Objectives of Section 2

- Describe the most common means of producing magnetic fields in electric machines
- Apply the right-hand rule to determine the direction of flux flow
- Relate the flux and flux density for uniform flux flow
- Apply Ampere's Law for a magnetic circuit with an assumed flux path
- Relate flux density, field intensity and permeability
- Define a magnetic equivalent circuit and relate the magnetic quantities to their electrical analogs
- Relate flux, reluctance and mmf for a magnetic equivalent circuit
- Explain why ferromagnetic materials are used to build electric machines
- Make calculations for magnetic circuits that do not contain an air gap
- Explain why we analyze circuits with air gaps
- Make calculations for magnetic circuits with an air gap

2.0 MAGNETISM AND MAGNETIC EQUIVALENT CIRCUITS

There is no way to explore the operation of electric machines without discussing magnetic fields. Our simplest experiences with magnetic fields are with magnets attracting to some metal surfaces and with two magnets either attracting or repelling one another. We will build from there. There are three basic ways to create a magnetic field:

1. A current-carrying conductor,
2. A permanent magnet,
3. A time-varying electric field.

Fortunately, the third does not directly pertain to our immediate discussion. For the permanent magnet illustrated in Figure 2.1a, closed, continuous lines of **magnetic flux** leave the North pole of the magnet and re-enter the South pole. We will soon see that these lines of flux are analogous to current in our electrical circuits. Where the lines of flux are dense is where the magnetic field is the strongest; where the lines are sparse is where the magnetic field is weakest. The units of flux are the Weber (Wb), and we will represent it with the symbol ϕ (phi). The lines of flux for the current-carrying conductor in Figure 2.1b are assigned by using Fleming's **Right Hand Rule:** grasp the conductor with the right hand with your thumb pointing in the direction of the current, your fingers will curl in the direction of the magnetic field. By winding the conductor in a coil, as depicted in Figure 2.1c, we notice that the resultant lines of flux resemble those of the permanent magnet—a North pole at the top and a South pole at the bottom. Such a coil is termed an **electromagnet** and is a key element in building an electric machine.

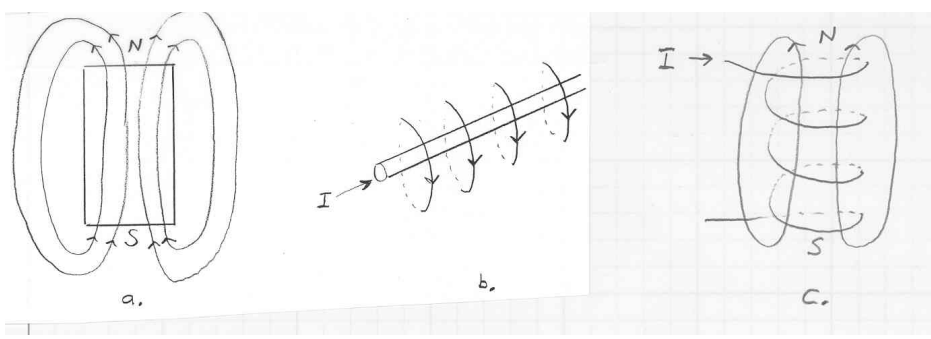

Figure 2.1: Lines of Magnetic Flux for (a) A Permanent Magnet; (b) Current-Carrying Conductor; (c) Electromagnet

2.1 Magnetic Circuit Analysis without an Air Gap

We ultimately want to quantify the amount of torque that a motor develops or the level of voltage that a generator produces. With these goals in mind, we need to build a library of terminology and relationships that will enable us to characterize magnetic systems. In this context, we will simplify a three-dimensional non-linear vector problem into a one-dimensional linear circuit-equivalent problem (you are free to cheer). The magnetic field

is generally quantified by the **flux density** ($\vec{B}$) or simply **B-field**. We can calculate the flux as being the surface integral of the flux density (think of this as integrating a rate of flow through a surface, equation below). For uniform flux flow (the B-field is uniformly distributed through the assumed surface), the nasty dot product and vector integration (below) becomes simply the product of the magnitude of the B-field and the cross-sectional area (A).

$$\phi = \int \vec{B} \cdot d\vec{S} \approx BA$$

Achieving a large B-field is key to our efforts to realize practical forces in a machine. The B-field is actually a vector quantity, having both direction and magnitude, and can be expressed in units of Tesla (T), Gauss (G), or less frequently as Wb/m^2 (we will use T). A typical magnitude of B-field in a conventional electric machine is 1-2T. The Navy is investigating super-conducting electric machines that may operate with B-fields between 4 and 6T (the goal here being lighter, more compact, high-power machinery).

Just as voltage is required to "push" current through an electric circuit, the **field intensity** is the magnetic force that pushes the flux density through a magnetic circuit. The field intensity, also referred to as the **H-field**, has units of A/m and is governed by **Ampere's Law.**

$$\oint \vec{H} \cdot d\vec{L} = i_{enclosed}$$

This equation looks evil but it's not. The circle in the integration symbol tells us that we must consider a closed path. Consider the magnetic circuit depicted in Figure 2.2a. It consists of a rectangular core and a coil of N turns wrapped about one of the legs (yes, this will soon become an electric machine, trust me). If the core is made of **ferromagnetic** material (such as iron, nickel, cobalt or some combination), it is reasonable to assume that most of the flux stays in the core and little leaks out into the air. Further, if we assume a mean path through the material adequately describes the flux (Figure 2.2b) and that the field intensity only acts along that path (no bends around the turns), then Ampere's Law simplifies: the dot product becomes multiplication and the integration becomes simply a summation over the various parts of the assumed path

$$\sum_{x_paths} H_{iron,x} L_x = H_{iron} L_1 + H_{iron} L_2 + H_{iron} L_3 + H_{iron} L_4 = NI$$

As we march along the mean path through the core, we cut through each of the N turns of the coil and thus, the enclosed current must be NI, where I is the coil current. The product of turns and current is typically called the magneto-motive-force or simply, **mmf**. Given the core dimensions listed in Figure 2.2b, Ampere's Law yields

$$H_{iron}\left(L_1 + L_2 + L_3 + L_4\right) = H_{iron} L_{mean_path} = NI = mmf$$

This result tells us that for our assumed path the field intensity depends directly on the current in the winding, but what about the flux and B-field? If the core material is

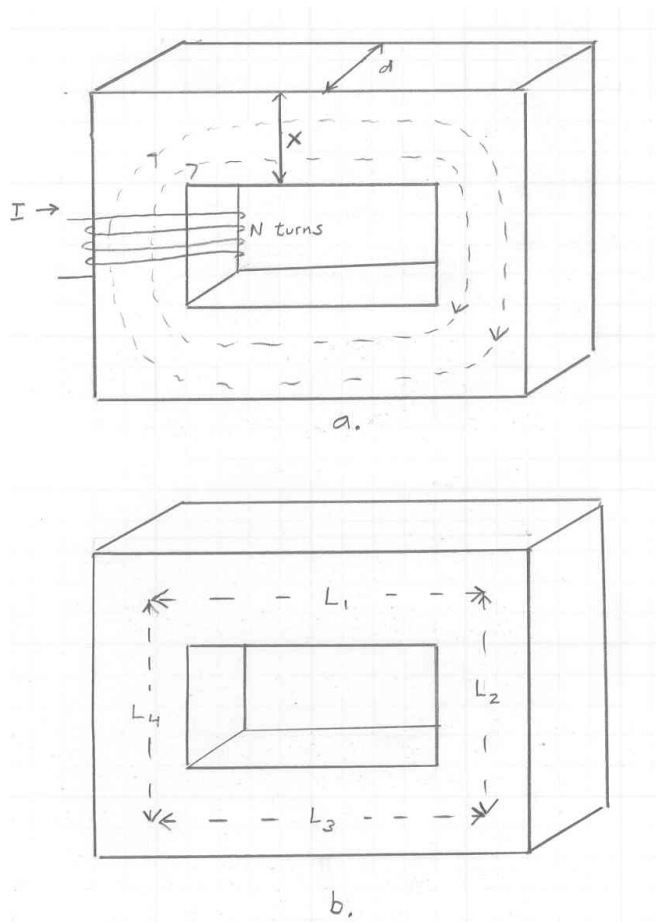

Figure 2.2: Simple Magnetic Circuit Illustrating (a) Lines of Flux for Positive Current; (b) Mean Flux Path

magnetically linear and isotropic (same properties in all directions, which iron is), then we can relate B to H via the **permeability** which we can divide into a constant term, the permeability of free space ($\mu_o = 4\pi x10^{-7} H/m$), and the relative permeability ($\mu_{r,iron}$ which is dimensionless).

$$B_{iron} = \mu_{r,iron}\mu_o H_{iron}$$

The relative permeability of air and other materials such as wood is very close to one; the relative permeability of ferromagnetic material is on the order of thousands or tens of thousands – the plot thickens! If we substitute the permeability relationship into our Ampere's Law result, we get

$$\frac{B_{iron}}{\mu_{r,iron}\mu_o} L_{mean_path} = NI$$

Then replace the B-field by its flux relationship, Ampere's Law yields

$$\frac{\phi_{iron}}{\mu_{r,iron}\mu_o A} L_{mean_path} = NI$$

Note, the cross-sectional area (A) in Figure 2.2 is the product xd . If we group the terms that depend on the core geometry and material properties, we get

$$\frac{L_{mean_path}}{\mu_{r,iron}\mu_o A}\phi_{iron} = \Re_{iron}\phi_{iron} = NI$$

The **reluctance** ($\Re_{iron} = \frac{L_{mean_path}}{\mu_{r,iron}\mu_o A}$) represents the material path's opposition to flux flow. It has units of inverse Henries. Thus, our result becomes

$$\Re\ \phi = mmf$$

This is *Magnetic's version of Ohm's law*: Reluctance times flux equals mmf. The mmf is analogous to voltage. It pushes the flux through the path, where flux is similar to current and must obey a comparable Kirchhoff's Current Law. The reluctance represents opposition to flux flow and is therefore analogous to resistance. Essentially, we applied the concept of adding reluctances in series in the development above. Iron (ferromagnetic) material is used in machines, because it has a high permeability ($\mu_{r,iron} >> 1$) and therefore a very low reluctance, meaning that it facilitates channeling the flux, but channeling it where? Well, now we have to introduce an air gap.

The actual relationship between the flux density (B) and the field intensity (H) is a bit more involved than that conveyed by $B_{iron} = \mu_{r,iron}\mu_o H_{iron}$. Consider the non-linear characteristic depicted in Figure 2.3. Note that the actual characteristic does "**saturate**." A sample of ferromagnetic material contains regions called **domains** (think of these as areas that possess a dipole moment (North and South poles), randomly distributed through the material). As a progressively larger magnetic field is applied, more and more of these domains align with this external field. Magnetic saturation occurs when no more domains remain to align. Another important characteristic shown in Figure 2.3 is the fact that ferromagnetic material exhibits **hysteresis** (this is the loop part of the characteristic). This hysteresis becomes important when the field intensity varies with time (say, when a sinusoidal current is applied to the coil so that the H-field is sinusoidal as well). Hysteresis occurs because the magnetic domains exhibit a resistance to shift their orientation. The area of the hysteresis loop is proportional to the amount of energy required to shift the orientation of the domains. Thus, we seek ferromagnetic materials with narrow hysteresis loops to minimize these losses. Further, we want to operate below magnetic saturation (in the 2 to 2.2T range) to avoid distortion and additional losses. For

convenience, we will assume that our ferromagnetic material is operated in the linear or non-saturated part of the B-H characteristic.

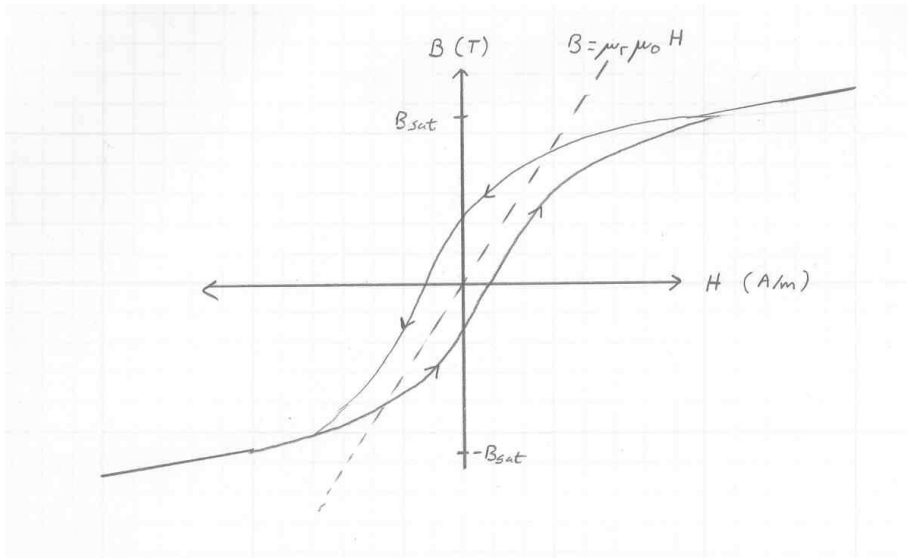

Figure 2.3: Representative B versus H Characteristic for Ferromagnetic Material

Example 2.1: Given a ferromagnetic core similar to the one shown in Figure 2.2a with dimensions $L_1 = 0.2m$, $L_2 = 0.1m$, $d = 0.02m$, and $x = 0.02m$. The relative permeability of the core material is 5000. The coil has 50 turns and carries a current of 1A. Determine the flux and flux density in the core material.

Solution. The reluctance is computed as follows

$$\Re = \frac{L_{mean_path}}{\mu_{r,iron}\mu_o A} = \frac{2(0.2m + 0.1m)}{5000(4\pi \times 10^{-7} H/m)(0.02m \times 0.02m)} = 238.7 \times 10^3 H^{-1}$$

The mmf is found from

$$mmf = NI = (50turns)(1A) = 50A - t$$

Magnetic's Ohm's Law then tells us that

$$\phi = \frac{mmf}{\Re} = \frac{50A - t}{238.7 \times 10^3 H^{-1}} = 209.5\mu Wb$$

The flux density is then found by assuming uniform flux flow:

$$B = \frac{\phi}{A} = \frac{209.5\mu Wb}{(.02m \times .02m)} = 0.523T$$

2.2 Magnetic Circuit Analysis with an Air Gap

Consider next that we introduce an air gap into the magnetic circuit as shown in Figure 2.4. The analysis is not much different except that we must now account for the fact that the flux must traverse material that is not ferromagnetic and that does not have its high relative permeability. Let's assume that our principle flux path is through the ferromagnetic core and air gap (this ignores leakage flux paths, but we will assume that these reluctance paths are much larger, approaching open circuits). We can apply Ampere's Law,

$$\sum_{paths} H_{path} L_{path} = H_{iron} L_{iron} + H_{gap} L_{gap} = NI$$

Substituting the permeability relationships,

$$\frac{B_{iron}}{\mu_{r,iron}\mu_o} L_{iron} + \frac{B_{gap}}{\mu_o} L_{gap} = NI$$

where we have used the fact that the air gap has a relative permeability of one. Next, relating flux (ϕ) to flux density (B),

$$\frac{\phi_{iron}}{\mu_{r,iron}\mu_o A_{iron}} L_{iron} + \frac{\phi_{gap}}{\mu_o A_{gap}} L_{gap} = NI$$

noting that the flux going into the air gap must be the flux coming back out (by our assumed path) and assuming that the cross-sectional area in the two media are the same (the flux does not bow out when crossing the air gap)

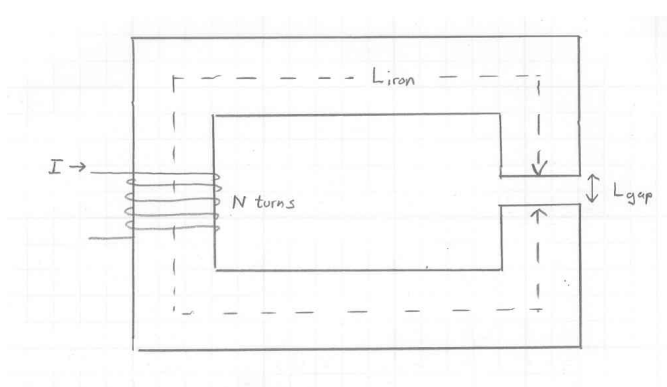

Figure 2.4: Magnetic Circuit with an Air Gap

$$\left(\frac{L_{iron}}{\mu_{r,iron}\mu_o A}\right)\phi + \left(\frac{L_{gap}}{\mu_o A}\right)\phi = NI$$

or by replacing the terms in parentheses by equivalent reluctances, we get

$$\left(\Re_{iron} + \Re_{gap}\right)\phi = NI$$

This equation shows us that we could have viewed this magnetic system as simply an mmf source driving flux through two series reluctances as shown in Figure 2.5. An electric machine is designed so that the reluctance of the air gap is much larger that the reluctance of the iron, so that most of the "source" mmf will drop across the air gap.

$$\phi \approx \frac{NI}{\Re_{gap}} = \frac{NI\mu_o A}{L_{gap}}$$

If you think about it (which I encourage), since $\phi = BA$, the above relationship tells us that

$$B = \frac{\phi}{A} \approx \frac{NI\mu_o}{L_{gap}}$$

If we know that we want this value between 1 and 2T, suddenly we have some guidance in how to design our machine; however, at this point, we have three parameters that we can control (turns N, current I, and air gap L_{gap}). It does at least start to point us to the fact that the dimensions of the machine and the material that it is constructed from are purposefully selected. OK, now what? We can place a ferromagnetic structure into our air gap, but how do we get it to move? Well, we must introduce a few more new concepts!

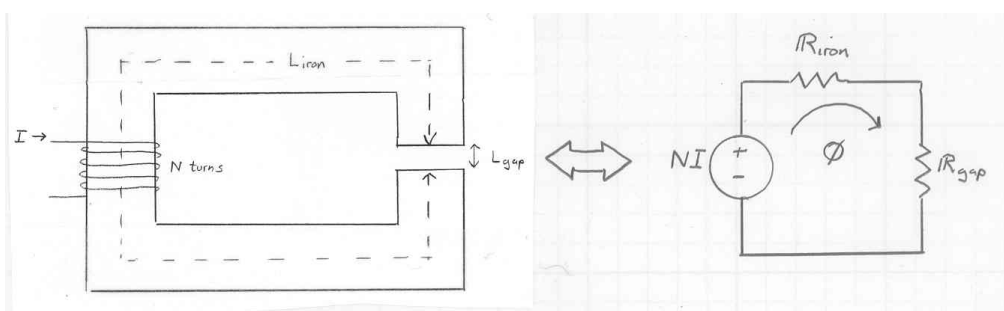

Figure 2.5: Circuit Equivalent for the Magnetic System with an Air Gap

Example 2.2: Suppose that the ferromagnetic core in the previous example has a 0.5cm air gap cut into it. Determine the reluctance of the air gap and the new number of turns that will ensure that we can achieve 1.25T in the gap with 10A of current.

Soultion. The air-gap reluctance is found via

$$\Re_{gap} = \frac{L_{gap}}{\mu_o A} = \frac{0.005m}{\left(4\pi \times 10^{-7} H/m\right)(0.02m \times 0.02m)} = 9.95 \times 10^{6} H^{-1}$$

Note, this value is nearly 42 times larger than the reluctance of the iron (which we will assume has not changed much by shortening the mean path by 0.5cm). Thus, the purpose of the iron is to channel the flux to the air gap so that most of the mmf is dropped here. To achieve the desired air-gap flux density, we consider

$$B = 1.25T \approx \frac{mmf / \Re_{gap}}{A} = \frac{NI\mu_o}{L_{gap}} = \frac{N(10A)\left(4\pi \times 10^{-7} H/m\right)}{0.005m}$$

The solution gives

$$N = 497.3 turns \approx 498 turns$$

Clearly, to create the larger value of flux (and flux density) flowing through a much larger reluctance path, we need to substantially increase our mmf source. In this case, we achieve this by both increasing the current and the turns (in each case, by a factor of 10).

Example 2.3: Consider the axial view (straight on) of a rotating machine presented in Figure 2.6a. Let's assume that the diameter of the rotor is 5cm, the axial length of the rotor is 10cm, and that the air gaps are 1mm. Further, we will assume that the permeability of the stator, rotor and pole iron is infinite so the detailed reluctance path of Figure 2.6b reduces to that of Figure 2.6c. Consider that we wrap a winding about the stator poles and choose to impress 0.5A in that winding (composed of copper wire with a resistivity of $1.724 \times 10^{-8} \Omega - m$ and a cross-sectional area of $0.2047 \times 10^{-6} m^2$, an area representative of AWG #24 wire).

a. How many turns must we wrap to achieve an air-gap flux density of 1T
b. Approximate the losses in this coil.

Solution. a. We attack this problem with the magnetic equivalent circuit shown in Figure 2.6c where

$$\phi_{gap} = \frac{mmf}{2\Re_{gap}} = \frac{NI}{2\frac{L_{gap}}{\mu_o A_{gap}}} = \frac{NI\mu_o A_{gap}}{2L_{gap}}$$

We can then calculate the B-field in the air gap as

$$B_{gap} = \frac{\phi_{gap}}{A_{gap}} = \frac{NI\mu_o}{2L_{gap}}$$

Solving for the turns gives

$$N = \frac{B_{gap} 2L_{gap}}{I\mu_o} = \frac{(1T)(0.002m)}{(0.5A)(4\pi \times 10^{-7} H/m)} = 3183 turns$$

This is a lot of turns! How can we modify the design to require fewer turns?

b. To get the winding losses, we need to figure out how much conductor we need to create the 3183 turns (no half turns allowed!). Consider the distance around one pole: across the front part of the pole is approximately one rotor diameter or 5cm; the axial distance (into the sheet) is 10cm. Thus, the length of one turn is

$$L_{turn} = 0.05m + 0.1m + 0.05m + 0.1m = 0.3m$$

The total length of wire is then the number of turns times the length/turn or

$$L_{wire} = NL_{turn} = 3183 turns(0.3m/turn) = 954.9m$$

To find the resistance of the wire, we apply our resistance formula

$$R_{wire} = \frac{\rho_{wire} L_{wire}}{A_{wire}} = \frac{(1.724 \times 10^{-8} \Omega - m)(954.9m)}{0.823 \times 10^{-6} m^2} = 80.42\Omega$$

Finally, to establish the losses in the coil we evaluate

$$P = I^2 R_{wire} = (0.5A)^2 (80.42\Omega) = 20.1W$$

And we would require a voltage of $V = IR_{wire} = (0.5A)(80.42\Omega) = 40.21V$ to achieve the required current. We start to see a bit of the iterative design process. If this power or voltage is not acceptable, we could adjust the current and turns and repeat the process until the voltage or power is what we desire. Sweet!

Now, to figure out how we can create forces and torques!

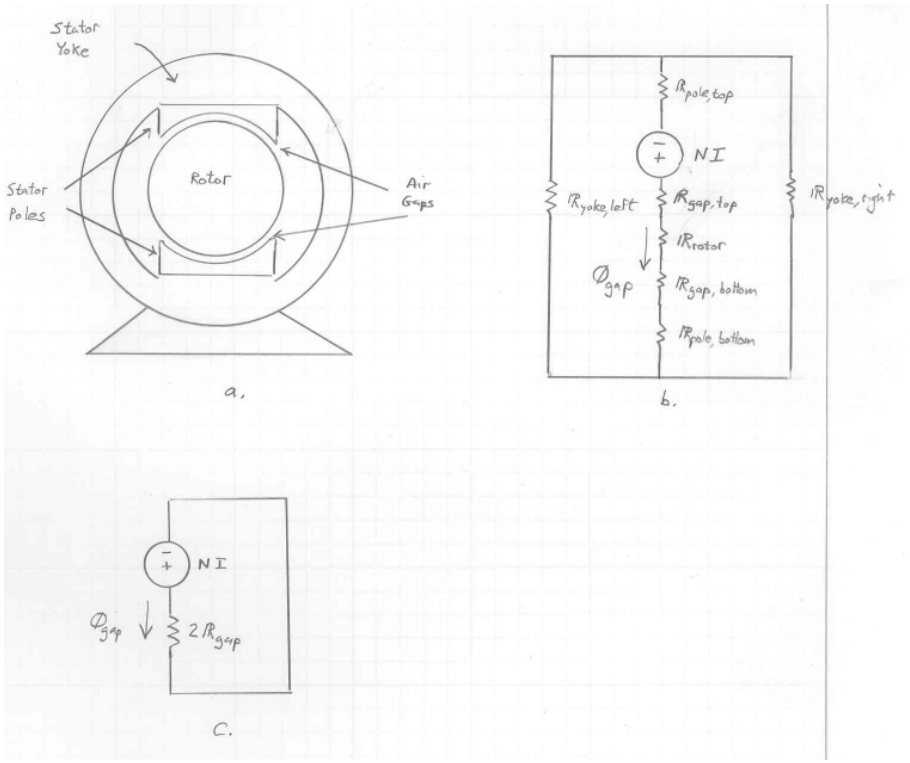

Figure 2.6: (a) Axial View of Rotating Machine; (b) Magnetic Equivalent Circuit Considering All Relevant Reluctance Paths; (c) Magnetic Equivalent Circuit Assuming that the Reluctance of the Iron Paths is Zero