① Text 15-31

$$g(t) \longleftrightarrow \frac{s+1}{s(s+4)}$$

(a) $$g(2t) \longleftrightarrow \frac{1}{2}\,\frac{\frac{s}{2}+1}{\frac{s}{2}\left(\frac{s}{2}+4\right)}\,\frac{4}{4} = \boxed{\frac{(s+2)}{s(s+8)}}$$

(b) $$\frac{d}{dt}g(t) \longleftrightarrow \frac{\cancel{s}(s+1)}{\cancel{s}(s+4)} - g(0^-)$$

Note: initial value Theorem can give $g(0)$

$$g(0^+) = \lim_{s\to\infty} sG(s) = \lim_{s\to\infty} \frac{\cancel{s}(s+1)}{\cancel{s}(s+4)} = 1$$

so

$$\frac{d}{dt}g(t) \longleftrightarrow \frac{s+1}{s+4} - 1 = \frac{s+1-(s+4)}{s+4}$$

$$= \boxed{\frac{-3}{s+4}}$$

(c) $$g(t-4) \longleftrightarrow \boxed{\frac{s+1}{s(s+4)}e^{-4s}}$$

(d) $$g(t) * g(t) \longleftrightarrow G(s)G(s) = \boxed{\frac{(s+1)^2}{s^2(s+4)^2}}$$

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(2) Text 15-34

$$e^{-4t}u(t) \longleftrightarrow G(s) =$$

(a) $G(s/3) \xleftrightarrow{\text{complex freq scaling}} \frac{1}{(\frac{1}{3})} g\left(\frac{t}{1/3}\right) = 3e^{-4(3t)}u(3t)$

$a = \frac{1}{3}$

$$= \boxed{3e^{-12t}u(t)}$$

note: $u(3t)$ plots the same as $u(t)$

(b) $G(s-2) + G(s+2) \longleftrightarrow e^{2t}g(t) + e^{-2t}g(t)$

Frequency shifting

$$= \left[e^{2t}e^{-4t} + e^{-2t}e^{-4t}\right]u(t)$$

$$= \boxed{\left[e^{-2t} + e^{-6t}\right]u(t)}$$

(c) $\frac{G(s)}{s} \longleftrightarrow \int_0^t g(\tau)\,d\tau = \int_0^t e^{-4\tau}u(\tau)\,d\tau$

$$= \int_0^t e^{-4\tau}\,d\tau$$

$$= -\frac{1}{4}e^{-4\tau}\Big|_{\tau=0}^{t}$$

$$= -\frac{1}{4}e^{-4t} - -\frac{1}{4}$$

$$= \boxed{\frac{1}{4}\left(1 - e^{-4t}\right)u(t)}$$

note: integral was zero for $t \geq 0$

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(3) Text 15-48c, d

(c) $x(t) = u(t), \quad H(s) = \frac{3s}{s+2}$

$$Y(s) = H(s)X(s) = \frac{3\cancel{s}}{s+2}\,\frac{1}{\cancel{s}} = \frac{3}{s+2}$$

$$y(t) = \boxed{3e^{-2t}u(t)}$$

(d) $x(t) = u(t) \quad H(s) = \frac{5s}{s^2+2s+2}$

$$Y(s) = X(s)H(s) = \frac{1}{\cancel{s}}\,\frac{5\cancel{s}}{s^2+2s+2} = \frac{5}{s^2+2s+2}$$

$$= \frac{5}{(s^2+2s+1)+1}$$

$$= \frac{5}{(s+1)^2+1^2}$$

$$\boxed{y(t) = 5e^{-t}\sin(t)\,u(t)}$$

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