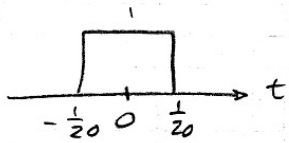
① $x(t) = \text{rect}(10t)$



$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t} = \int_{-\frac{1}{20}}^{\frac{1}{20}} e^{-j2\pi f t}\, dt$$

$$= -\frac{1}{j2\pi f} e^{-j2\pi f t}\Big|_{t=-\frac{1}{20}}^{\frac{1}{20}} = \frac{-1}{\pi f}\left(\frac{1}{2j}\right)\left(e^{-j2\pi f \frac{1}{20}} - e^{j2\pi f \frac{1}{20}}\right)$$

$$= \frac{1}{\pi f}\, \frac{1}{2j}\left(e^{j\pi f/10} - e^{-j\pi f/10}\right)$$

$$= \frac{1}{\pi f} \sin(\pi f/10) = 1/10\, \frac{\sin(\pi f/10)}{\pi f/10}$$

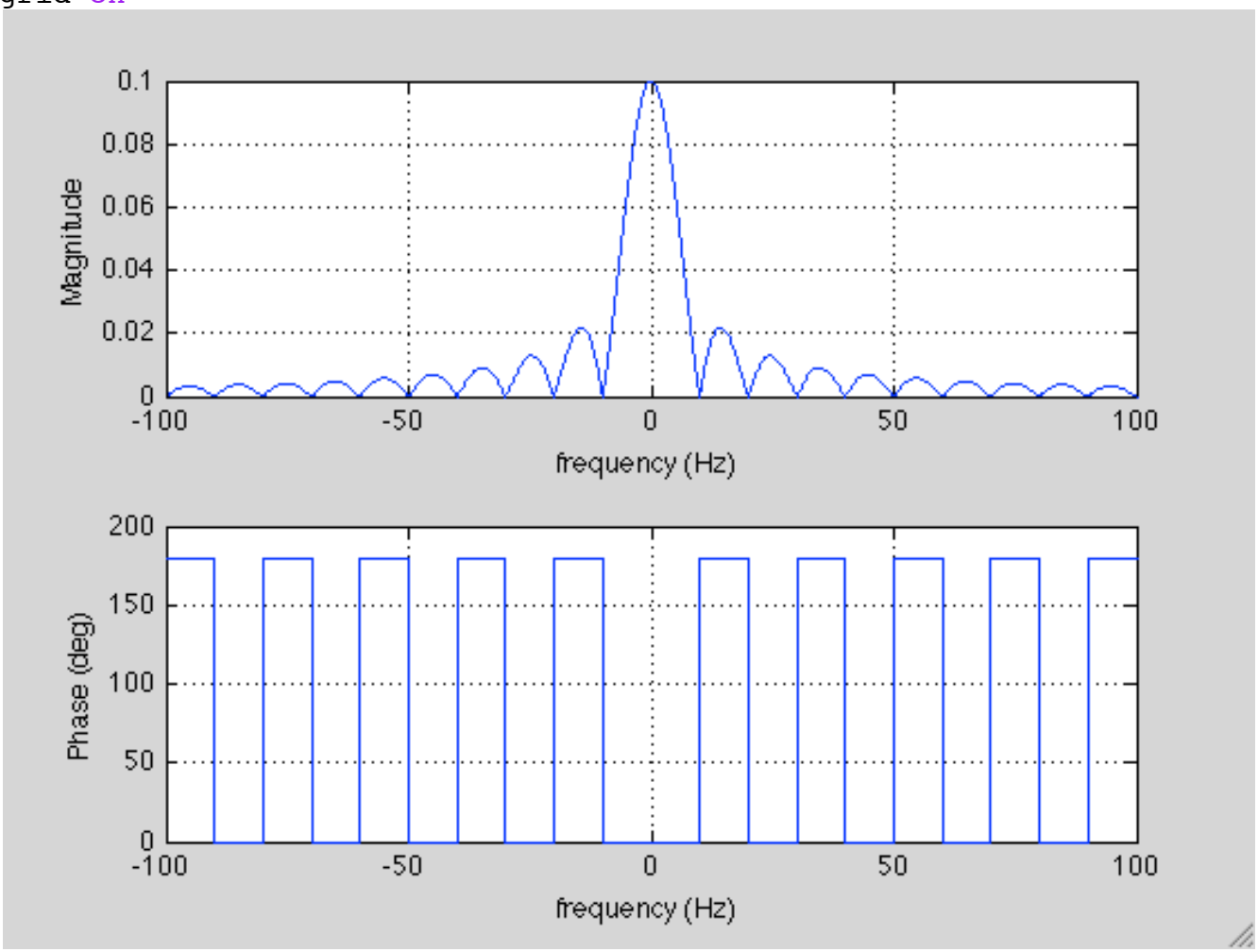
$$= \boxed{\frac{1}{10} \operatorname{sinc}(f/10)}$$

- The phase plot is 0° when the sinc has a positive value, and 180° when the sinc has a negative value.

- This is the frequency content of $x(t)$ over <u>all</u> time. In problem set 23, we were analyzing frequency only over a short time. We computed the coefficients of complex exponentials, which, when added together would result in that same rectangle over that same time. Those coefficient values followed a sinc shape.

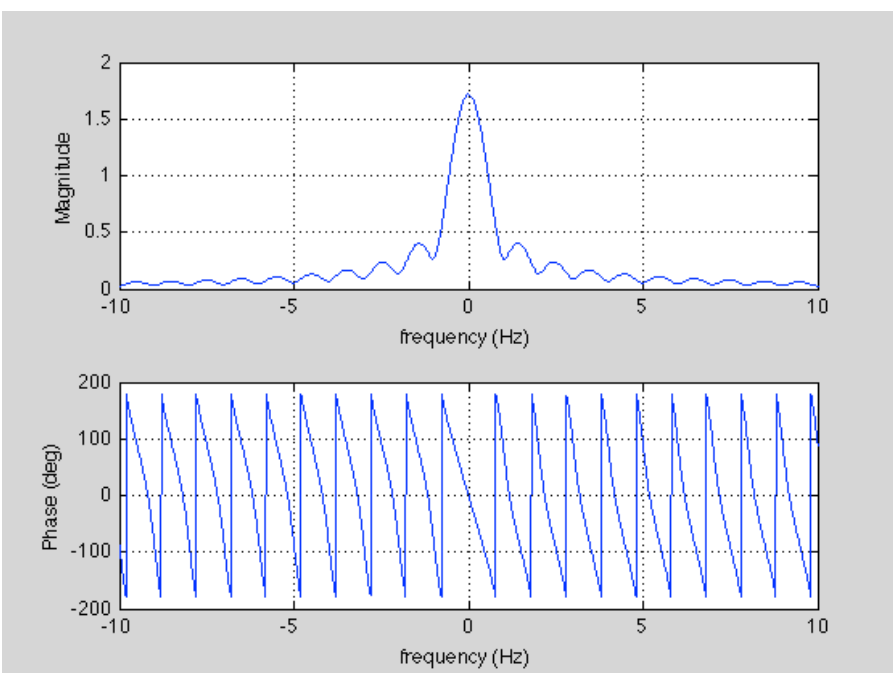
```
% EE322 PS25 Problem 1
f=-100:.01:100;
X=1/10*sinc(f/10);
figure(1),subplot(2,1,1)
plot(f,abs(X)),xlabel('frequency (Hz)'), ylabel('Magnitude'),grid on

subplot(2,1,2)
plot(f,angle(X)*180/pi),xlabel('frequency (Hz)'),ylabel('Phase
(deg)')
grid on
```



② $$X(f) = \int_0^1 e^{t} e^{-j2\pi f t}\,dt = \int_0^1 e^{(1-j2\pi f)t}\,dt$$

$$= \frac{1}{1-j2\pi f}\left(e^{(1-j2\pi f)t} \Big|_{t=0}^{1} \right)$$

$$= \boxed{\frac{1}{1-j2\pi f}\left(e^{1-j2\pi f} - 1 \right)}$$

```
% EE322 PS25 Problem 2
f=-10:.01:10;
X=1./(1-j*2*pi*f).*(exp(1-j*2*pi*f)-1);
figure(2),subplot(2,1,1)
plot(f,abs(X)),xlabel('frequency (Hz)'), ylabel('Magnitude'),grid on

subplot(2,1,2)
plot(f,angle(X)*180/pi),xlabel('frequency (Hz)'),ylabel('Phase
(deg)')
grid on
```