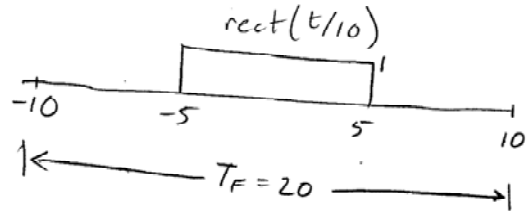
EE322 Fall 2008 PS26-Solutions

(1)



<u>soln</u> $T_F = 20$ so $f_F = \frac{1}{20}$

$$X[k] = \frac{1}{T_F}\int_{t_0}^{t_0+T_F} x(t)\, e^{-j2\pi(kf_F)t}\, dt$$

$$= \frac{1}{20}\int_{-10}^{10} 1\cdot e^{-j2\pi(k\cdot\frac{1}{20})t}\, dt = \frac{1}{20}\int_{-5}^{5} e^{-j2\pi(\frac{k}{20})t}\, dt$$

$$= \frac{1}{\cancel{20}}\left(-\frac{1}{j2\pi k\frac{1}{\cancel{20}}}\right) e^{-j2\pi(\frac{k}{20})t}\Big|_{t=-5}^{t=5} = -\frac{1}{j2\pi k}\left[e^{-j2\pi\frac{k\cdot 5}{20}} - e^{-j2\pi\frac{k}{20}(-5)}\right]$$

$$= -\frac{1}{\pi k}\frac{1}{2j}\left[e^{-j\frac{\pi k}{2}} - e^{+j\frac{\pi k}{2}}\right]$$

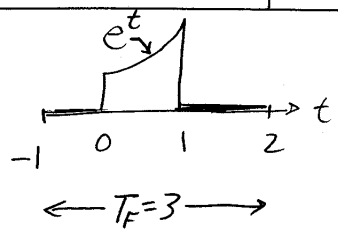
$$X[k] = \frac{1}{\pi k}\left[\frac{e^{+j\frac{\pi k}{2}} - e^{-j\frac{\pi k}{2}}}{2j}\right] \qquad \boxed{\frac{1}{\pi k}\sin\left(\frac{\pi k}{2}\right)}$$

or

$$X[k] = \frac{1}{2}\,\frac{2}{\pi k}\sin\left(\frac{\pi k}{2}\right) = \frac{1}{2}\,\frac{\sin(\pi k/2)}{(\pi k/2)}$$

$$= \boxed{\frac{1}{2}\,\mathrm{sinc}\left(\frac{\pi k}{2}\right)}$$

(2)



$T_F = 3$ so $f_F = \frac{1}{3}$

$$X[k] = \frac{1}{T_F}\int_{t_0}^{t_0+T_F} x(t)\, e^{-j2\pi(kf_F)t}\, dt$$

$$= \frac{1}{3}\int_{-1}^{2} e^{t} e^{-j2\pi(kf_F)t}\, dt = \frac{1}{3}\int_{0}^{1} e^{(1-j2\pi k/3)t}\, dt$$

$$= \frac{1}{3}\frac{1}{(1-j2\pi k/3)}\, e^{(1-j2\pi k/3)t}\Big|_{t=0}^{t=1}$$

$$\boxed{X[k] = \frac{1}{3-j2\pi k}\left[e^{1-j2\pi k/3} - 1\right]}$$

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