found to be

$$\phi_{mag} = \frac{N_1 I_1}{R_{mag}} \quad (1.2)$$

The flux linkage for coil 1 is then evaluated as

$$\lambda_1 = N_1 \phi_1 = \left(\frac{N_1^2}{R_{leak}} + \frac{N_1^2}{R_{mag}} \right) I_1 = (L_{leak} + L_{mag}) I_1 = L_1 I_1 \quad (1.3)$$

The reluctance of the air gap is given by

$$R_g = \frac{g}{A \mu_o} \quad (1.4)$$

where g is the length of the air gap, A is the gap cross-sectional area, and μ_o is the permeability of free space, $4\pi \times 10^{-7} H/m$. Thus as the gap is increased, $R_g \uparrow$, so $R_{mag} \uparrow$ and $L_{mag} \downarrow$. To understand how this will potentially impact the current, we need to consider the electrical characteristics of coil 1. If the coil resistance is negligible, electrically the circuit would be analyzed using the following circuit in the frequency domain

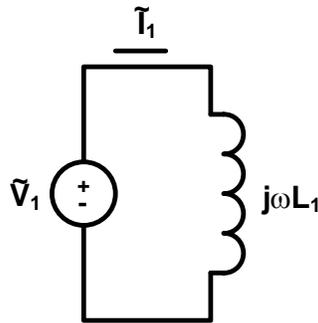


Figure 3: Electrical Equivalent Circuit (Ignoring Coil Resistance)

Ohm's Law then predicts that $\tilde{I}_1 = \frac{\tilde{V}_1}{j\omega L_1}$ or in terms of magnitudes $|\tilde{I}_1| = \frac{|\tilde{V}_1|}{2\pi f L_1}$. Thus if $R_g \uparrow$, it follows that $L_1 \downarrow$, which implies that for fixed $|\tilde{V}_1|$ it must follow that $|\tilde{I}_1| \uparrow$.

Q: What about the relationship for the induced voltage in the second coil?

Since $\phi_{mag} = \frac{N_1 I_1}{R_{mag}}$, if I_1 is sinusoidal, then ϕ_{mag} must also be sinusoidal. Let's assume that $\phi_{mag} = \phi_{mag, pk} \sin \omega t$ where $\phi_{mag, pk}$ is the peak value of the flux and ω the angular frequency of the source. Then Faraday's Law predicts that the induced voltage in coil #2 will be

$$E_2 = N_2 \frac{d\phi_{mag}}{dt} = N_2 \omega \phi_{mag, pk} \cos \omega t \quad (1.5)$$

The peak value of this induced voltage must then be

$$E_{2, pk} = N_2 (2\pi f) \phi_{mag, pk} \quad (1.6)$$

which has an RMS value of

$$E_{2, rms} = \frac{E_{2, pk}}{\sqrt{2}} = \left(\frac{2\pi}{\sqrt{2}} \right) N_2 f \phi_{mag, pk} = 4.44 N_2 f \phi_{mag, pk} \quad (1.7)$$

In a similar manner, the coil #1 voltage can be related to the flux through it by

$$E_{1, rms} = 4.44 N_1 f \phi_{1, pk} \quad (1.8)$$

This final equation is important since it says that if the applied voltage is fixed (and the number of turns and applied frequency is fixed), then the flux flowing through coil #1 must be fixed. If coil #1 has 275 turns, we apply $30V_{rms}$, and the frequency is 60Hz, establish the anticipated flux through that coil

$$\phi_{1, pk} = \underline{\hspace{2cm}}$$

OK, part of our experiment today will entail collecting data so that we can exercise both our magnetic equivalent circuit model and our proposed electrical circuit model. Let's get to it!

Step1: Using the Digital Multimeter (DMM) at your workstation measure the resistance of the 275-turn coil between terminals 1 and 2 (this should be a small value):

$$R_{12} = \underline{\hspace{2cm}}$$

Step 2: Build the circuit in Figure 4

- Place one of the 133mm laminated bars *with the hook* through one coil module (labeled A)
 - Place the flux meter “probe end” around this same 133mm laminated bar; set the meter to the $1000\mu Wb$ scale
 - Arrange the four bars together on the base-plate initially with *one spacer*, providing an air gap of 0.75mm
 - Ensure that the power supply variable voltage control knob is zero and that the supply is OFF
 - Attach the variable voltage power supply terminals to the power analyzer plugs (PM300: yellow V1 & A1 and black V1)
 - Make the connections to Coil A (PM300: black A1 to terminal 2 and power supply neutral N to coil terminal 1)
 - Couple the flux meter output to the scope. Apply averaging to the signal to facilitate measuring the peak value.
- **HAVE THE INSTRUCTOR VERIFY YOUR CONNECTIONS:**

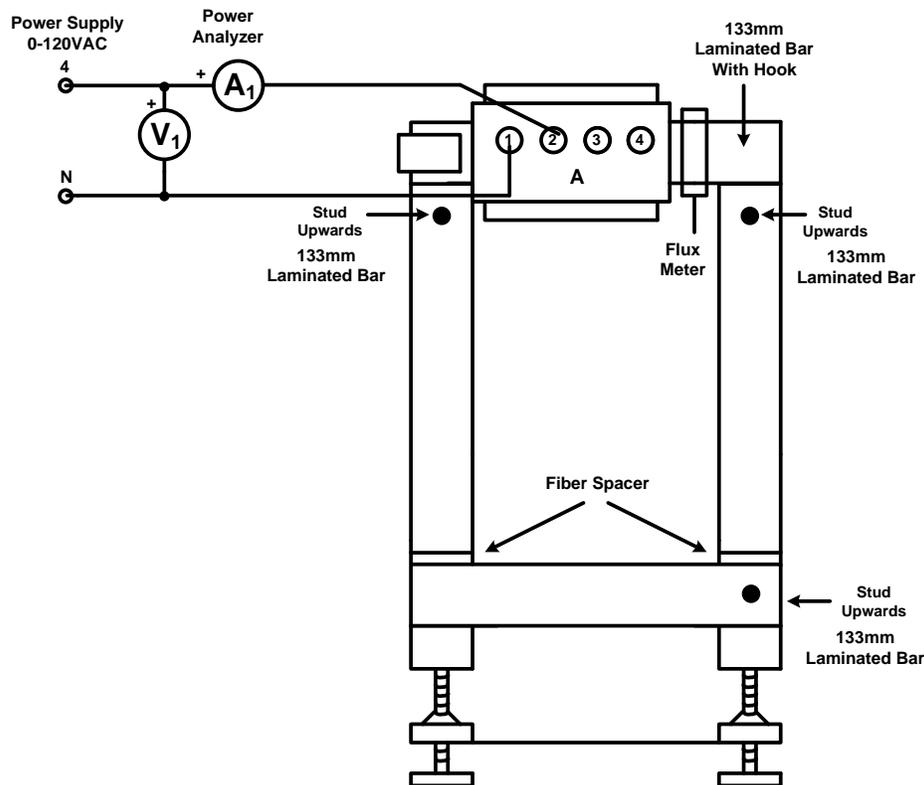


Figure 4: Magnetic Circuit with Air Gap

Step 3: Energize the power supply. Adjust the power supply voltage so that the power analyzer voltmeter (V_1) reads as close to **30V** as possible. Record the data in Table 1

Table 1

g (mm)	V_1 (V)	I_1 (A)	W_1 (W)	ϕ_1 (μWb)	ϕ_{mag} (μWb)
0.75 (1 spacer)					

Step 4: De-energize the power supply. **DO NOT TURN THE VOLTAGE TO ZERO.** Move the flux meter to the center of the 133mm bar separated by the spacers, as shown in Figure 5. Re-energize the power supply and record the value of magnetizing flux in Table 1 (note these values are peak values)

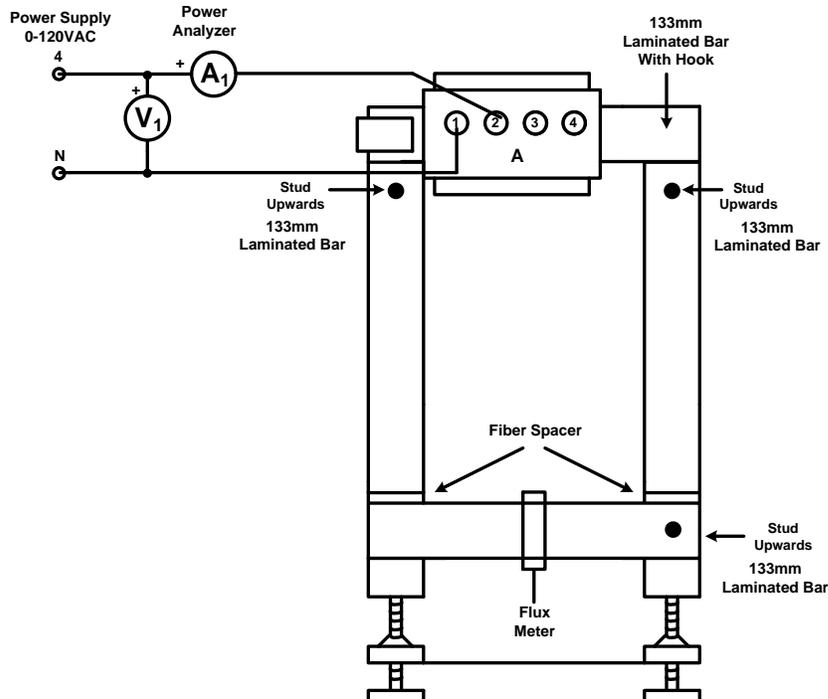


Figure 5: Magnetic Circuit with Flux Meter Measuring Magnetizing Flux

Step 5: De-energize the power supply and make the following calculations:

- Use Figure 2b and KCL to estimate the leakage flux
- Establish the mmf source value in Figure 2b. The coil turns are listed on the coil and remember to use the PEAK value of current, not the rms value that the meter gives you!

- Use Magnetic's Ohm's Law to estimate the value of leakage reluctance (this will be a large value)
- Use Magnetic's Ohm's Law to estimate the value of magnetizing reluctance (this will also be a large value)
- Establish the reluctance introduced by the TWO air gaps. Assume that the cross-sectional area of the air gap will be the same as that of the steel bars. The cross-sectional area of a laminated bar can be found in Figure 6.
- Using equation (1.1) and the value of twice the air gap reluctance, determine the reluctance of the steel (this should be smaller than the air gap reluctance)
- Using the dimensions of the part of the circuit consisting of steel shown in Figure 6, establish an expression for the reluctance of the steel (this should be a function of μ_i , the relative permeability of the iron material). Set this expression equal to your answer above to establish a value for μ_i

HAVE THE INSTRUCTOR CHECK YOUR CALCULATIONS: _____

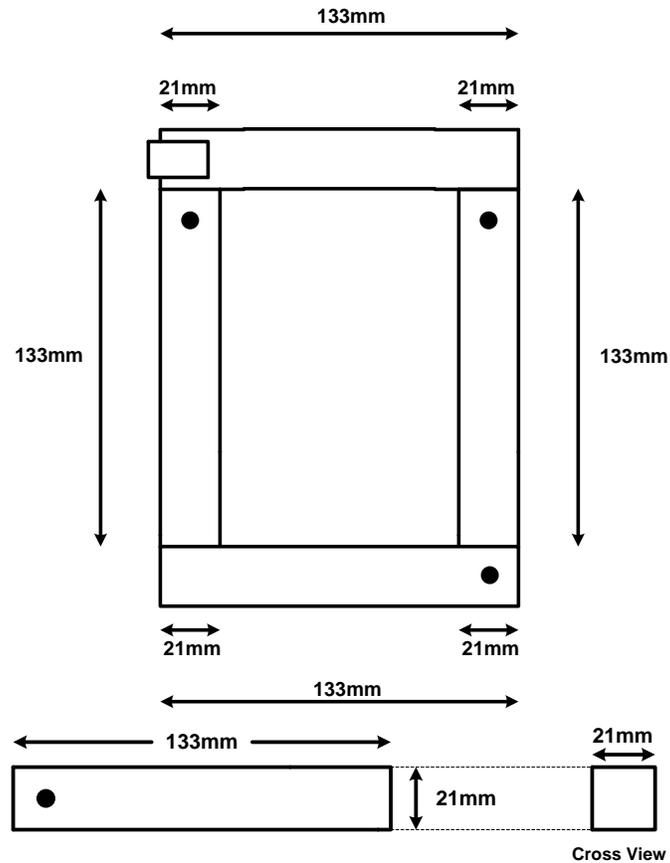


Figure 6: Dimensions of Steel Part of Circuit

- Evaluate the coil inductance using equation (1.3)
- Use the circuit of Figure 3 to estimate the rms current (use the measured rms voltage and remember that the frequency of the power supply is 60Hz). How does this value compare to your measured value of Table 1?

- Using the resistance measured in Step 1, compute the winding copper losses.
- Estimate the core losses based on your measured input power and the copper losses

Step 6: Insert a second spacer in the air gap. Energize the power supply and record the data in Table 2. With the power supply de-energized, relocate the flux meter as required to secure the two flux values.

Table 2

g (mm)	V_1 (V)	I_1 (A)	W_1 (W)	ϕ_1 (μWb)	ϕ_{mag} (μWb)
1.50 (2 spacers)					

Step 7: De-energize the power supply and make the following calculations

- Establish the new magnetizing reluctance based on your previously determined reluctance of the steel and the new air gap reluctance
- Based on the new measured current and the known turns, establish the new mmf
- Calculate the anticipated flux through the air gap using the above two values. Compare this with your measured value.

- Calculate the leakage flux by using the previously determined leakage reluctance and the calculated mmf value above.
- Evaluate the leakage flux using your Table 2 data and KCL. If this value is different than the one above, hypothesize about whether the leakage reluctance has increased or decreased from our previous test.

Step 8: Let's next consider magnetic induction. Add the second coil to the circuit as shown in Figure 7 by placing coil B about the steel bar separated by the spacers. Using a handheld Fluke meter, measure the induced voltage for the three conditions described in Table 3. For the rightmost column, compute the voltage based on equation (1.7) and your previously measured values of flux.

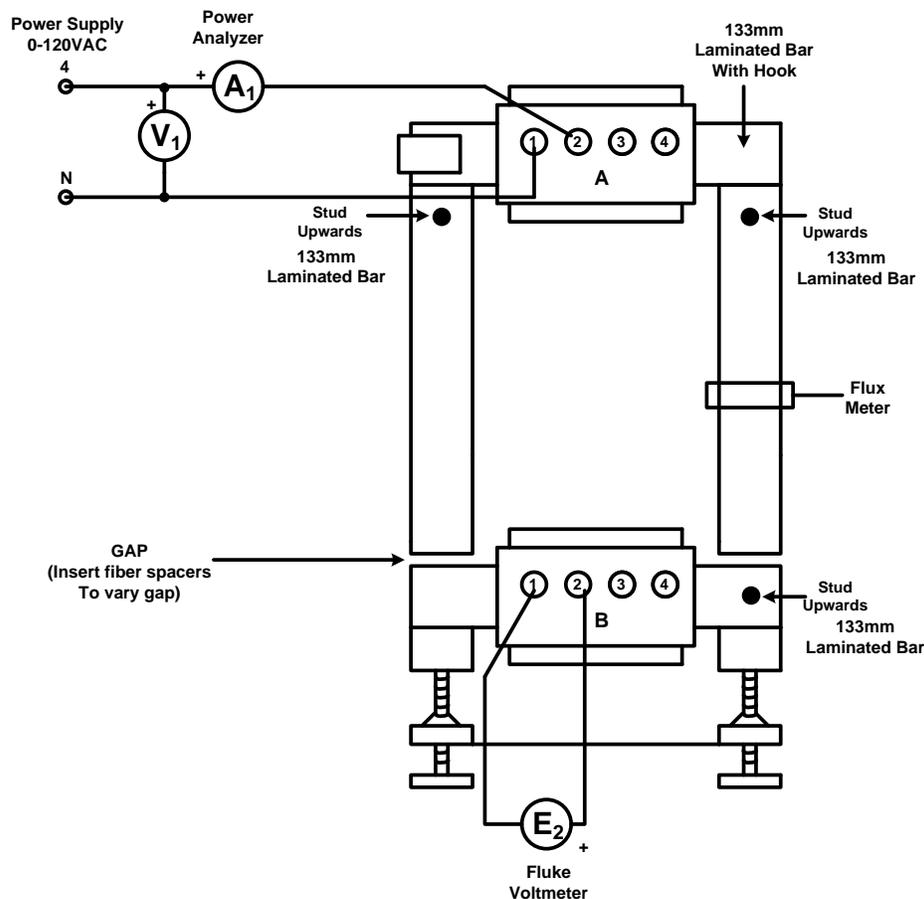


Figure 7: Circuit with Second Coil

Table 3.

g (mm)	E_2 Measured (V)	E_2 Calculated (V)
0		
0.75		
1.5		

Step 9: Disconnect the circuit and return the parts to where you originally found them.