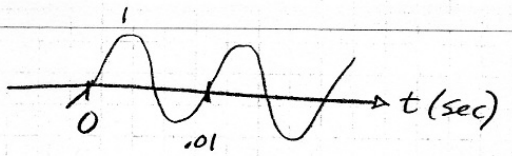
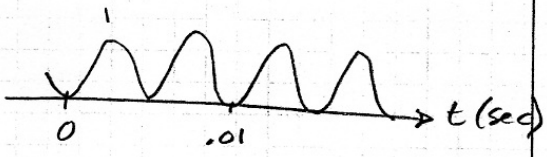
(1) a) $x(t) = \sin(2\pi 100 t)$

$f_0 = 100$ Hz

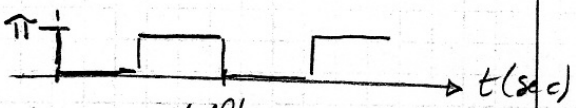
$T_0 = 0.01$ sec



$|x(t)|$



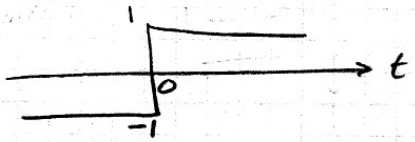
$\angle x(t)$ (rad)



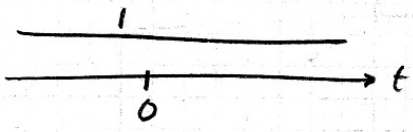
note: phase could be $\pm\pi$ when sine is negative

b) $x(t) = j\,\mathrm{sgn}(t)$ note: the j causes a phase shift of $\frac{\pi}{2}$ radians to $\mathrm{sgn}(t)$, but will not affect magnitude.

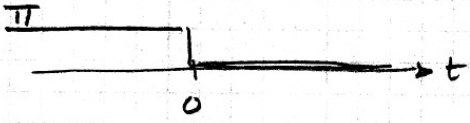
$\mathrm{sgn}(t)$:



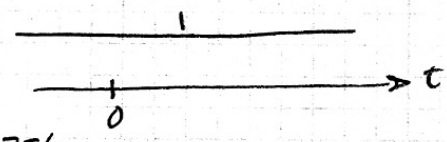
$|\mathrm{sgn}(t)|$



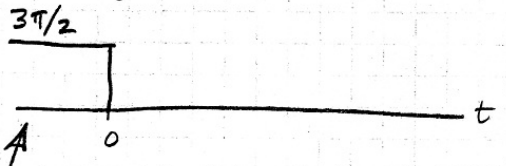
$\angle \mathrm{sgn}(t)$



so $|j\,\mathrm{sgn}(t)|$



$\angle j\,\mathrm{sgn}(t)$



note: this phase could be $-\frac{\pi}{2}$ also

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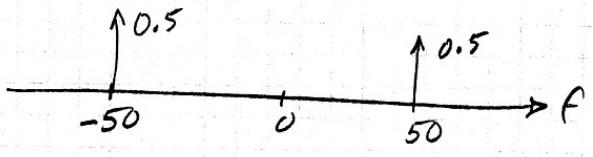
c) $\underline{X}(f) = -0.5j\,[\delta(f-50) - \delta(f+50)]$

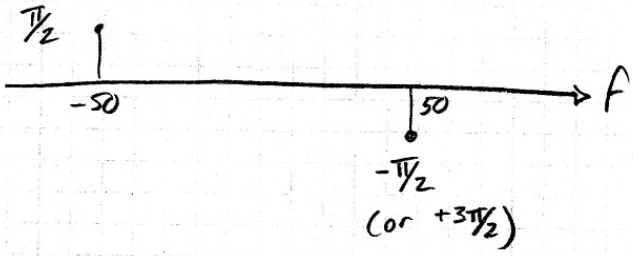
here, the 0.5 scales the area of the impulses, the negative signs introduce a phase shift of π, and the j introduces a phase shift of $\frac{\pi}{2}$

$$\underline{X}(f) = -0.5j\,\delta(f-50) + 0.5j\,\delta(f+50)$$

$|\underline{X}(f)|$:



$\angle \underline{X}(f)$



(2) $x(t) = \overbrace{\cos(2\pi 10t)}^{\text{real part}} + j\overbrace{\sin(2\pi 10t)}^{\text{imag part}}$

$$|x(t)| = \sqrt{\cos(2\pi 10t)^2 + \sin(2\pi 10t)^2} = \boxed{1} \quad (\text{trig identity: } \cos^2 A + \sin^2 A = 1)$$

$$\angle x(t) = \tan^{-1}\left(\frac{\sin(2\pi 10t)}{\cos(2\pi 10t)}\right) = \tan^{-1}(\tan(2\pi 10t))$$

$$= \boxed{2\pi 10t}$$

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(3) $2e^{j20t}$

(a) mag funct: 2

(b) phase funct: $20t$

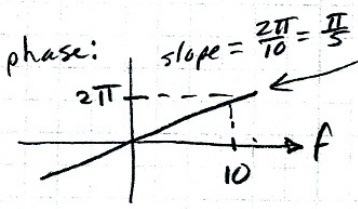
(c) real funct: $2\cos(20t)$

(d) imag funct: $2\sin(20t)$

(4) any complex exponential can be written as

$$R(f)\,e^{j\theta(f)}$$

here, $R(f) = 3$ = magnitude



The equation for the phase plot is $\theta(f) = \frac{\pi}{5}f = 0.2\pi f$

so the answer is $\boxed{3\,e^{j0.2\pi f}}$

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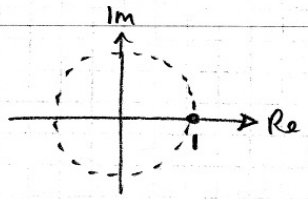
Bonus

(5) $x^6 = 1$

rewrite 1 as $e^{j2\pi n}$ (n is an integer)

(magnitude is 1, phase is any integer multiple of 2π)

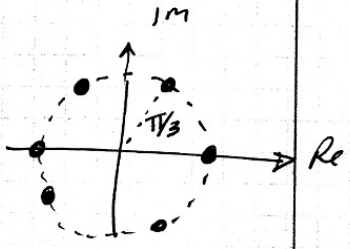


so $x^6 = e^{j2\pi n}$ take the $1/6^{th}$ root

$$x = \left(e^{j2\pi n}\right)^{1/6} = e^{j\frac{2\pi n}{6}}$$

- if we let $n = 0, 1, 2, 3, 4, 5$, these plot as six points around the unit circle on the complex plane.
- if you let n be any other integer value, it will plot on top of one of those six points already identified.
- see if these solve $x^6 = 1$:

n	x_n	$(x_n)^6$
0	$e^{j0} = 1$	$1^6 = 1$
1	$e^{j\frac{2\pi}{6}}$	$\left(e^{j\frac{2\pi}{6}}\right)^6 = e^{j2\pi} = 1$
2	$e^{j4\pi/6}$	$\left(e^{j4\pi/6}\right)^6 = e^{j4\pi} = 1$
3	$e^{j6\pi/6} = -1$	$(-1)^6 = 1$
4	$e^{j8\pi/6}$	$\left(e^{j8\pi/6}\right)^6 = e^{j8\pi} = 1$
5	$e^{j10\pi/6}$	$\left(e^{j10\pi/6}\right)^6 = e^{j10\pi} = 1$



each of these solve $x^6 = 1$

The 6 roots of 1 are evenly spaced around the unit circle, separated by $\pi/3$ radians in phase.

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