

## EE322 Fall 2012: Lab 8-Continuous-time Fourier Series using Hardware

To introduce you to the hardware available to you, this lab will have you create and analyze how well the Fourier series theory follows real life. In particular, you will use the computation of the Fourier series of a 50 kHz square wave, then compare it to the actual frequency content of the square wave created using a function generator (use the Agilent 33220A Function/Arbitrary Waveform Generator). You will use an oscilloscope (use the LeCroy oscilloscope--displays time signals) and a spectrum analyzer (use the HP 8591E Spectrum Analyzer--displays frequency content) to analyze the signal in both the time and frequency domain.

### 200 kHz Sinusoid

- By inspection, the Fourier series representation of a 200 kHz cosine with amplitude 1 Volt (note: this corresponds to a 2 Volt peak-to-peak value):  $x(t) = \cos(2\pi 200,000t)$  is  $X[k] = \frac{1}{2} \delta[k+1] + \frac{1}{2} \delta[k-1]$ .

$$X[+1] = \frac{1}{2} \text{ corresponding to a frequency of } +200 \text{ kHz}$$

$$X[-1] = \frac{1}{2} \text{ corresponding to a frequency of } -200 \text{ kHz}$$

- Using the Agilent 33220A Function/Arbitrary Waveform Generator, create a 200 kHz sine wave with a *peak-to-peak* voltage of 2 Volts. This can be easily done by pushing the following sequence of buttons: **“Freq”, “2”, “0”, “0”, “kHz”**. (Note: Make sure you’re changing the frequency and not the period.) The default signal amplitude is set to 100mV peak-to-peak, which can be changed by depressing the “Ampl” button.

Important: Check to ensure the function generator’s output impedance is set to match the Oscilloscope and the spectrum analyzer.

--On the function generator, push the following sequence of buttons as needed:

**“Utility”, “Output Setup”, “Load”, and 50 Ω** should appear in the window.

--On the Oscilloscope, for the channel that is connected to the function generator, check that the input impedance is selected to be 50 Ω vice 1 MΩ.

Be sure to push the “Output” button on the function generator to enable the output.

Pass this signal using a T-coax connector to the oscilloscope.

- Use the oscilloscope’s horizontal sec/div and vertical volts/div to verify the period, frequency and amplitude of the signal the function generator is creating.

Period: \_\_\_\_\_ Frequency: \_\_\_\_\_ Amplitude: \_\_\_\_\_

- Connect the other end of the T-coax connector to the input of the spectrum analyzer and **disconnect** the coaxial cable from the oscilloscope. The spectrum analyzer will display a 1.8GHz range of frequencies, which is too large to view the 200 kHz sinusoid. Using the “FREQUENCY” button, adjust the *Start Frequency* to 150 kHz. This can be done by depressing the following sequence of buttons: **“FREQUENCY”, “Start Frequency”, “1”, “5”, “0”, “kHz”**. Similarly, adjust the *Stop Frequency* to 250 kHz.

- Change the spectrum analyzer from a log (decibel) scale to a linear scale using the following sequence of buttons: **“AMPLITUDE”, “Scale Log/Lin”** (until the word “LIN” is underlined).

- If the peak of the sinusoid’s spectrum is too high to show in the window, adjust the reference amplitude level until it appears: **“AMPLITUDE”, “REF LVL”**, then adjust the round knob as needed.

- Using the marker (press “MKR”, then turn the round white knob, measure the frequency and amplitude of the peak...the frequency and amplitude will be displayed when using the marker.

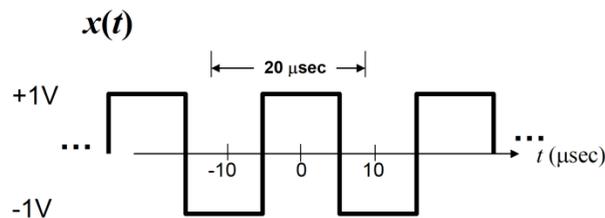
Frequency: \_\_\_\_\_ Amplitude: \_\_\_\_\_

The frequency should be approximately the same as in part (b). However, the amplitude in the frequency domain should be less than the amplitude in the time domain. This is due to the fact that the spectrum analyzer is displaying the rms voltage vice the peak voltage. Note also that the spectrum analyzer only displays positive frequencies.

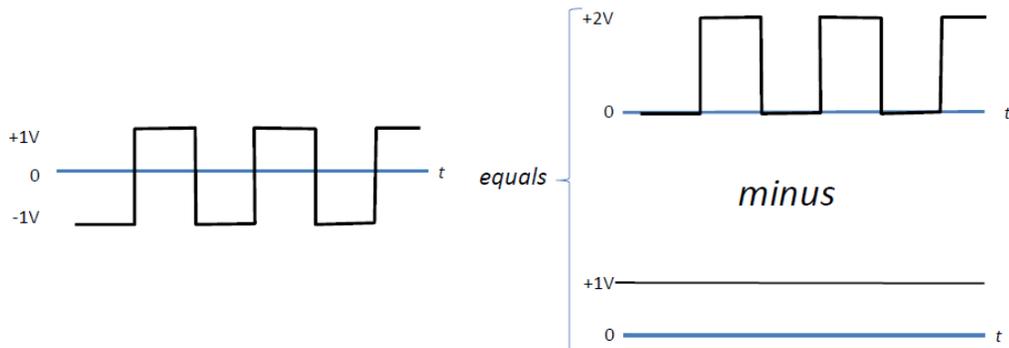
- e. On the function generator, vary the frequency of the sinusoid and watch the changes on the spectrum analyzer. Do you get what you expect?

### 50 kHz Square Wave

1. On a separate sheet of paper, determine the Fourier series Harmonic function  $X[k]$  of the 50 kHz square wave shown below. Note that it ranges from -1 Volt to +1 Volt (which is a 2 Volt peak-to-peak square wave), and has a duty cycle of 50% (that is, it is high for  $\frac{1}{2}$  the period, then low for the other half). The simplest way to proceed is to treat this square wave as the difference of a square wave that ranges between 0V and 2V and a constant 1V, as shown in the 2<sup>nd</sup> figure. Show your work on a separate sheet. What is the fundamental period of the square wave? What is the fundamental frequency?



To simplify the calculations, note:



Write your results below:

$T_F =$  \_\_\_\_\_  $f_F =$  \_\_\_\_\_

$X[k] =$  \_\_\_\_\_

2. The equation you come up with should have a *sinc* in it...the sinc is called the “envelope” of the frequency spectrum, because its values control the magnitude of the spectrum. Fill in the table below for  $k = 0, 1, 2, \dots, 9$ .

Recall that each value of  $X[k]$  corresponds to a delta function in the frequency domain, and these delta functions are separated by the fundamental frequency. The last column in the table is used to normalize the values so the largest peak height is 1.0...this allows a direct comparison with hardware results, which may be off by a scale factor.

<b>k (multiple of fundamental frequency)</b>	<b>  X[k]  </b>	$\frac{ X[k] }{\text{Max }  X[k] }$
<b>0 (0 Hz)</b>	<b>0</b>	<b>0</b>
<b>1 (50 kHz)</b>		<b>1.0</b>
<b>2 (100 kHz)</b>		
<b>3 (150 kHz)</b>		
<b>4 (200 kHz)</b>		
<b>5 (250 kHz)</b>		
<b>6 (300 kHz)</b>		
<b>7 (350 kHz)</b>		
<b>8 (400 kHz)</b>		
<b>9 (450 kHz)</b>		

3. Now use the function generator to create a 50 kHz square wave with peak-to-peak amplitude of 2 Volts.
- Attach the function generator **only to the oscilloscope**, and adjust the time scale on the oscilloscope to view a few periods of the square wave. Insure that the intended waveform is observed using the cursors.
  - Disconnect the signal from the oscilloscope, and view the signal on the spectrum analyzer. The square wave by default has no DC value, so adjust the range of frequencies displayed on the spectrum analyzer to display 25 kHz to 475 kHz.
  - To make the peaks stand out more on the spectrum analyzer, adjust the reference voltage until the largest peak takes up most of the display vertically (i.e., so that the max peak goes from near the bottom to the top).
  - Fill in the following table to record the frequency and amplitude of the peaks at multiples of the fundamental frequency. Some peaks may not be visible (too small in amplitude to be displayed). The last column is used to normalize the values so the largest peak height is 1.0...this allows a direct comparison with theory.

Use the marker to measure the frequency and amplitude of visible peaks:

<b>Multiples of Fundamental Frequency</b>	<b>Frequency</b>	<b>Amplitude</b>	$\frac{\text{Amplitude}}{\text{Max Amplitude}}$
<b>50 kHz</b>			<b>1.0</b>
<b>100 kHz</b>			
<b>150 kHz</b>			
<b>200 kHz</b>			
<b>250 kHz</b>			
<b>300 kHz</b>			
<b>350 kHz</b>			
<b>400 kHz</b>			
<b>450 kHz</b>			

- d. Sketch the spectrum analyzer display below, labeling the axes.
- e. Compare the two tables above. When comparing the normalized values from theory to the normalized values from the hardware display, how well does theory match real-life?
- f. What do you expect to happen when you change the frequency of the square wave to 80 kHz? Does the Fourier series Harmonic function  $X[k]$  change? Change the frequency of the square wave on the function generator and see what you get on the spectrum analyzer...do you get what you expect?

- g. Set the fundamental frequency on the function generator back to 50 kHz, and see what happens when you change the duty cycle on the function generator to 25%. Which peaks disappear (which multiples of the fundamental frequency)? Change the duty cycle to 20%. Which peaks disappear (which multiples of the fundamental frequency)? Do you see a pattern?

**Lab writeup: Turn in these sheets which include your answers to all questions.**