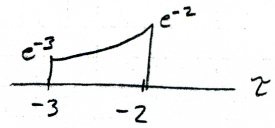
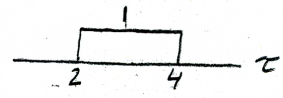
(1) $x(t) = \mathrm{rect}\left(\frac{t-3}{2}\right)$ $\quad h(t) = e^{t}\left[u(t+3) - u(t+2)\right]$

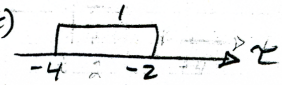
$h(\tau)$:



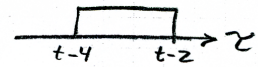
$x(\tau)$



$x(-\tau)$

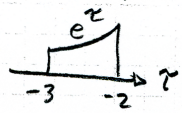


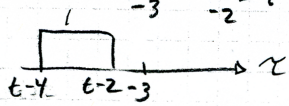
$x(t-\tau)$



<u>case 1: no overlap left</u>

$h(\tau)$



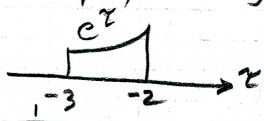


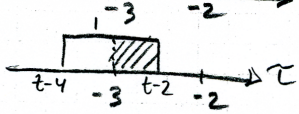
$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau = \boxed{0}$$

for $\boxed{t \le -1}$

<u>Case 2: overlap w/ leading edge</u>

$h(\tau)$



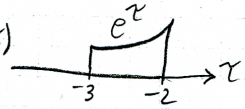


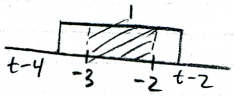
$$y(t) = \int_{-3}^{t-2} e^{\tau}\,d\tau = e^{\tau}\Big|_{-3}^{t-2} = \boxed{e^{t-2} - e^{-3}}$$

for $\boxed{-1 \le t \le 0}$

<u>case 3: complete overlap</u>

$h(\tau)$



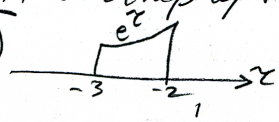


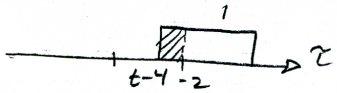
$$y(t) = \int_{-3}^{-2} e^{\tau}\,d\tau = e^{\tau}\Big|_{-3}^{-2} = \boxed{e^{-2} - e^{-3}}$$

for $\boxed{0 \le t \le 1}$

<u>case 4: overlap w/ trailing edge</u>

$h(\tau)$





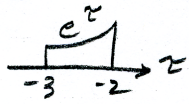
$$y(t) = \int_{t-4}^{-2} e^{\tau}\,d\tau = e^{\tau}\Big|_{t-4}^{-2} = \boxed{e^{-2} - e^{t-4}}$$

for $\boxed{1 \le t \le 2}$

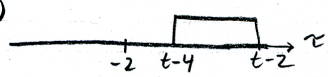
TOPS. 35500

<u>Case 5</u>: no overlap right

$h(\tau)$



$x(t-\tau)$



$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \boxed{0}$$

for $\boxed{t \geq 2}$

overall; $y(t) = \begin{cases} 0, & t \leq -1 \\ e^{t-2} - e^{-3}, & -1 \leq t \leq 0 \\ e^{-2} - e^{-3}, & 0 \leq t \leq 1 \\ e^{-2} - e^{t-4}, & 1 \leq t \leq 2 \\ 0, & t \geq 2 \end{cases}$

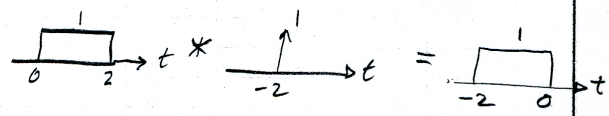
(2) $x(t) = \text{rect}\left(\frac{t-1}{2}\right)$ $\quad h(t) = \delta(t+2) + u(t)$

$$x(t) * h(t) = \text{rect}\left(\tfrac{t-1}{2}\right) * \left(\delta(t+2) + u(t)\right)$$

$$= \text{rect}\left(\tfrac{t-1}{2}\right) * \delta(t+2) + \text{rect}\left(\tfrac{t-1}{2}\right) * u(t)$$

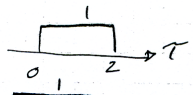
do in two parts:

1- $\text{rect}\left(\frac{t-1}{2}\right) * \delta(t+2) =$

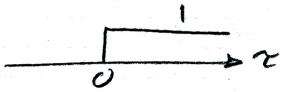


2- $\text{rect}\left(\frac{t-1}{2}\right) * u(t)$

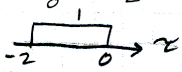
$x(\tau)$



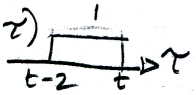
$u(\tau)$



$x(-\tau)$

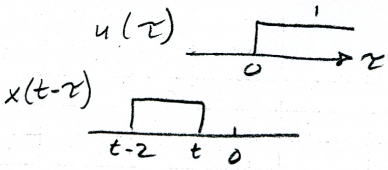


$x(t-\tau)$



TOPS. 35500

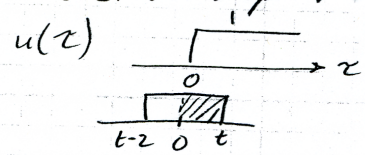
case 1: no overlap left



$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, x(t-\tau)\, d\tau = \boxed{0}$$

for $\boxed{t \le 0}$

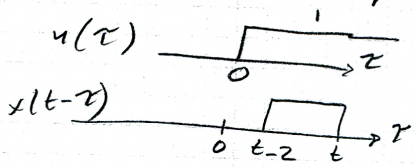
Case 2: overlap w/ leading edge



$$y(t) = \int_0^t 1\, d\tau = \tau \Big|_0^t = \boxed{t}$$

for $\boxed{0 \le t \le 2}$

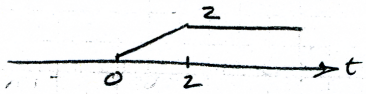
case 3: complete overlap



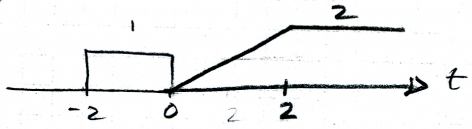
$$y(t) = \int_{t-2}^{t} 1\, d\tau = \tau \Big|_{t-2}^{t} = \boxed{2}$$

for $\boxed{t \ge 2}$

$x(t) * u(t)$ looks like:



so $h(t) * x(t)$ looks like



(3) BIBO stable if $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |\delta(t-1) + 0.5\,\delta(t-2) + \mathrm{rect}(t-4)|\, dt$$

$$= 1 + .5 + 1 = 2.5 < \infty$$

$\Rightarrow$ $\boxed{\text{BIBO stable}}$

(4) $h(t) = -t^{-3} u(t-1)$

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{1}^{\infty} |t^{-3}|\,dt = -3t^{-4}\Big|_{1}^{\infty} = -\frac{3}{\infty^4} - -3$$

$$= 0 - -3 = 3 < \infty$$

$\Rightarrow$ $\boxed{\text{BIBO stable}}$

TOPS. 35500