Table of Laplace Transform Pairs

F.1 CAUSAL FUNCTIONS

$$\delta(t) \xleftrightarrow{\mathcal{L}} 1 \qquad \text{all } s$$

$$\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s} \qquad \mathrm{Re}(s) > 0$$

$$\mathrm{u}_{-n}(t) = \underbrace{\mathrm{u}(t) * \cdots * \mathrm{u}(t)}_{(n-1)\,\text{convolutions}} \xleftrightarrow{\mathcal{L}} \frac{1}{s^n} \qquad \mathrm{Re}(s) > 0$$

$$t\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s^2} \qquad \mathrm{Re}(s) > 0$$

$$e^{-\alpha t}\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+\alpha} \qquad \mathrm{Re}(s) > -\alpha$$

$$t^n\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{n!}{s^{n+1}} \qquad \mathrm{Re}(s) > 0$$

$$te^{-\alpha t}\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{1}{(s+\alpha)^2} \qquad \mathrm{Re}(s) > -\alpha$$

$$t^n e^{-\alpha t}\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{n!}{(s+\alpha)^{n+1}} \qquad \mathrm{Re}(s) > -\alpha$$

$$\sin(\beta t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{\beta}{s^2+\beta^2} \qquad \mathrm{Re}(s) > 0$$

$$\cos(\beta t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2+\beta^2} \qquad \mathrm{Re}(s) > 0$$

$$e^{-\alpha t}\sin(\beta t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{\beta}{(s+\alpha)^2+\beta^2} \qquad \mathrm{Re}(s) > -\alpha$$

$$e^{-\alpha t}\cos(\beta t)\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{s+\alpha}{(s+\alpha)^2+\beta^2} \qquad \mathrm{Re}(s) > -\alpha$$

$$e^{-\alpha t}\left[A\cos(\beta t)+\left(\frac{B-A\alpha}{\beta}\right)\sin(\beta t)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{As+B}{(s+\alpha)^2+\beta^2}$$

$$e^{-\alpha t}\left[\sqrt{A^2+\left(\frac{B-A\alpha}{\beta}\right)^2}\cos\left(\beta t-\tan^{-1}\left(\frac{B-A\alpha}{A\beta}\right)\right)\right]\mathrm{u}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{As+B}{(s+\alpha)^2+\beta^2}$$

$$e^{-(C/2)t}\left[A\cos\left(\sqrt{D-\left(\frac{C}{2}\right)^2}\,t\right)+\frac{2B-AC}{\sqrt{4D-C^2}}\sin\left(\sqrt{D-\left(\frac{C}{2}\right)^2}\,t\right)\right]\mathrm{u}(t)$$
$$\overset{\mathcal{L}}{\longleftrightarrow} \frac{As+B}{s^2+Cs+D}$$

$$e^{-(C/2)t}\left[\sqrt{A^2+\left(\frac{2B-AC}{\sqrt{4D-C^2}}\right)^2}\cos\left(\sqrt{D-\left(\frac{C}{2}\right)^2}\,t-\tan^{-1}\left(\frac{2B-AC}{A\sqrt{4D-C^2}}\right)\right)\right]\mathrm{u}(t)$$
$$\overset{\mathcal{L}}{\longleftrightarrow} \frac{As+B}{s^2+Cs+D}$$

F.2 ANTICAUSAL FUNCTIONS

$$-\mathrm{u}(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s} \qquad \mathrm{Re}(s)<0$$

$$-e^{-\alpha t}\mathrm{u}(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha} \qquad \mathrm{Re}(s)<-\alpha$$

$$-t^n\mathrm{u}(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{n!}{s^{n+1}} \qquad \mathrm{Re}(s)<0$$

F.3 NONCAUSAL FUNCTIONS

$$e^{-\alpha|t|} \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+\alpha}-\frac{1}{s-\alpha} \qquad -\alpha<\mathrm{Re}(s)<\alpha$$

$$\mathrm{rect}(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{e^{s/2}-e^{-(s/2)}}{s} \qquad \text{all } s$$

$$\mathrm{tri}(t) \overset{\mathcal{L}}{\longleftrightarrow} \left(\frac{e^{s/2}-e^{-(s/2)}}{s}\right)^2 \qquad \text{all } s$$