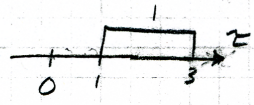
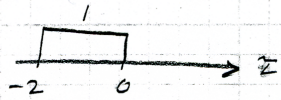
① $x(t) = \text{rect}\left(\frac{t-2}{2}\right)$ $\quad h(t) = \text{rect}\left(\frac{t+1}{2}\right)$

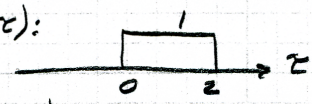
$x(\tau)$:



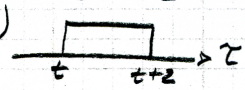
$h(\tau)$:



$h(-\tau)$:

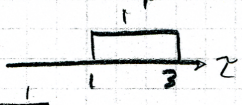


$h(t-\tau)$

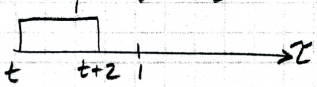


case 1: no overlap to the left

$x(\tau)$



$h(t-\tau)$

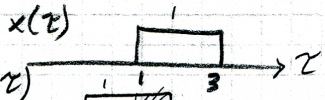


$$y(t) = \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau = \boxed{0}$$

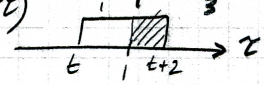
for $\boxed{t \le -1}$

case 2: overlap w/ leading edge

$x(\tau)$



$h(t-\tau)$

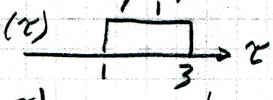


$$y(t) = \int_{1}^{t+2} 1 \cdot 1\,dt = t\Big|_{1}^{t+2} = t+2-1 = \boxed{t+1}$$

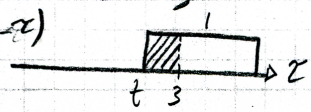
for $\boxed{-1 \le t \le 1}$

case 3: overlap w/ trailing edge

$x(\tau)$



$h(t-\tau)$

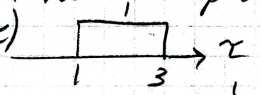


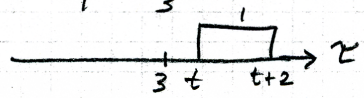
$$y(t) = \int_{t}^{3} 1 \cdot 1\,dt = t\Big|_{t}^{3} = \boxed{3-t}$$

for $\boxed{1 \le t \le 3}$

case 4: no overlap to the right

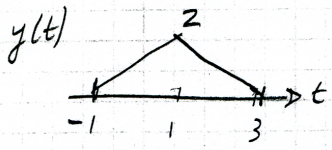
$x(\tau)$



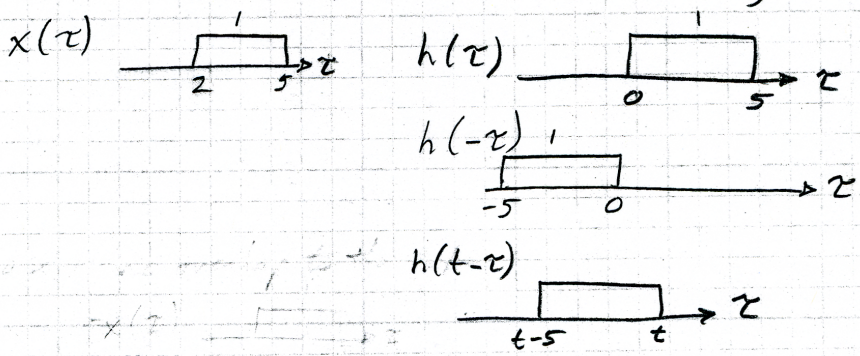


$\boxed{y(t) = 0}$, for $\boxed{t \ge 3}$

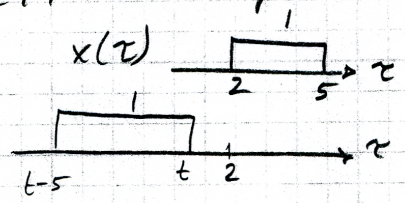
overall: $y(t) = \begin{cases} 0, & t \le -1 \\ t+1, & -1 \le t \le 1 \\ 3-t, & 1 \le t \le 3 \\ 0, & t \ge 3 \end{cases}$



(2) $x(t) = \mathrm{rect}\left(\frac{t-3}{2}\right)$, $h(t) = u(t) - u(t-5)$



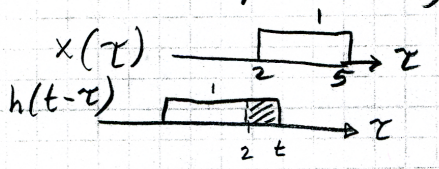
case 1: no overlap, left



$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \boxed{0}$

for $\boxed{t \le 2}$

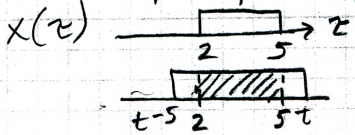
case 2: overlap w/leading edge



$y(t) = \int_2^t 1 \cdot 1\,dt = t\Big|_2^t = \boxed{t-2}$

for $\boxed{2 \le t \le 5}$

case 3: complete overlap



$y(t) = \int_2^5 1 \cdot 1\,dt = t\Big|_2^5 = 5-2 = \boxed{3}$

for $\boxed{5 \le t \le 7}$

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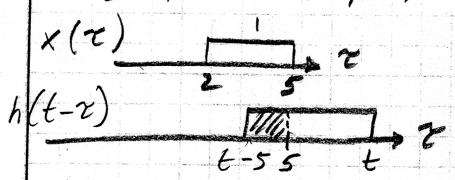
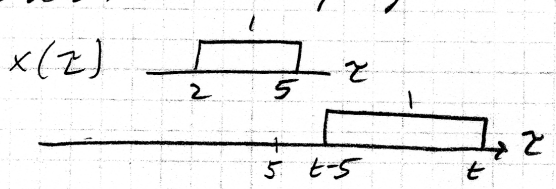
Case 4: overlap w/ trailing edge



$$y(t) = \int_{t-5}^{5} 1 \cdot 1 \, dt = t \Big|_{t-5}^{5} = 5 - (t-5) = \boxed{10-t}$$

for $\boxed{7 \le t \; 10}$

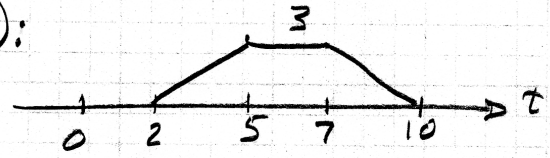
case 5: no overlap right



$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau = \boxed{0}$$

$t \ge 10$

overall,

$$y(t) = \begin{cases} 0, & t \le 0 \\ t-2, & 2 \le t \le 5 \\ 3, & 5 \le t \le 7 \\ 10-t, & 7 \le t \le 10 \\ 0, & t \ge 10 \end{cases}$$

y(t):