Name: Key

Section:

EE322 Fall 2010 Exam 2: Part 1

	Problem	Possible Points	Score
Part 1	1	25	
	2	25	
	3	25	
	4	25	
Part 2	5	10	
	6	20	
Total		130	

- You will have the first 75 minutes of the lab period to take Part 1 of the exam.
- For this portion of the exam, you are allowed to use 1 page, single-side of notes or whatever you want to write on it, and an FE Exam-approved calculator.

  Indicate the manufacturer and model of the calculator you will use for this exam, or indicate "None":

  Calculator: ______________________

- You ***must show your work*** to get full credit for problems. Expect to lose points if you don't.
- Label your sketches (axes and functions) carefully, including units if applicable. Expect to lose points if you don't.
- If you finish Part 1 of the exam before it is called for, turn it in and pick up Part 2 (MATLAB).
- Sign the statement below:

I will not discuss this exam with anyone until after 6th period today. ______________________
(Signature)

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1. (25 pts) Fourier Transforms/Properties. Answer the following, showing your work.

a. $x(t) = \frac{e^{-5t} - e^{-3t}}{2} u(t)$. Find $X(f)$.

$$\frac{e^{-at} - e^{-bt}}{b-a} u(t) \longleftrightarrow \frac{1}{(j2\pi f + a)(j2\pi f + b)}$$

$$\frac{e^{-5t} - e^{-3t}}{2} u(t) \longleftrightarrow \boxed{\frac{-1}{(j2\pi f + 5)(j2\pi f + 3)}}$$

c. $X(f) = \mathrm{sinc}^2(10f) \cdot e^{-j20\pi f}$. Find $x(t)$.

$$\mathrm{tri}\left(\frac{t}{10}\right) \longleftrightarrow 10\,\mathrm{sinc}^2(10f)$$

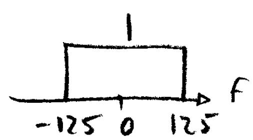
$$\mathrm{tri}\left(\frac{t-10}{10}\right) \longleftrightarrow 10\,\mathrm{sinc}^2(10f)\, e^{-j2\pi f \cdot 10}$$

$$\text{so } \boxed{\frac{1}{10}\mathrm{tri}\left(\frac{t-10}{10}\right)} \longleftrightarrow \mathrm{sinc}^2(10f)\, e^{-j20\pi f}$$

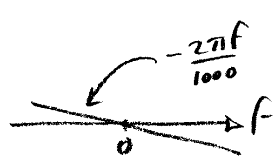
d. An ideal low pass filter has impulse response $rect\left(\frac{f}{250}\right) \cdot e^{-j2\pi f/1000}$. The input to the filter is $\sin(2\pi 20t - 0.1\pi) + \cancel{4\cos(2\pi 240t + 0.2\pi)}$. What is the output of the filter?

filtered out

$|H(f)|$



$\angle H(f)$



for 20 Hz, $|H(20)| = 1$

$$\angle H(20) = -\frac{2\pi(20)}{1000} = -0.04\pi$$

$$y(t) = \sin(2\pi 20t - 0.1\pi - 0.04\pi)$$

$$= \boxed{\sin(2\pi 20t - 0.14\pi)}$$

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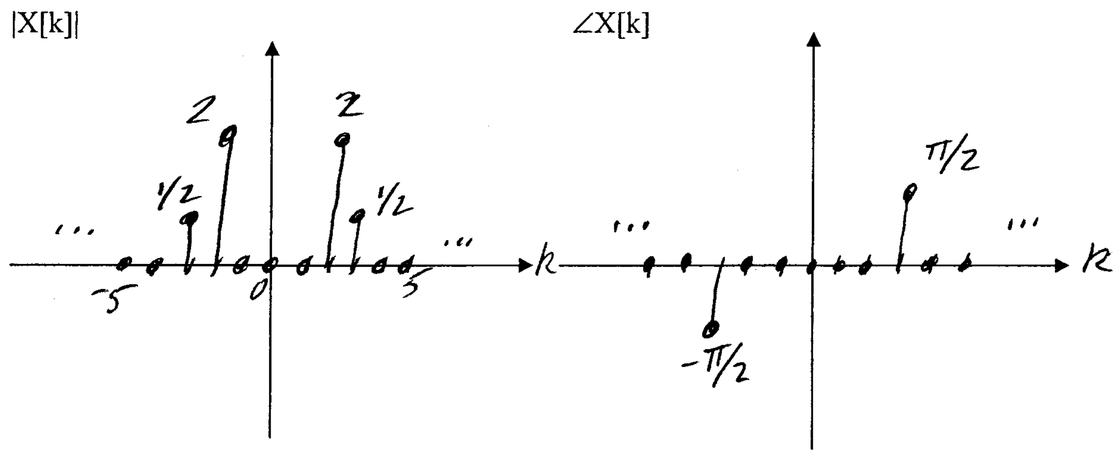
(25 pts) Continuous-time Fourier Series. Let T_f =0.001 sec, and a harmonic function be given by:

$$X[k] = -\frac{1}{2j}\delta[k-3] + \frac{1}{2j}\delta[k+3] + 2\delta[k+2] + 2\delta[k-2].$$

$0.5j$ $\quad$ $-0.5j$

a. Plot the magnitude and phase of X[k] on the axes below. Label your plots.



b. Find the equation for the time signal that corresponds to this harmonic function and fundamental period in terms of sinusoids.

$$f_F = \frac{1}{T_F} = 1000\,Hz$$

$$x(t) = -\frac{1}{2j}e^{j2\pi 3\cdot 1000t} + \frac{1}{2j}e^{j2\pi(-3)1000t} + 2e^{j2\pi(-2)1000t} + 2e^{j2\pi 2\cdot 100}$$

$$= -\frac{1}{2j}\left(e^{j2\pi 3000t} - e^{-j2\pi 3000t}\right) + 4\cdot\frac{1}{2}\left(e^{j2\pi 2000t} + e^{-j2\pi 200}\right)$$

$$= -\sin(2\pi 3000t) + 4\cos(2\pi 2000t)$$

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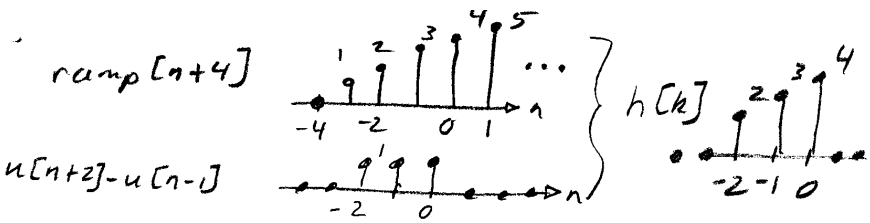
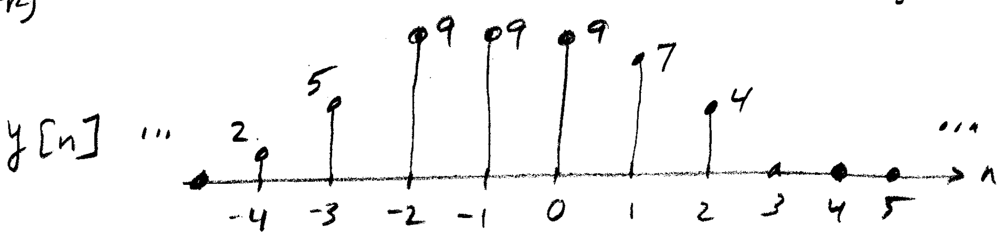


3. (25 pts) Discrete Systems.

   a. A discrete system has impulse response h[n] = ramp[n+4] (u[n+2] – u[n-1]). The input to the system is given by $rect_2[n]$. Determine the system output y[n].

k	-5	-4	-3	-2	-1	0	1	2	3	4	5	
$x[k]$				1	1	1	1	1				
$h[-4-k]$		4	3	2								$y[-4] = 2$
$h[-3-k]$			4	3	2							$y[-3] = 5$
$h[-2-k]$				4	3	2						$y[-2] = 9$
$h[-1-k]$					4	3	2					$y[-1] = 9$
$h[0-k]$						4	3	2				$y[0] = 9$
$h[1-k]$							4	3	2			$y[1] = 7$
$h[2-k]$								4	3	2		$y[2] = 4$



   b. A system is described by y[n] = 0.4y[n-1] + x[n]. Using this equation, determine an equation for the system's impulse response h[n].

n	$x[n]=\delta[n]$	$0.4y[n-1]$	$y[n]=h[n]$
0	1	0	1
1	0	0.4	0.4
2	0	$(0.4)^2$	$(0.4)^2$
3	0	$(0.4)^3$	$(0.4)^3$
			⋮

$$\boxed{h[n]=(0.4)^n u[n]}$$

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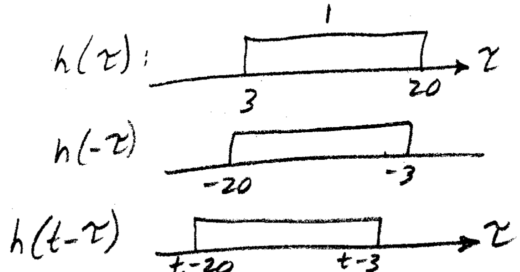
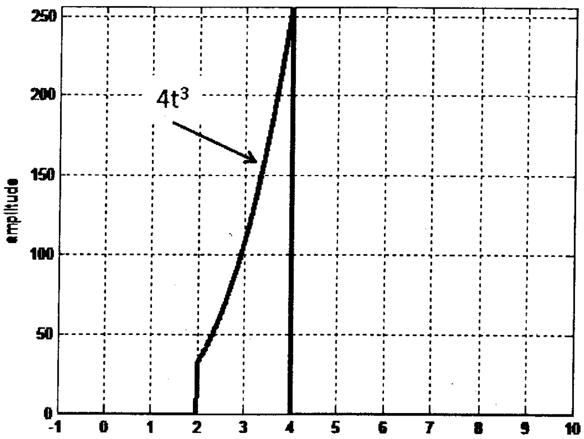
4. (25 pts) The impulse response of a certain system is $h(t) = u(t-3) - u(t-20)$.

Suppose the input to the system is the signal shown to the right, defined as:

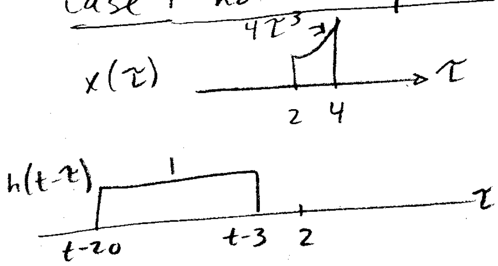
$$x(t) = \begin{cases} 4t^3, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}.$$





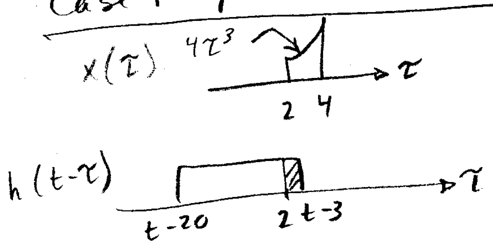
a. Use the convolution integral to find the equation for the output of this system with this input. Use the next page of the exam to show your work if needed. Write your overall final equation on the next page in the space provided.

Case 1 no overlap left



$y(t) = 0$, for $t-3 < 2$, or $t < 5$

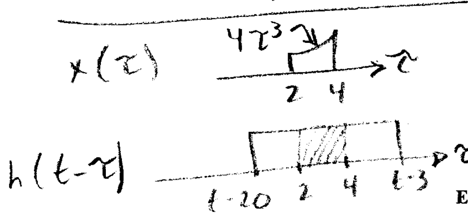
Case 1 partial overlap leading edge



$$y(t) = \int_2^{t-3} 4\tau^3 d\tau = \tau^4 \Big|_2^{t-3} = (t-3)^4 - 16$$

$5 \le t \le 7$

case 3 complete overlap



$$y(t) = \int_2^4 4\tau^3 d\tau = \tau^4 \Big|_2^4 = 256 - 16 = 240$$

$7 \le t \le 22$

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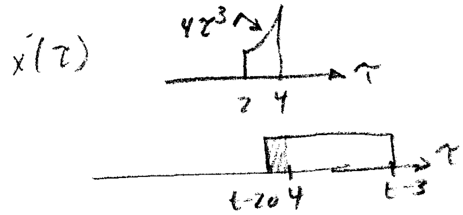
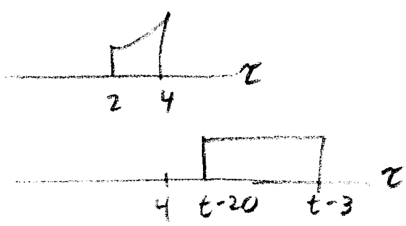
case 4: overlap w/ trailing edge



$$y(t) = \int_{t-20}^{4} 4\tau^3 d\tau = \tau^4 \Big|_{t-20}^{4} = \underline{256 - (t-20)^4}$$

$\underline{22 \le t \le 24}$

case 5: no overlap right



$y(t) = 0, \quad t > 24$

Your answer: $y(t) = \begin{cases} 0, & t < 5 \\ (t-3)^4 - 16, & 5 \le t \le 7 \\ 240, & 7 \le t \le 22 \\ 256 - (t-20)^4, & 22 \le t \le 24 \\ 0, & t > 24 \end{cases}$

b. What is the value of $y(10)$? $y(10) =$ 240

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