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Name: Key

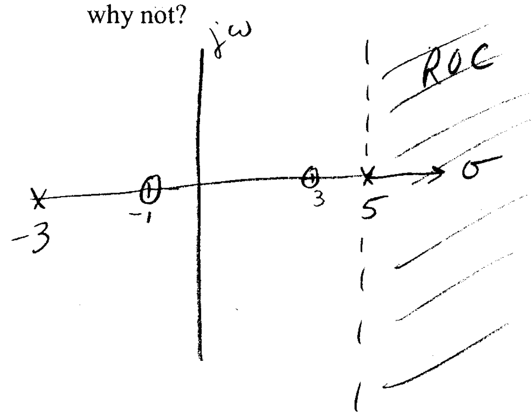
EE322 Final Exam Review Worksheet
(Use additional paper as necessary)

1. (Laplace Transform) Find the poles and zeros of a system with transfer function given by:

$$H(s)=\frac{\left(s^2-2s-3\right)}{(s+3)(s-5)}. \quad = \frac{(s-3)(s+1)}{(s+3)(s-5)}$$

zeros: $s=3,-1$ poles: $s=-3,5$

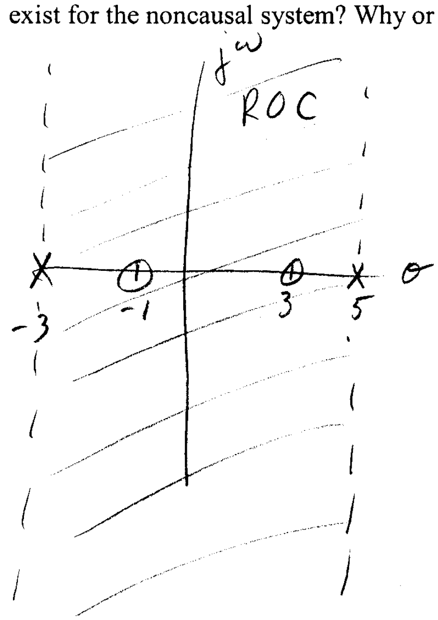
a. Plot the ROC if this represents a causal system. Does the continuous-time Fourier transform exist for the causal system? Why or why not? Is this system stable? Why or why not?



• FT does not exist – the jω axis is not within the ROC

• Not stable – H(s) has a pole in RHP (right half plane)

b. Plot the ROC if this is a noncausal system. Does the continuous-time Fourier transform exist for the noncausal system? Why or why not?

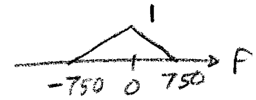


• FT does exist – jω axis is in the ROC

2. (Sampling) Suppose you wish to sample the analog signal $x(t) = 750\,\mathrm{sinc}^2(750t)$. What is the Nyquist rate? Sketch the frequency spectrum if this signal is sampled with the impulse train function: $\cancel{4000}\,\delta_{1/2000}(t)$.

0.001

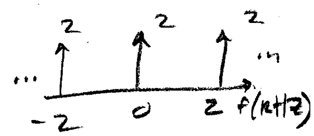
$$750\,\mathrm{sinc}^2(750t) \longleftrightarrow \cancel{750}\,\frac{1}{\cancel{750}}\,\mathrm{tri}\left(\frac{f}{750}\right)$$

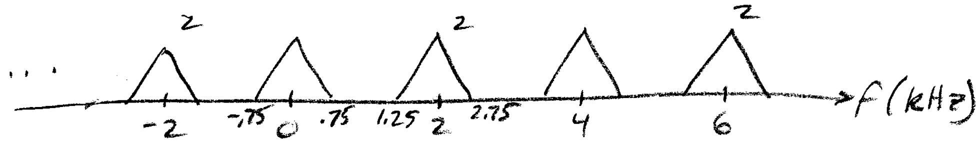


$$\delta_{T_0}(t) \longleftrightarrow f_0\,\delta_{f_0}(f)$$

so $\delta_{\frac{1}{2000}}(t) \longleftrightarrow 2000\,\delta_{2000}(f)$

and $0.001\,\delta_{\frac{1}{2000}}(t) \longleftrightarrow 0.001(2000)\,\delta_{2000}(f)$





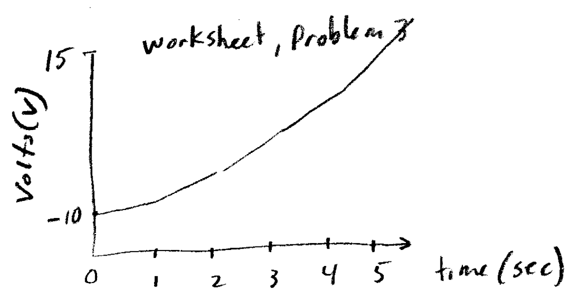
3. (MATLAB) Given the following MATLAB code, when the code is run, what is displayed? Be specific (e.g., if there is an error, state what it is; if there is a plot, sketch the plot; if values are displayed in the command window, describe them).

```
t=0:1:5;
x=t.^t;
x=x-10;
x(3)
x(5)
figure(1), plot(t,x)
xlabel('time (sec)'), ylabel('Volts (V)')
title('Worksheet, Problem 3')
```

x = [1 1 4 27 256 3125]

x = [-9 -9 -6 17 246 3115]

- in command window, will give the value of x(3) = -6, and x(5) = 246
- will produce the figure that looks like this:



4. (Fourier Transform) Find the inverse CTFT (i.e., the time signal $x(t)$) corresponding to:

a. $X(f) = \dfrac{3}{(2+j2\pi f)^2}$.

tables: $te^{-at}u(t) \longleftrightarrow \dfrac{1}{(a+j2\pi f)^2}$

$$\frac{3}{(2+j2\pi f)^2} \longleftrightarrow \boxed{3te^{-2t}u(t)}$$

b. $X(f) = \dfrac{3}{(2+j2\pi f)^2}e^{-j4\pi f}$.

$x(t-t_0) \longleftrightarrow X(f)e^{-j2\pi f t_0}$

$$\frac{3}{(2+j2\pi f)^2}e^{-j2\pi f\cdot 2} \longleftrightarrow \boxed{3(t-2)e^{-2(t-2)}u(t-2)}$$

5. (Fourier Transform) If $x(t) = \dfrac{4e^{-4t}-2e^{-2t}}{2}u(t)$, find the CTFT of:

$$X(f) = \frac{j2\pi f}{(4+j2\pi f)(2+j2\pi f)}$$

a. $\dfrac{d}{dt}x(t)$ $\boxed{\dfrac{(j2\pi f)^2}{(4+j2\pi f)(2+j2\pi f)}}$

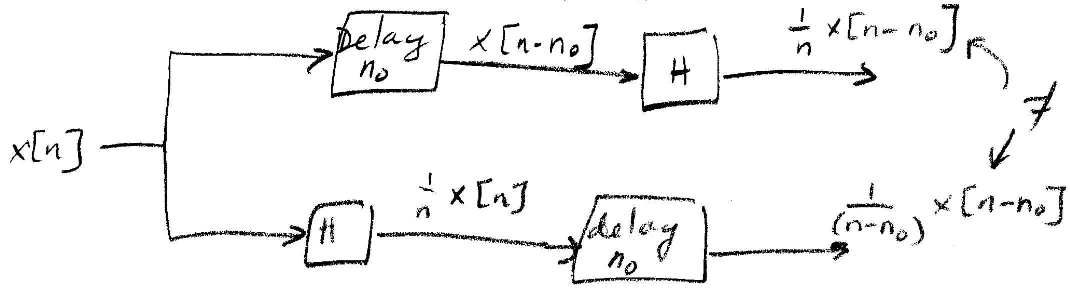
b. $x(t-3)$ $\dfrac{j2\pi f\, e^{-j6\pi f}}{(4+j2\pi f)(2+j2\pi f)}$

6. (System Properties) Find the impulse response of the system defined by the difference equation: $y[n] = x[n+2] - x[n-1] + 3x[n] + 0.5x[n-10]$.

let $x[n] = \delta[n]$

$$\boxed{h[n] = \delta[n+2] - \delta[n-1] + 3\delta[n] + 0.5\,\delta[n-10]}$$

7. (System Properties) Demonstrate whether the system $y[n] = \frac{1}{n}x[n]$ is time-invariant or not.



2 outputs not equal $\Rightarrow$

$\boxed{\text{NOT time invariant}}$

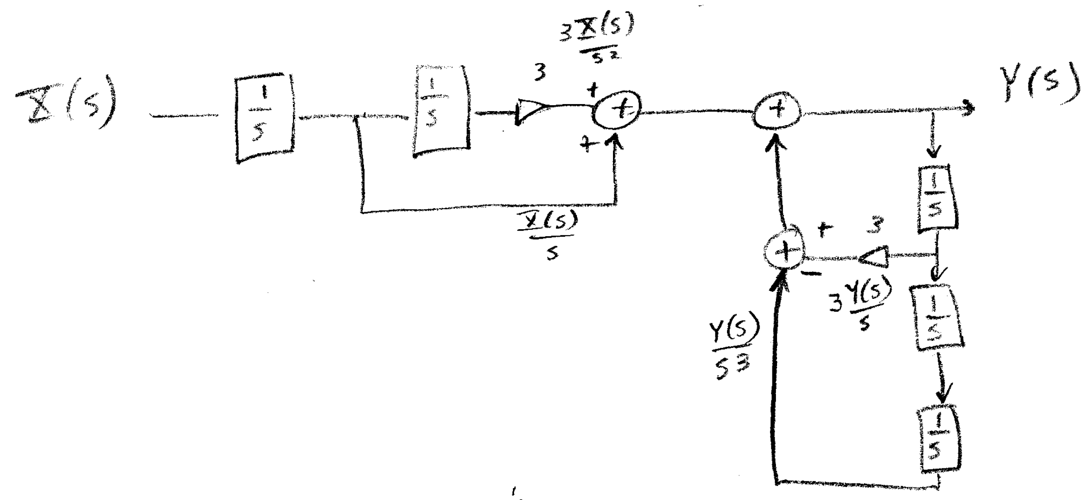
8. (Laplace Transform) A system has the transfer function $H(s)=\dfrac{s(s+3)}{s^3-3s^2+1}$. Draw a block diagram for the system using integrators, and find the differential equation that relates the input $x(t)$ to the output $y(t)$.

$$H(s)=\frac{Y(s)}{X(s)}=\frac{s^2+3s}{s^3-3s^2+1}$$

so $s^2X(s)+3sX(s)=s^3Y(s)-3s^2Y(s)+Y(s)$

divide by s^3 and solve for $Y(s)$:

$$Y(s)=\frac{X(s)}{s}+\frac{3X(s)}{s^2}+\frac{3Y(s)}{s}-\frac{Y(s)}{s^3}$$



9. (System Properties) Is the system $y(t)=\int_{-\infty}^{t}x(t)dt$ causal? Why or why not? BIBO stable? Why or why not?

causal - does not depend on future inputs

Not BIBO stable - let $x(t)=1$ (bounded),

$$y(t)=\int_{-\infty}^{t}1\,dt=\infty \text{ (not bounded)}$$

BIBO stable?

$Y(s)=\frac{X(s)}{s}$, so $H(s)=\frac{1}{s}$

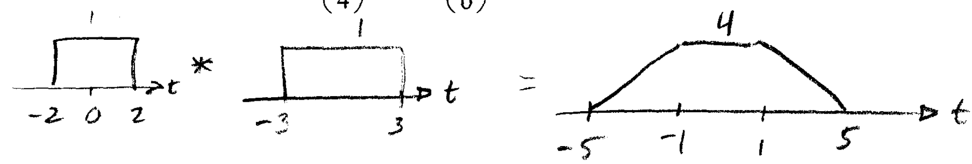
$h(t)=u(t)$

$\int_{-\infty}^{\infty}|h(t)|dt=\int_{0}^{\infty}1\,dt=\infty$ Not BIBO stable

10. (Signals) Evaluate: $\int_{-33}^{80}6\delta(t+2)dt$. $=6\int_{-\infty}^{\infty}\delta(t+2)dt=6$

11. (Continuous Convolution) Plot the result of convolving the continuous functions below.

$$rect\left(\frac{t}{4}\right) * rect\left(\frac{t}{6}\right).$$



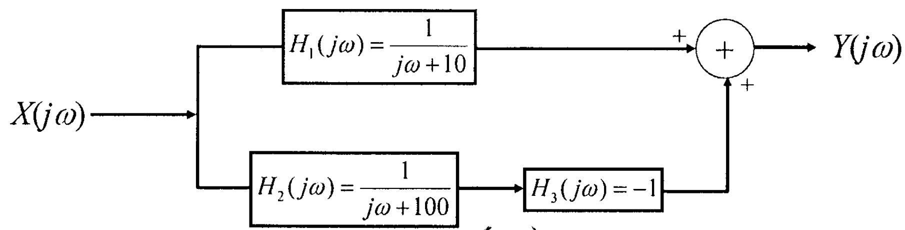
12. (Discrete Convolution) Find the discrete convolution of x[n] and y[n], where
$x[n] = -\delta[n+1] + \delta[n]$ and $y[n] = \delta[n+1] + 2\delta[n] + \delta[n-1]$.

$$y[n] * \left(-\delta[n+1] + \delta[n]\right) = -y[n+1] + y[n]$$

index n:	-3	-2	-1	0	1	2
$y[n]$			1	2	1	
$-y[n+1]$		-1	-2	-1		
$y[n]-y[n-1]$		-1	-1	1	1	

$$x[n] * y[n] = -\delta[n+2] - \delta[n+1] + \delta[n] + \delta[n-1]$$

13. (Frequency response) A system is shown below. Sketch the Bode magnitude of the frequency response. Label your corner frequencies and slopes. What kind of filter is this?



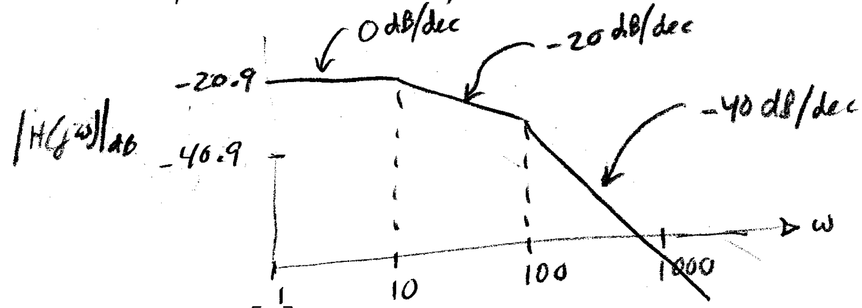
$$Y(j\omega) = \underbrace{\left[H_1(j\omega) + H_2(j\omega) H_3(j\omega) \right]}_{H(j\omega)} X(j\omega)$$

$$H(j\omega) = \frac{1}{j\omega+10} - \frac{1}{j\omega+100} = \frac{j\omega+100 - j\omega - 10}{(j\omega+10)(j\omega+100)}$$

$$= \frac{90}{(j\omega+10)(j\omega+100)} = \frac{90}{10\left(\frac{j\omega}{10}+1\right) 100\left(\frac{j\omega}{100}+1\right)} = \frac{.09}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)}$$

Gain = .09, or $20(\log_{10}(.09)) = -20.9\,dB$

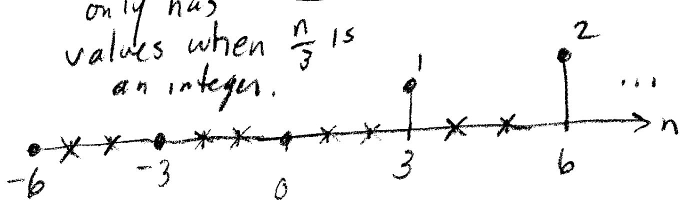
pole at $\omega = 10$, $\omega = 100$



$\boxed{\text{LPF}}$

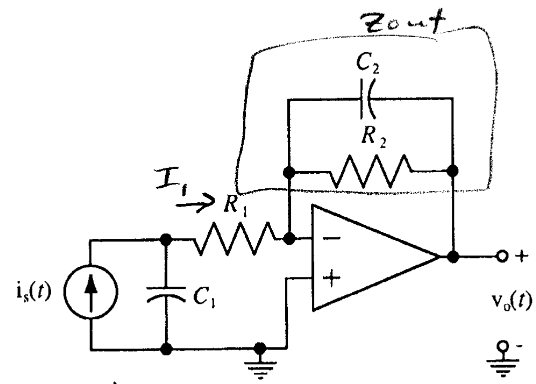
14. (Signals) Plot $ramp\left[\frac{n}{3}\right]$.

only has values when $\frac{n}{3}$ is an integer.



15. A system is shown below, where the input is $i_s(t)$ and the output is $v_o(t)$. Use the following values for components:

$R_1 = 100\ \text{k}\Omega$
$R_2 = 10\ \text{k}\Omega$
$C_1 = 10\ \mu\text{F}$
$C_2 = 1\ \mu\text{F}$



a. Determine the system's transfer function (H(s)).

b. What type of filter is this (HPF, LPF, etc) ?

c. Determine the system's impulse response, $h(t)$.

Use additional paper as needed.

Since no current flows into Op Amp, the current thru R1 is the same current that flows thru Zout

- Current flowing through R1 (current divider):

$$I_1(s) = \frac{\frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} I_s(s) = \frac{I_s(s)}{1 + sR_1C_1}$$

- Since the $V_- = V_+ = 0V$ (ideal op amp), $V_o(s) = -I_1(s)\, Z_o(s)$

$$V_o(s) = -I_1(s) \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = -\frac{I_s(s) \quad R_2 \frac{1}{sC_2}}{(1+sR_1C_1)(R_2 + \frac{1}{sC_2})}$$

or $$H(s) = \frac{V_o(s)}{I_s(s)} = \frac{-R_2 \frac{1}{sC_2}}{(1+sR_1C_1)(R_2 + \frac{1}{sC_2})}$$

$$= \frac{-\frac{R_2}{sC_2}}{R_2 + \frac{1}{sC_2} + sR_1R_2C_1 + \frac{R_1C_1}{C_2}} \cdot \frac{\left(\frac{s}{R_1R_2C_1}\right)}{\left(\frac{s}{R_1R_2C_1}\right)}$$

$$= -\frac{1}{R_1C_1C_2} \cdot \frac{1}{s^2 + \frac{1}{R_2C_2}s + \frac{1}{R_1C_1}s + \frac{1}{R_1R_2C_1C_2}}$$

$$= -1e6 \frac{1}{s^2 + 101s + 100} = -1e6 \frac{1}{(s+1)(s+100)}$$

(b) two poles – lowpass filter

(c) $H(s) = -1e6 \dfrac{1}{(s+1)(s+100)}$

$$= \frac{10{,}101}{s+1} - \frac{10{,}101}{s+100}$$

$$\boxed{h(t) = 10{,}101 e^{-t} u(t) - 10{,}101 e^{-100t} u(t)}$$