Name: Key

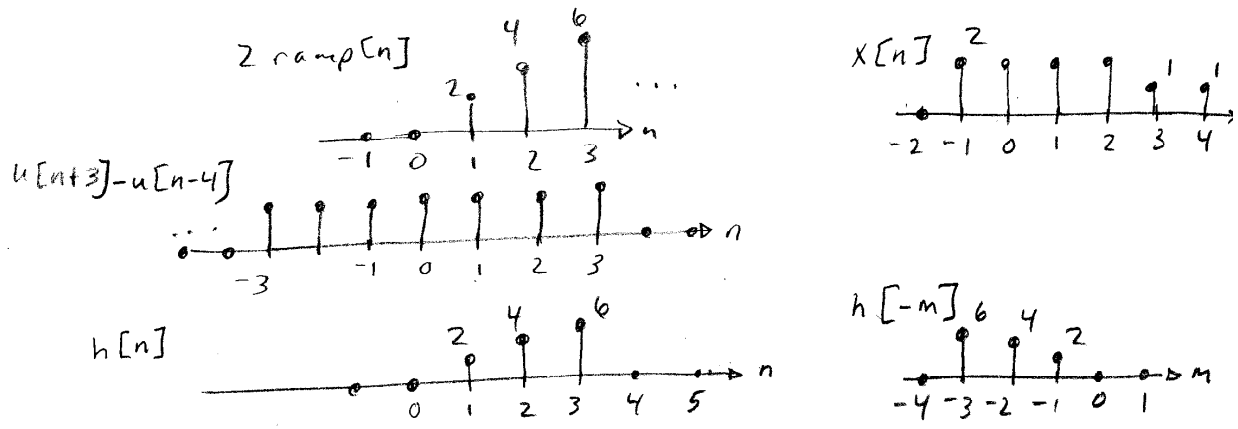
Section:

EE322 Fall 2008 Exam 2

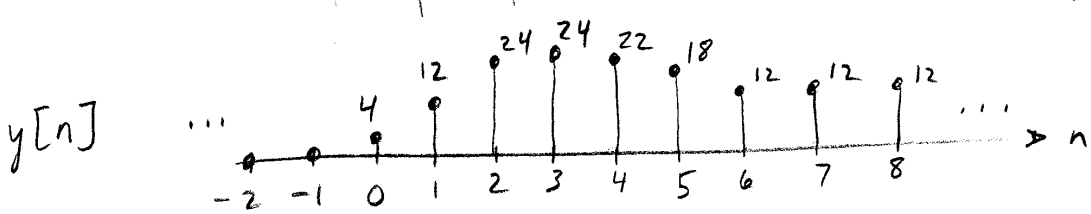
Problem	Possible Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

- You will have the entire lab period (1 hr 50 min) to take this exam.
- You are allowed to use 1 page, single-side of notes or whatever you want to write on it, and a calculator, and the Table of Fourier transforms and properties that will be supplied to you.
- You ***must show your work*** to get full credit for problems. Expect to lose points if you don't.
- Label your sketches (axes and functions) carefully. Expect to lose points if you don't.

1. (20 pts) Given a system with impulse response $h[n] = 2 \cdot \text{ramp}[n] \cdot (u[n+3] - u[n-4])$, plot the system output $y[n]$ when the input is $x[n] = 2u[n+1] - u[n-3]$.



Index m	-4	-3	-2	-1	0	1	2	3	4	5	6	
x[m]				2	2	2	2	1	1	1	1 ...	
h[0-m]		6	4	2								y[0] = 4
h[1-m]			6	4	2							y[1] = 12
h[2-m]				6	4	2						y[2] = 24
h[3-m]					6	4	2					y[3] = 24
h[4-m]						6	4	2				y[4] = 22
h[5-m]							6	4	2			y[5] = 18
h[6-m]								6	4	2		y[6] = 12



2. (20 pts) The Fourier series Harmonic function for a certain signal $x(t)$ is given by: $X[k] = 4 + \delta[k-3] - 0.2j\delta[k+2] + \delta[k+3] + 0.2j\delta[k-2]$. The fundamental period of the signal is 0.01 sec. Determine the signal $x(t)$ and write it in terms of sinusoids.

a. What is the fundamental frequency in this problem?

$$f_F = \frac{1}{T_F} = \boxed{100\ Hz}$$

b. Write $x(t)$ as a sum of complex exponentials.

$$x(t) = 4e^{j0} + 1e^{j2\pi(3)100t} + 1e^{j2\pi(-3)100t} + 0.2j\,e^{j2\pi(2)100t} - 0.2j\,e^{j2\pi(-}$$

$$= 4 + e^{j2\pi 300t} + e^{-j2\pi 300t} + 0.2je^{j2\pi 200t} - 0.2je^{-j2\pi 200t}$$

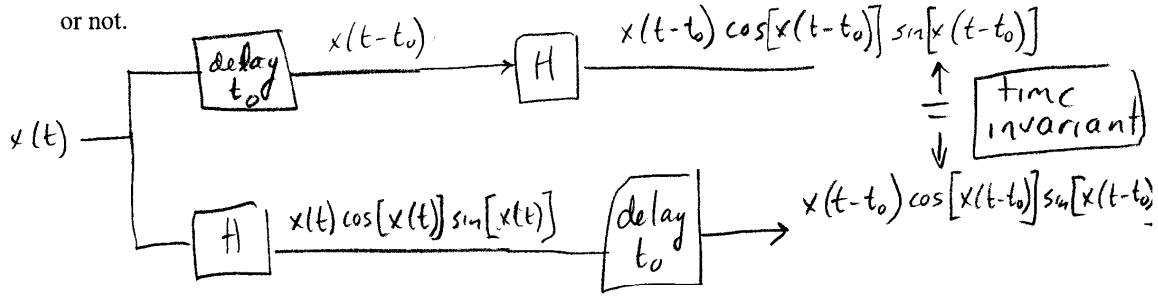
$$= 4 + 2\left[\frac{e^{j2\pi 300t} + e^{-j2\pi 300t}}{2}\right] + \left(-\frac{2}{5}\right)\left[\frac{e^{j2\pi 200t} - e^{-j2\pi 200t}}{2j}\right]$$

c. Write $x(t)$ as a sum of sinusoids.

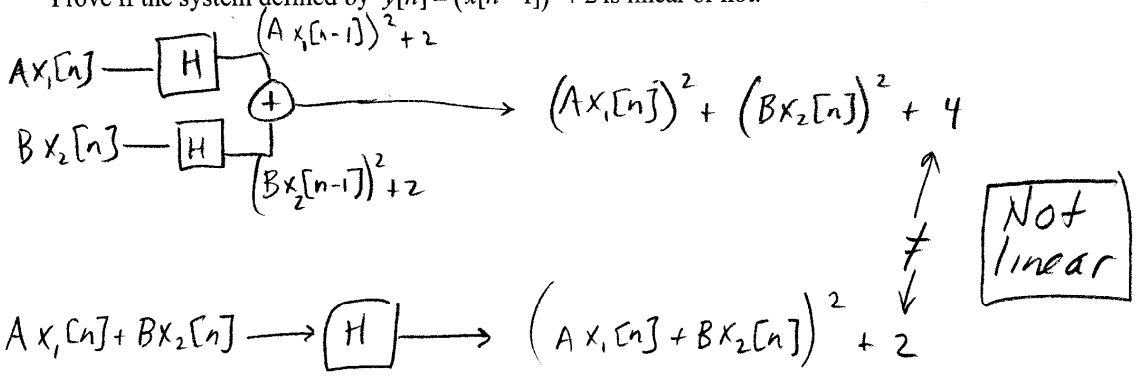
$$\boxed{x(t) = 4 + 2\cos 2\pi 300t - \frac{2}{5}\sin 2\pi 200t}$$

3. (20 pts) System Properties.

a. Prove whether or not the system defined by $y(t) = x(t) \cdot \cos(x(t)) \cdot \sin(x(t))$ is time invariant or not.



b. Prove if the system defined by $y[n] = (x[n-1])^2 + 2$ is linear or not.



c. Is the system defined by $y(t) = x(t/3)$ causal? Why or why not?

No - for negative values of t, output depends on future inputs, e.g. $y(-3) = x(-1)$

(time $t = -1$ occurs after time $t = -3$)

d. A system is defined by: $y[n] = x[n] + 0.3y[n-2]$. Find the impulse response $h[n]$ and prove whether the system is BIBO stable or not.

n	$x[n]=\delta[n]$	$.3y[n-2]$	$y[n]=h[n]$
0	1	0	1
1	0	0	0
2	0	.3	.3
3	0	0	0
4	0	$(.3)^2$	$(.3)^2$
5	0	0	0
6	0	$(.3)^3$	$(.3)^3$
			$\vdots$

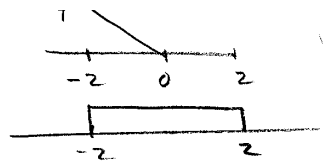
$$\sum_{n=-\infty}^{\infty} |h[n]| = 1 + .3 + .3^2 + .3^3 + \dots = \frac{1}{1-.3}$$

$$= \frac{1}{.7} = 1.4286 < \infty, \text{ so}$$

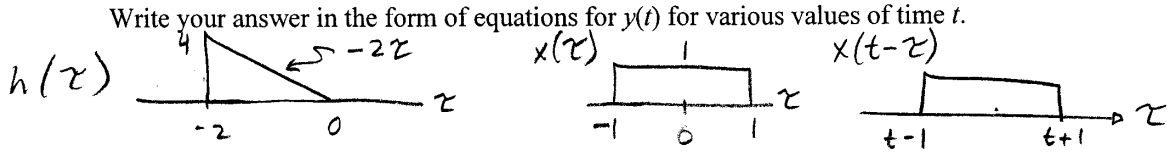
BIBO stable

$2\,\mathrm{ramp}(-t)$

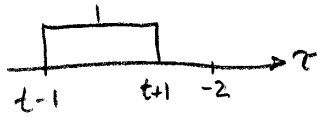
$\mathrm{rect}(t/4)$



4. (20 pts total): Given a system with impulse response $h(t) = 2 \cdot \mathrm{ramp}(-t) \cdot \mathrm{rect}(t/4)$, using the continuous convolution integral, find the system output $y(t)$ when the input is $x(t) = \mathrm{rect}(t/2)$. Write your answer in the form of equations for $y(t)$ for various values of time t.

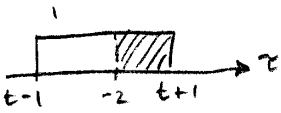


case 1: no overlap to the left



$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau = \underline{0} \qquad \underline{t \le -3}$$

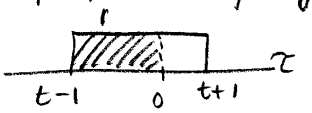
case 2: overlap w/ leading edge



$$y(t) = \int_{-2}^{t+1} -2\tau\,d\tau = -\tau^2\Big|_{-2}^{t+1}$$

$$= \underline{-(t+1)^2 + 4} \qquad \underline{-3 \le t \le -1}$$

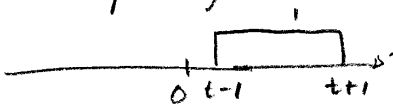
case 3: overlap w/ trailing edge



$$y(t) = \int_{t-1}^{0} -2\tau\,d\tau = -\tau^2\Big|_{t-1}^{0}$$

$$= \underline{(t-1)^2} \qquad \underline{-1 \le t \le 1}$$

case 4: no overlap to right

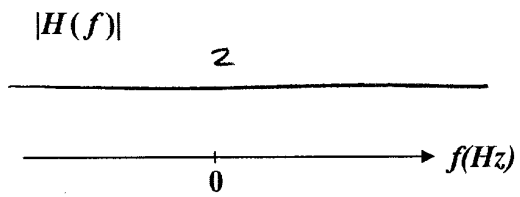


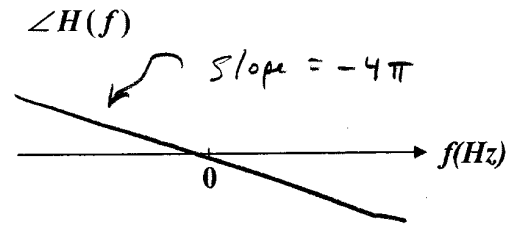
$$\underline{y(t) = 0} \qquad t > 1$$

$$\boxed{y(t) = \begin{cases} 0, & t \le -3 \\ -(t+1)^2 + 4, & -3 \le t \le -1 \\ (t-1)^2 & -1 \le t \le 1 \\ 0 & t > 1 \end{cases}}$$

(20 points total) A system's impulse response $h(t)$ has a Fourier transform given by: $H(f) = 2e^{-j4\pi f}$

a. Sketch the magnitude and phase of $H(f)$ on the axes below. Clearly label your plots.





b. What is the impulse response of this system?

$H(f) = 2e^{-j2\pi f \cdot 2}$

$h(t) =$ $\delta(t-2)$

c. Is this a causal system? Why or why not?

yes - output at any time only depends on a past input