Properties of the Continuous-Time Fourier Transform

Linearity

$$\alpha\,\mathrm{x}(t)+\beta\,\mathrm{y}(t)\xleftrightarrow{\mathcal{F}}\alpha\,\mathrm{X}(f)+\beta\,\mathrm{Y}(f)$$

Time Shifting

$$\mathrm{x}(t-t_0)\xleftrightarrow{\mathcal{F}}\mathrm{X}(f)e^{-j2\pi f t_0}$$

Frequency Shifting

$$\mathrm{x}(t)e^{+j2\pi f_0 t}\xleftrightarrow{\mathcal{F}}\mathrm{X}(f-f_0)$$

Time Scaling

$$\mathrm{x}(at)\xleftrightarrow{\mathcal{F}}\frac{1}{|a|}\mathrm{X}\left(\frac{f}{a}\right)$$

Frequency Scaling

$$\frac{1}{|a|}\mathrm{x}\left(\frac{t}{a}\right)\xleftrightarrow{\mathcal{F}}\mathrm{X}(af)$$

Transform of a Conjugate

$$\mathrm{x}^*(t)\xleftrightarrow{\mathcal{F}}\mathrm{X}^*(-f)$$

Multiplication-Convolution Duality

$$\mathrm{x}(t)*\mathrm{y}(t)\xleftrightarrow{\mathcal{F}}\mathrm{X}(f)\mathrm{Y}(f)$$

$$\mathrm{x}(t)\mathrm{y}(t)\xleftrightarrow{\mathcal{F}}\mathrm{X}(f)*\mathrm{Y}(f)$$

Time Differentiation

$$\frac{d}{dt}\left(\mathrm{x}(t)\right)\xleftrightarrow{\mathcal{F}}j2\pi f\,\mathrm{X}(f)$$

Modulation

$$\mathrm{x}(t)\cos(2\pi f_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2}\left[\mathrm{X}(f - f_0) + \mathrm{X}(f + f_0)\right]$$

Transforms of Periodic Signals

$$\mathrm{x}(t) = \sum_{k=-\infty}^{\infty} \mathrm{X}[k] e^{-j2\pi(kf_F)t} \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \sum_{k=-\infty}^{\infty} \mathrm{X}[k]\delta(f - kf_0)$$

Integral Definition of an Impulse

$$\int_{-\infty}^{\infty} e^{-j2\pi xy} dy = \delta(x)$$

Parseval's Theorem

$$\int_{-\infty}^{\infty} \left|\mathrm{x}(t)\right|^2 dt = \int_{-\infty}^{\infty} \left|\mathrm{X}(f)\right|^2 df$$

Duality

$$\mathrm{X}(t) \xleftrightarrow{\mathcal{F}} \mathrm{x}(-f) \quad \text{and}$$

$$\mathrm{X}(-t) \xleftrightarrow{\mathcal{F}} \mathrm{x}(f)$$

Integration

$$\int_{-\infty}^{t} \mathrm{x}(\lambda)d\lambda \xleftrightarrow{\mathcal{F}} \frac{\mathrm{X}(f)}{j2\pi f} + \frac{1}{2}\mathrm{X}(0)\delta(f)$$

Total-Area Integral

$$\mathrm{X}(0) = \left[\int_{-\infty}^{\infty} \mathrm{x}(t) e^{-j2\pi ft} dt\right]_{f\to 0} = \int_{-\infty}^{\infty} \mathrm{x}(t)dt$$

$$\mathrm{x}(0) = \left[\int_{-\infty}^{\infty} \mathrm{X}(f) e^{+j2\pi ft} df\right]_{t\to 0} = \int_{-\infty}^{\infty} \mathrm{X}(f)df$$