① Text, 10-3b

$$x(t) = \delta(t+\tfrac{1}{2}) - \delta(t-\tfrac{1}{2})$$

$$X(f) = \int_{-\infty}^{\infty} [\delta(t+\tfrac{1}{2}) - \delta(t-\tfrac{1}{2})]\, e^{-j2\pi f t}\, dt$$

Recall: $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

$$= e^{-j2\pi f(-\frac{1}{2})} - e^{-j2\pi f(+\frac{1}{2})}$$

$$= e^{j\pi f} - e^{-j\pi f}$$

$$= \frac{2j}{2j}\left(e^{j\pi f} - e^{-j\pi f}\right) = \boxed{2j\sin(\pi f)}$$

② Text, 10-20b

$$g(t) = 8\,\mathrm{rect}(t/3)$$

$$G(f) = \int_{-\infty}^{\infty} 8\,\mathrm{rect}(t/3)\, e^{-j2\pi f t}\, dt = 8\int_{-1.5}^{1.5} e^{-j2\pi f t}\, dt$$

$$= 8\left(-\frac{1}{j2\pi f}\right)\left(e^{-j2\pi f t}\Big|_{t=-1.5}^{1.5}\right)$$

$$= 8\left(-\frac{1}{j2\pi f}\right)\left(e^{-j3\pi f} - e^{+j3\pi f}\right) = \frac{8}{\pi f}\left(\frac{e^{j3\pi f} - e^{-j3\pi f}}{2j}\right)$$

$$= 8\,\frac{\sin(3\pi f)}{\pi f} = 8\cdot\frac{3}{3}\,\frac{\sin(3\pi f)}{\pi f} = \boxed{24\,\mathrm{sinc}(3f)}$$

(3) $2\cos(2\pi 20t) + \cos(2\pi 30t)$

$f_1 = 20$ $\quad$ $f_2 = 30$

$T_1 = \frac{1}{20}$ $\quad$ $T_2 = \frac{1}{30}$

$$T_F = \mathrm{Lcm}\left(\frac{1}{20}, \frac{1}{30}\right) = \frac{\mathrm{Lcm}\left(\frac{60}{20}, \frac{60}{30}\right)}{60} = \frac{\mathrm{Lcm}(3,2)}{60}$$

$$= \frac{6}{60} = \frac{1}{10}$$

so $f_F = 10$

$$2\cos 2\pi 20t + \cos 2\pi 30t =$$

$$2\,\frac{1}{2}\left(e^{j2\pi 20t} + e^{-j2\pi 20t}\right) + \frac{1}{2}\left(e^{j2\pi 30t} + e^{-j2\pi 3}\right.$$

$$= e^{j2\pi(2)10t} + e^{j2\pi(-2)10t} + \frac{1}{2}e^{j2\pi(3)10t} + \frac{1}{2}e^{j2\pi(-3)10t}$$

$X[2] = 1$ $\quad$ $X[-2] = 1$ $\quad$ $X[3] = \frac{1}{2}$ $\quad$ $X[-3] = \frac{1}{2}$

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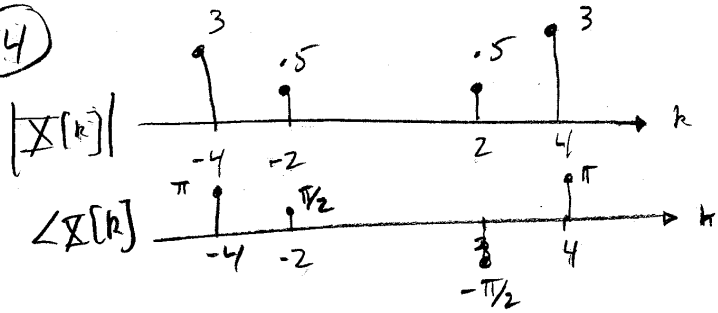
(4)



$f_F = 100\,Hz$

$X[-4] = 3e^{j\pi} = -3$

$X[4] = 3e^{j\pi} = -3$

$X[-2] = \frac{1}{2}e^{j\pi/2} = \frac{j}{2}$

$X[2] = \frac{1}{2}e^{-j\pi/2} = \frac{-j}{2}$

$X[k] = 0$ for all other k

$$x(t) = X[-4]e^{j2\pi(-4)f_F t} + X[4]e^{j2\pi(4)f_F t} + X[-3]e^{j2\pi(-2)f_F t} + X[3]e^{j2\pi(2)f_F t}$$

$$= -3e^{-j2\pi 400t} - 3e^{+j2\pi 400t} + \frac{j}{2}e^{-j2\pi 200t} - \frac{j}{2}e^{j2\pi 200t}$$

$$= -3\left(e^{j2\pi 400t} + e^{-j2\pi 400t}\right) + \frac{j}{2}\left(e^{-j2\pi 200t} - e^{+j2\pi 200t}\right)$$

$$= -6\cos(2\pi 400t) + \frac{j}{j}\frac{j}{2}\left(e^{-j2\pi 200t} - e^{+j2\pi 200t}\right)$$

$$= -6\cos(2\pi 400t) + \frac{e^{j2\pi 200t} - e^{-j2\pi 200t}}{2j}$$

$$\boxed{= -6\cos(2\pi 400t) + \sin(2\pi 200t)}$$