Properties of the Unilateral (One-sided) Laplace Transform

Linearity	$\alpha g(t) + \beta h(t) \xleftrightarrow{\mathcal{L}} \alpha G(s) + \beta H(s)$
Time shifting	$g(t - t_0) \xleftrightarrow{\mathcal{L}} G(s)e^{-st_0} \qquad t_0 > 0$
Complex-frequency shifting	$e^{at}g(t) \xleftrightarrow{\mathcal{L}} G(s - a)$
Time scaling	$g(at) \longleftrightarrow \frac{1}{a}G\left(\frac{s}{a}\right) \qquad a > 0$
Frequency scaling	$\frac{1}{a}g\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} G(as) \qquad a > 0$
Time differentiation once	$\frac{d}{dt}(g(t)) \xleftrightarrow{\mathcal{L}} sG(s) - g(0^-)$
Time differentiation twice	$\frac{d^2}{dt^2}(g(t)) \xleftrightarrow{\mathcal{L}} s^2G(s) - sg(0^-) - \frac{d}{dt}(g(t))_{t=0^-}$
Complex frequency differentiation	$-tg(t) \xleftrightarrow{\mathcal{L}} \frac{d}{ds}(G(s))$
Multiplication–convolution duality	$g(t) * h(t) \xleftrightarrow{\mathcal{L}} G(s)H(s)$
	$g(t)h(t) \xleftrightarrow{\mathcal{L}} \frac{1}{j2\pi}\int_{\sigma-j\infty}^{\sigma+j\infty} G(w)H(s-w)\,dw$
Integration	$\int_{0^-}^{t} g(\tau)\,d\tau \xleftrightarrow{\mathcal{L}} \frac{G(s)}{s}$
Initial value theorem	$g(0^+) = \lim_{s\to\infty} sG(s)$
Final value theorem	$\lim_{t\to\infty} g(t) = \lim_{s\to 0} sG(s) \qquad$ if $\lim_{t\to\infty} g(t)$ exists

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Some of the properties of the bilateral Laplace transform are not the same as the corresponding properties of the unilateral Laplace transform. The properties are summarized below without proof. The proofs are similar to the corresponding property proofs for the unilateral Laplace transform. One significant difference is that the region of convergence must be more carefully taken into account when applying the properties of the bilateral transform. Let $\mathrm{G}(s) = \mathcal{L}(\mathrm{g}(t))$ and $\mathrm{H}(s) = \mathcal{L}(\mathrm{h}(t))$ and let the ROC of G be R_G and let the ROC of H be R_H.

Linearity	$\alpha \mathrm{g}(t) + \beta \mathrm{h}(t) \xleftrightarrow{\mathcal{L}} \alpha \mathrm{G}(s) + \beta \mathrm{H}(s)$ $\mathrm{ROC} = R_G \cap R_H$	
Time shifting	$\mathrm{g}(t - t_0) \xleftrightarrow{\mathcal{L}} \mathrm{G}(s) e^{-s t_0}$	$\mathrm{ROC} = R_G$
Time scaling	$\mathrm{g}(at) \xleftrightarrow{\mathcal{L}} \frac{1}{\lvert a \rvert} \mathrm{G}\left(\frac{s}{a}\right)$	$\mathrm{ROC} = \frac{R_G}{a}$
Complex-frequency shifting	$e^{s_0 t} \mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \mathrm{G}(s - s_0)$	$\mathrm{ROC} = R_G$ shifted to the right by s_0
Frequency scaling	$\frac{1}{\lvert a \rvert} \mathrm{g}\left(\frac{t}{a}\right) \xleftrightarrow{\mathcal{L}} \mathrm{G}(as)$	$\mathrm{ROC} = a R_G$
Time differentiation	$\frac{d}{dt}(\mathrm{g}(t)) \xleftrightarrow{\mathcal{L}} s\mathrm{G}(s)$	$\mathrm{ROC} = R_G$ at least
Complex-frequency differentiation	$-t\mathrm{g}(t) \xleftrightarrow{\mathcal{L}} \frac{d}{ds}(\mathrm{G}(s))$	$\mathrm{ROC} = R_G$
Convolution	$\mathrm{g}(t) * \mathrm{h}(t) \xleftrightarrow{\mathcal{L}} \mathrm{G}(s)\mathrm{H}(s)$ $\mathrm{ROC} = R_G \cap R_H$ at least	
Integration	$\int_{-\infty}^{t} \mathrm{g}(\tau)\, d\tau \xleftrightarrow{\mathcal{L}} \frac{\mathrm{G}(s)}{s}$ $\mathrm{ROC} = R_G \cap \{\mathrm{Re}(s) > 0\}$ at least	