Name: Key

Section: ________________

EE322 Fall 2009 Exam 2, Part 1

	Problem	Possible Points	Score
Part 1	1	25	
	2	25	
	3	25	
	4	25	
Part 2	5	10	
	6	15	
	Total	125	

- You will have the entire lab period to take this exam.
- You are allowed to use 1 page, single-side of notes or whatever you want to write on it, and a calculator. Fourier transform tables and Fourier properties tables will be supplied.
- You ***must show your work*** to get full credit for problems. If using a specific property or transform from the Fourier tables provided, be sure to indicate which ones you are using.
- Label your sketches (axes and functions) carefully.

1. (25 pts) Midn 2/c W.T. Door has created a senior design project that is a system with an impulse response $h(t) = \delta(t-1) + 0.5\delta(t-10)$. To test his system, he will input to it the signal obtained by measuring the voltage across a discharging capacitor. The initial voltage across the capacitor is 10V, and when the test starts, the input signal is given by $x(t) = te^{-3t}u(t)$.

a. Write the mathematical expression for system frequency response, $H(f)$.

From tables, $\delta(t) \longleftrightarrow 1$

$\delta(t-1) \longleftrightarrow e^{-j2\pi f}$

$.5\delta(t-10) \longleftrightarrow .5e^{-j20\pi f}$

$$\boxed{H(f) = e^{-j2\pi f} + .5e^{-j20\pi f}}$$

b. Write the mathematical expression for the Fourier transform of the input signal, $X(f)$.

From table, $te^{-at}u(t) \longleftrightarrow \frac{1}{(j2\pi f + a)^2}$

so $x(t) = te^{-3t}u(t) \longleftrightarrow \boxed{\frac{1}{(j2\pi f + 3)^2} = X(f)}$

c. Write the mathematical expression for the Fourier transform of the output, $Y(f)$.

$$Y(f) = H(f)X(f) = \frac{e^{-j2\pi f}}{(j2\pi f+3)^2} + \frac{0.5e^{-j20\pi f}}{(j2\pi f+3)^2}$$

d. Write the mathematical expression for the output signal, $y(t)$.

$e^{-j2\pi f} \Rightarrow$ delay of 1, $e^{-j20\pi f} \Rightarrow$ delay of 10

$$\boxed{y(t) = (t-1)e^{-3(t-1)}u(t-1) + 0.5(t-10)e^{-3(t-10)}u(t-10)}$$

$T_1 = \frac{1}{4}$ $\quad$ $T_2 = \frac{1}{8}$

$\sin 2\pi 8t$

2. (25 pts) Continuous-time Fourier Series. Let $x(t) = 2\cos 2\pi 4t + \sin 16\pi t$.

   a. Determine the fundamental period and fundamental frequency of $x(t)$.

$$T_o = \text{Lcm}\left(\frac{1}{4}, \frac{1}{8}\right) = \frac{\text{LCm}\left(\frac{8}{4}, \frac{8}{8}\right)}{8} = \frac{\text{LCm}(2,1)}{8} = \frac{2}{8} = \boxed{\frac{1}{4}}$$

$$f_o = 4\,Hz$$

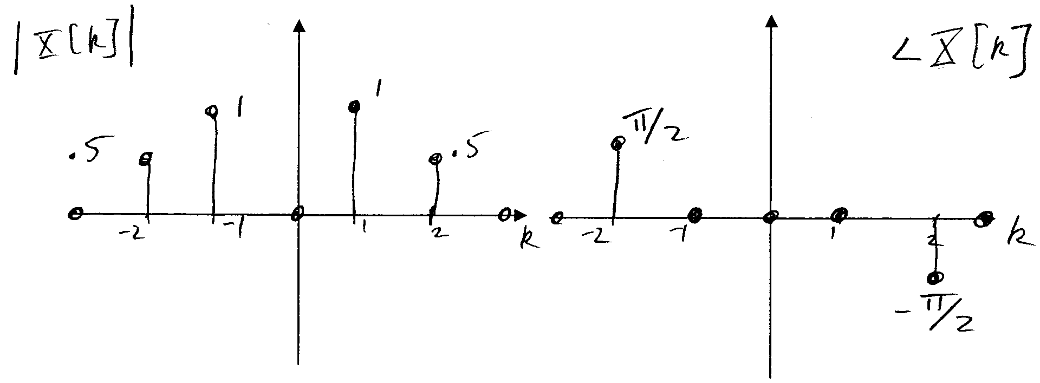
   b. Find the equation for the Fourier Series Harmonic function ($X[k]$) in terms of delta functions.

$$2\cos 2\pi 4t + \sin 2\pi 8t = \cancel{2}\frac{e^{j2\pi(1\cdot 4)t}}{\cancel{2}} + \frac{\cancel{2}e^{j2\pi(-1\cdot 4)t}}{\cancel{2}} + \frac{1}{2j}e^{j2\pi(2\cdot 4)t} - \frac{1}{2j}e^{j2\pi(-2\cdot 4)t}$$

$$X[1] = 1,\ X[-1] = 1,\ X[2] = \frac{1}{2j},\ X[-2] = -\frac{1}{2j}$$

$$X[k] = \delta[k-1] + \delta[k+1] + \frac{1}{2j}\delta[k-2] - \frac{1}{2j}\delta[k+2]$$

   c. Plot the magnitude and phase of X[k] on the axes below. Label your plots.



   d. This question is unrelated to the other questions on this page. Suppose you had a different signal that you computed the Harmonic function for, and in this case $X[k] = \dfrac{1+e^{\frac{j\pi k}{2}}}{1-e^{\frac{j\pi k}{2}}}$. What is the magnitude and phase of $X[1]$?

|X[k]|= 1

∠X[k]= $\pi/2$

$$X[1] = \frac{1+j}{1-j}$$

$$|X[1]| = \frac{\sqrt{1^2+1^2}}{\sqrt{1^2+1^2}} = 1$$

$$\angle X[1] = \tan^{-1}\left(\frac{1}{1}\right) - \tan^{-1}\left(\frac{-1}{1}\right) = \frac{\pi}{4} - -\frac{\pi}{4} = \frac{\pi}{2}$$

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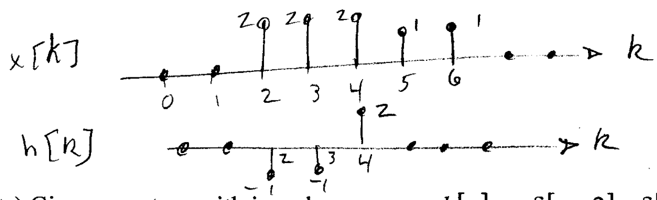
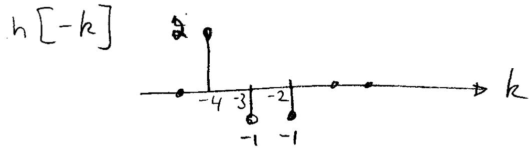


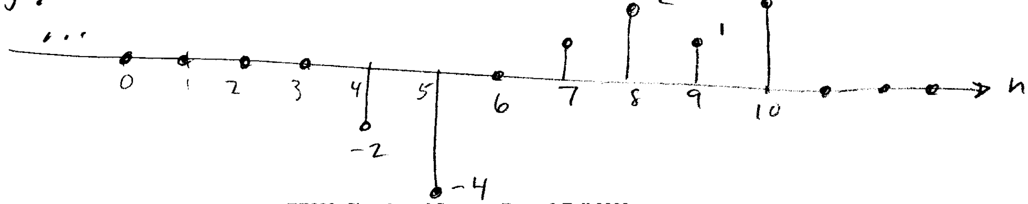
3. (25 pts) Given a system with impulse response $h[n] = -\delta[n-2] - \delta[n-3] + 2\delta[n-4]$, find the system output $y[n]$ when the input is $2u[n-2] - u[n-5]. - u[n-7]$.



note: must shift this right by 4 to have any overlap.

index	-1	0	1	2	3	4	5	6	7	8	9	
x[k]				2	2	2	1	1				
h[4-k]		2	-1	-1								y[4] = -2
h[5-k]			2	-1	-1							y[5] = -4
h[6-k]				2	-1	-1						y[6] = 0
h[7-k]					2	-1	-1					y[7] = 4-2-1 = 1
h[8-k]						2	-1	-1				y[8] = 4-1-1 = 2
h[9-k]							2	-1	-1			y[9] = 2-1 = 1
h[10-k]								2	-1	-1		y[10] = 2
h[11-k]									2	-1	-1	y[11] = 0 ⋮

y[n]:



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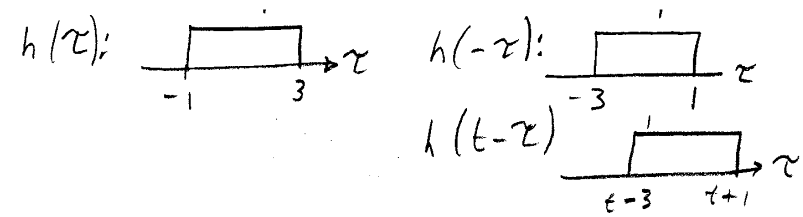
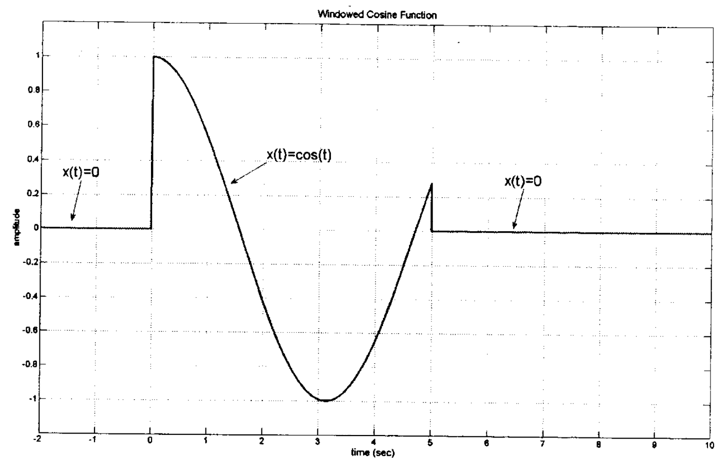


4. (25 pts) The impulse response of a certain system is $h(t) = u(t+1) - u(t-3)$. Suppose the input to the system is the windowed cosine signal shown to the right,

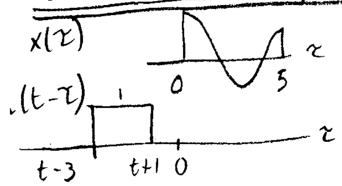
$$x(t) = \begin{cases} \cos(t), & 0 \le t \le 5 \\ 0, & \text{otherwise} \end{cases}.$$



(a) Use the convolution integral to find the equation for the output of this system with this input.

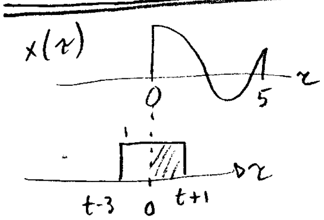
Use the back of this page if you need more room to answer.

case 1 no overlap to left



$y(t) = 0, \quad$ for $t < -1$

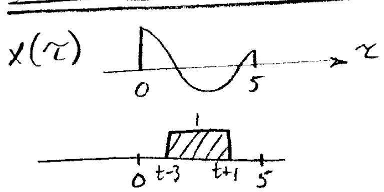
case 2 overlap w/ leading edge



$$y(t) = \int_0^{t+1} \cos(t)\,dt = \sin(t)\Big|_0^{t+1} = \sin(t+1)$$

for $-1 \le t \le 3$

case 3 complete overlap



$$y(t) = \int_{t-3}^{t+1} \cos(t)\,dt = \sin(t)\Big|_{t-3}^{t+1} = \sin(t+1) - \sin(t-3)$$

For $3 \le t \le 4$

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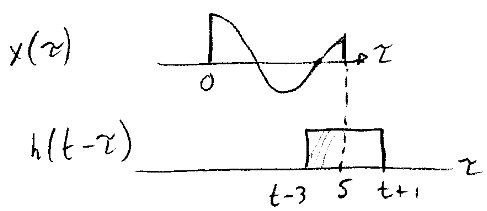
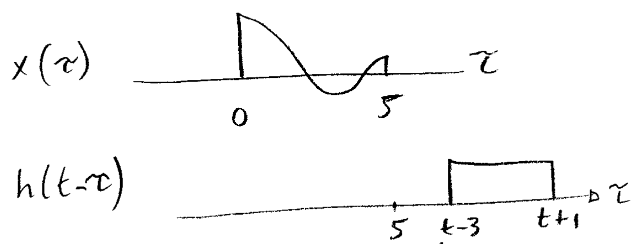
case 4 : overlap w/ trailing edge



$$y(t) = \int_{t-3}^{5} \cos(t)\,dt = \sin(t)\Big|_{t-3}^{5} = \sin(5) - \sin(t-3)$$

for $4 \le t \le 8$

case 5: no overlap to the right



$y(t) = 0$ for $t \ge 8$

overall;

$$y(t) = \begin{cases} 0, & t < -1 \\ \sin(t+1), & -1 \le t \le 3 \\ \sin(t+1) - \sin(t-3), & 3 \le t \le 4 \\ \sin(5) - \sin(t-3), & 4 \le t \le 8 \\ 0, & t > 8 \end{cases}$$

(b) Find $y(3.3) = \sin(3.3+1) - \sin(3.3-3) = \boxed{-1.2117}$