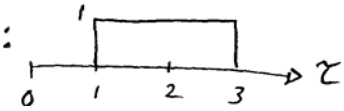
Some Continuous-time Convolution Background

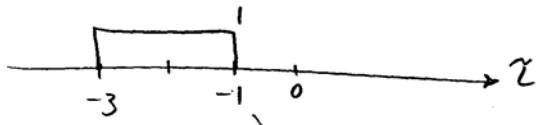
(1) Time shifting a time-reversed signal

let $x(t) = \text{rect}\left(\frac{t-2}{2}\right)$

$x(\tau)$:



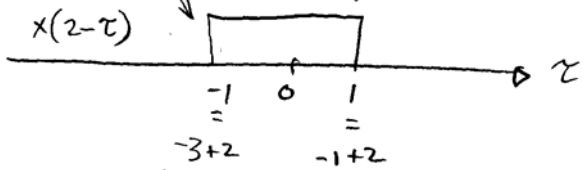
so $x(-\tau)$:



"leading edge" of rectangle is at -1
"Trailing edge" is at -3

– what does it look like if I shift it right by 2 time units? To do this, add $+2$ to the time axis

$x(2-\tau)$

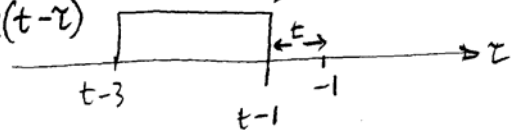


"leading edge" of rectangle is at 1
"trailing edge" is at -1

– what does $x(-\tau)$ look like if I shift it by t time units

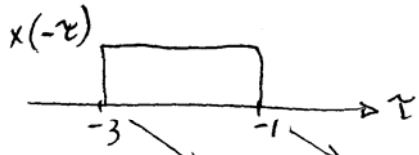
$x(-\tau)$

$x(t-\tau)$

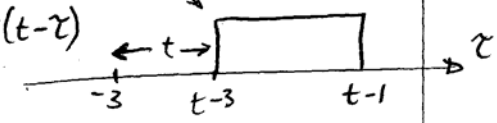


(if t is <0)

$x(-\tau)$



$x(t-\tau)$



(if $t>0$)

Important point: (for this problem) leading edge of this rectangle is at $t-1$, trailing edge of this rectangle is at $t-3$

(2) Convolution in CT

(A) $x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = x(t) * h(t) = h(t) * x(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

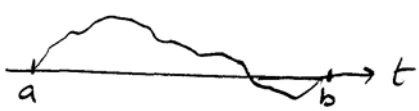
- for example, the output at time 0 is found by setting $t=0$ in this equation

$$y(0) = \int_{-\infty}^{\infty} x(\tau)\, h(-\tau)\, d\tau$$

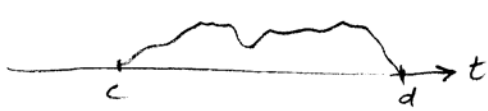
- to find the output at all times, must carry out the integration for every time t.

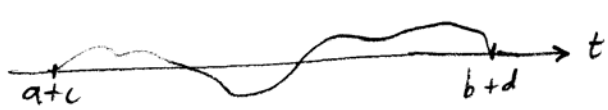
(B) A convolution fact:

- if this is signal $x(t)$ (starts at time a, ends at time b)



and this is signal $z(t)$: (starts at time c, ends at time d)



- Then $x(t) * z(t)$ will produce a result that starts at time $a+c$ and ends at time $b+d$

$x(t) * z(t)$: