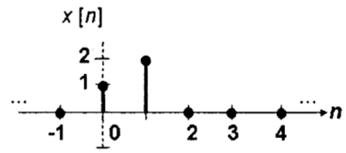
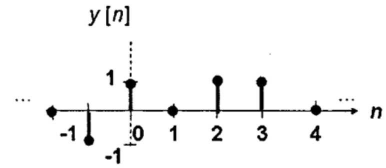
EE322 Fall 2012 Discrete Convolution Worksheet-Solutions

1. Evaluate the convolution of $z[n] = x[n]*y[n]$, given plots of $x[n]$ and $y[n]$ as shown to the right:



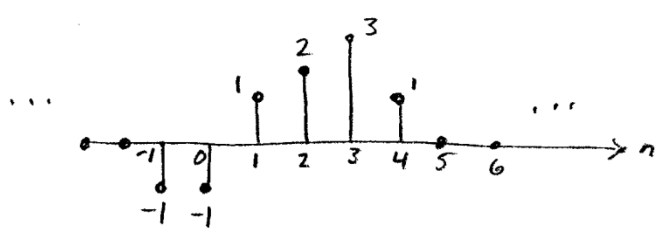


k	-3	-2	-1	0	1	2	3	4	5
$y[k]$			-1	1	0	1	1		
$x[-k]$			2	1					

shift $x[-k]$ left by 1 to begin

k	-3	-2	-1	0	1	2	3	4	5	
$y[k]$			-1	1	0	1	1			
$x[-1-k]$		2	1							$z[-1] = -1$
$x[0-k]$			2	1						$z[0] = -2+1 = -1$
$x[1-k]$				2	1					$z[1] = 2$
$x[2-k]$					2	1				$z[2] = 1$
$x[3-k]$						2	1			$z[3] = 1+2 = 3$
$x[4-k]$							2	1		$z[4] = 2$

$$z[n] = -\delta[n+1] - \delta[n] + 2\delta[n-1] + \delta[n-2] + 3\delta[n-3] + 2\delta[n-4]$$



2. Find the impulse response of a system that can be described by the difference equation:

$$y[n] = 3x[n+1] - 2x[n] + x[n-2].$$

Recall that finding the impulse response involves setting the input to be a unit impulse and monitoring the output.

let $x[n] = \delta[n]$

then $\boxed{h[n] = 3\delta[n+1] - 2\delta[n] + \delta[n-2]}$

Is this system BIBO stable?

$$\sum_{n=-\infty}^{\infty} |h[n]| = 3 + 2 + 1 = 6 < \infty$$

$\boxed{\text{BIBO stable}}$

3. Using a table, find the impulse response of a system that can be described by the difference equation: $y[n] = x[n] - 1.1y[n-1]$. Write your answer in a closed form expression.

n	$x[n] = \delta[n]$	$-1.1y[n-1]$	$y[n] = h[n]$
-1	0	0	0
0	1	0	1
1	0	-1.1	-1.1
2	0	$(-1.1)^2$	$(-1.1)^2$
3	0	$(-1.1)^3$	$(-1.1)^3$
⋮	⋮	⋮	⋮

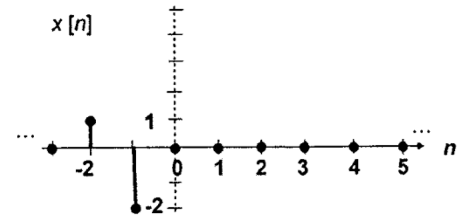
$\boxed{h[n] = (-1.1)^n u[n]}$

Is this system BIBO stable?

$$\sum_{n=-\infty}^{\infty} |h[n]| = 1.1 + (1.1)^2 + (1.1)^3 + \dots = \infty$$

$\boxed{\text{Not BIBO stable}}$

4. Evaluate the convolution
$y[n] = x[n] * u[n]$, given $x[n]$ as in the figure to the right, and $u[n]$ the unit step function.



k	-4	-3	-2	-1	0	1
x[k]			1	-2		
u[-n]	1	1	1	1	1	

shift u[n] 2 units left to begin

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k	-4	-3	-2	-1	0	1
x[k]			1	-2		
u[-2-k]	1	1	1			
u[-1-k]	1	1	1	1		
u[0-k	1	1	1	1	1	

$y[-2] = 1$

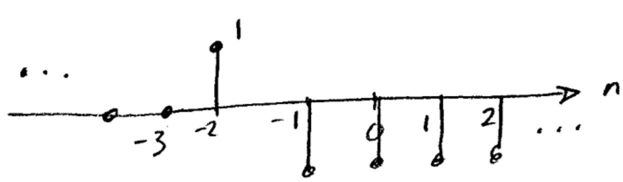
$y[-1] = 1-2 =$

$y[0] = 1-2 =$

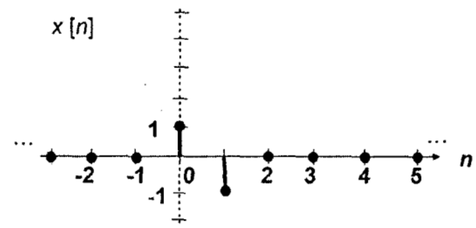
⋮

$$y[n] = \begin{cases} 0, & n < -2 \\ 1, & n = -2 \\ -1, & n \geq -1 \end{cases}$$

y[n]:



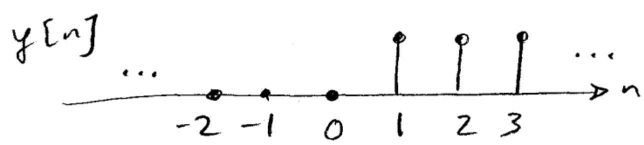
4. Evaluate the convolution $y[n] = x[n] * ramp[n]$, given $x[n]$ as in the figure to the right, and $ramp[n]$ the unit ramp function.



k	-3	-2	-1	0	1	2	3	4	
$ramp[k]$					1	2	3	4	
$g[0-k]$			-1	1					$y[0] = 0$
$g[1-k]$				-1	1				$y[1] = 1$
$g[2-k]$					-1	1			$y[2] = -1+2 = 1$
$g[3-k]$						-1	1		$y[3] = -2+3 = 1$

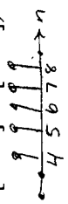
$$y[n] = \begin{cases} 0, & n \leq 0 \\ 1, & n \geq 1 \end{cases}$$

$$y[n] = u[n-1]$$

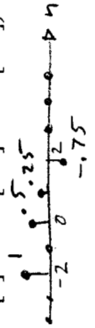


6. Evaluate the convolution $y[n] = \underbrace{(u[n-4]-u[n-9])}_{\text{call this } x[n]} * \underbrace{(\delta[n+2]+0.5\delta[n]+0.25\delta[n-1]-0.75\delta[n-2])}_{\text{call this } h[n]}$.

$x[n]$:



$h[n]$



k	-3	-2	-1	0	1	2	3	4	5	6	7	
$h[k]$		1	0	0.5	0.25	-0.75						
$x[2-k]$	1	1										$y[2]=1$
$x[3-k]$	1	1	1									$y[3]=1$
$x[4-k]$	1	1	1	1								$y[4]=1+.5=1.5$
$x[5-k]$	1	1	1	1	1							$y[5]=1+.5+.25=1.75$
$x[6-k]$		1	1	1	1	1						$y[6]=1+.5+.25-.75=1$
$x[7-k]$			1	1	1	1	1					$y[7]=.75-.75=0$
$x[8-k]$				1	1	1	1	1				$y[8]=.75-.75=0$
$x[9-k]$					1	1	1	1	1			$y[9]=.25-.75=-0.5$
$x[10-k]$						1	1	1	1	1		$y[10]=-.75$
$x[11-k]$							1	1	1	1	1	$y[11]=0$

$$y[n] = \delta[n-2] + \delta[n-3] + 1.5\delta[n-4] + 1.75\delta[n-5] + \delta[n-6] - 0.5\delta[n-9] - 0.75\delta[n-10]$$

$y[n]$: