① Text 15-2a,b,c

(a) $x(t) = e^{t}u(t)$

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt = \int_{0}^{\infty} e^{t}e^{-st}dt = \int_{0}^{\infty} e^{(1-s)t}dt$$

$$= \frac{1}{1-s}e^{(1-s)t}\Big|_{t=0}^{\infty} = \frac{1}{1-s}e^{(1-\sigma)t}e^{-j\omega t}\Big|_{t=0}^{\infty}$$

$$= \frac{1}{1-s}\Big[\underbrace{e^{(1-\sigma)\infty}e^{-j\omega\infty}}_{=0 \text{ if } 1-\sigma<0} - \underbrace{e^{(1-\sigma)0}e^{-j\omega 0}}_{1}\Big] = -\frac{1}{1-s}$$

so ROC is $1-\sigma<0$, or $\sigma>1$

$$\boxed{X(s) = \frac{1}{s-1} \quad \text{ROC: } Re(s) > 1}$$

(b) $x(t) = e^{2t}\cos(200\pi t)u(t)$

$$= e^{2t}\Big[\frac{1}{2}e^{j200\pi t} + \frac{1}{2}e^{-j200\pi t}\Big]u(t)$$

$$= \Big[\frac{1}{2}e^{(2+j200\pi)t} + \frac{1}{2}e^{(2-j200\pi)t}\Big]u(t)$$

$$X(s) = \int_{-\infty}^{\infty} \frac{1}{2}e^{(2+j200\pi)t}e^{-st}u(t) + \int_{-\infty}^{\infty} \frac{1}{2}e^{(2-j200\pi)t}e^{-st}u(t)\,dt$$

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$$= \underbrace{\frac{1}{2}\int_0^\infty e^{(2+j200\pi-s)t}\,dt}_{\text{integral \#1}} + \underbrace{\frac{1}{2}\int_0^\infty e^{(2-j200\pi-s)t}\,dt}_{\text{integral \#2}}$$

integral #1: $\frac{1}{2}\int_0^\infty e^{(2+j200\pi-s)t}\,dt = \frac{1}{2(2+j200\pi-s)} e^{(2+j200\pi-\sigma-j\omega)t}\Big|_{t=0}^{\infty}$

$$= \frac{1}{2(2+j200\pi-s)}\left[e^{(2-\sigma)t}\, e^{(j200\pi-j\omega)t}\right]\Big|_{t=0}^{\infty}$$

$$= \frac{1}{2(2+j200\pi-s)}\left[\underbrace{e^{(2-\sigma)\infty}\, e^{(j200\pi-j\omega)\infty}}_{=0 \text{ if } 2-\sigma<0} - e^{(2-\sigma)0}\, e^{(j200\pi-j\omega)0}\right]$$

so ROC is $\sigma > 2$

similarly for integral 2,

integral #2 $= \frac{1}{2(2-j200\pi+s)}$ ROC $\sigma > 2$

overall, $X(s) = \frac{1}{2}\left[\frac{1}{2+j200\pi-s} + \frac{1}{2-j200\pi-s}\right]$

ROC: $\sigma > 2$

OR by simplifying, $= \frac{1}{2}\,\frac{s-2}{(s-2)^2+(200\pi)^2}$

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(c) $x(t) = \text{ramp}(t) = t\,u(t)$

$$X(s) = \int_{-\infty}^{\infty} t\,u(t)e^{-st}\,dt = \int_{0}^{\infty} t\,e^{-st}\,dt$$

$$= \frac{e^{-st}}{(-s)^2}(-st-1)\Big|_{t=0}^{\infty}$$

$$= \underbrace{\frac{e^{-s\infty}}{(-s)^2}(-s\infty-1)}_{\substack{=0 \text{ as long} \\ \text{as } Re(s)>0}} - \underbrace{\frac{e^{-s0}}{(-s)^2}(-0\cdot t-1)}_{\frac{1}{s^2}}$$

$$\boxed{X(s) = \frac{1}{s^2} \quad Re(s) > 0}$$

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