EE322 Fall 2008 Lab 06: Complex Functions

Introduction

Typically, taking the Fourier or Laplace transforms of real signals will result in complex functions, as you'll see later in the course. These functions (as do all complex values) have a magnitude and phase. In addition, systems may have a complex frequency response. It is important for electrical engineers to be able to understand and plot the magnitude and phase of complex functions or signals, as they will continue to appear time after time in engineering endeavors.

Consider a complex Fourier transform *X*(*f*), where *X*(*f*) is composed of real and imaginary parts:

$$X(f) = A(f) + jB(f) = R(f)e^{j\theta(f)} \tag{1.1}$$

Here, the real part is *A*(*f*), the imaginary part is *B*(*f*), the magnitude is *R*(*f*) and the phase is $\theta(f)$.
Since this is a function of frequency *f*, the magnitude of this function is also a function and is given by:

$$R(f) = |X(f)| = \sqrt{\left(A(f)\right)^2 + \left(B(f)\right)^2}\ . \tag{1.2}$$

The phase of this function is given by:

$$\theta(f) = \angle X(f) = \tan^{-1}\left(\frac{B(f)}{A(f)}\right) \tag{1.3}$$

For example, if *C*(*f*)=30*f* - *j*5*f*,

$$R(f) = |30f - j5f| = \sqrt{(30f)^2 + (-5f)^2} = \sqrt{900f^2 + 25f^2} = \sqrt{925f^2} \tag{1.4}$$

and

$$\theta(f) = \angle\left(30f - j5f\right) = \tan^{-1}\left(\frac{-5f}{30f}\right) \tag{1.5}$$

Finally, if a complex function *X*(*f*) is given as follows, where *V*(*f*) and *W*(*f*) are complex functions:

$$X(f) = \frac{V(f)}{W(f)}, \text{ then } |X(f)| = \frac{|V(f)|}{|W(f)|} \text{ and } \angle X(f) = \angle V(f) - \angle W(f)\,. \tag{1.6}$$

Other Useful Relationships: if *A* and θ are real values, then

$$Ae^{j\theta} = A\cos\theta + jA\sin\theta\,,\ \text{Re}\left\{Ae^{j\theta}\right\} = A\cos\theta\,,\ \text{Imag}\left\{Ae^{j\theta}\right\} = A\sin\theta$$

See Appendix H in textbook for more useful relationships involving complex numbers.

MATLAB Simulations

A. Using MATLAB

Plotting these complex functions by hand can be tedious and prone to errors. However, MATLAB can be used to easily provide computations or plots of magnitude and phase for complex functions. The ***abs*** function is used to compute absolute values for real numbers or functions, and it will compute the magnitude for complex numbers or functions. The ***angle*** function will compute the phase of a complex number or function in *radians*.

For the example $C(f)=30f - j5f$, the magnitude (equation (1.4)) and phase (equation (1.5)) can be generated with:

```
>> f= -20:.01:20;  % range of frequency from -20 Hz to +20 Hz
>> mag=abs(30*f - j*5*f);      % magnitude
>> ph_rad=angle(30*f - j*5*f); % phase in radians
>> ph_deg = ph_rad*180/pi;     % phase in degrees
>> figure(1), subplot(2,1,1), plot(f,mag), grid on
>> xlabel('frequency (Hz)'), ylabel('Magnitude')
>> subplot(2,1,2), plot(f,ph_deg), grid on
>> xlabel('frequency (Hz)'), ylabel('Degrees')
>> title('Top: Magnitude, Bottom: Phase')
```

To compute the real and imaginary parts, use:

```
>> r1=real(30*f - j*5*f);      % real part
>> i1=imag(30*f - j*5*f);      % imaginary part
>> figure(2), subplot(2,1,1), plot(f,r1), grid on
>> xlabel('frequency (Hz)'), ylabel('Real Amplitude')
>> subplot(2,1,2), plot(f,i1), grid on
>> xlabel('frequency (Hz)'), ylabel('Imaginary Amplitude')
>> title('Top: Real Part, Bottom: Imaginary Part')
```

B. MATLAB Simulation

1. Using the equations provided in the Introduction section of this lab, compute (by hand) the equation for the magnitude and phase of the following function:

$$H(f)=\frac{1}{1+j2\pi f}$$

One way to generate a plot of the magnitude and phase would be to place your equations you just determined in m-files as is, and use the MATLAB *plot* command. But why bother doing that…instead, compute and plot your result using MATLAB's ***abs*** (which computes magnitude) and ***angle*** (which computes phase) functions for $-2 \le f \le 2$. Use a small frequency step such as 0.001Hz to get an accurate plot. Use the subplot command to plot the magnitude on the top plot, and phase (in DEGREES) right below it on the same figure. Turn on the grid (type ***help grid*** if needed). Be sure to LABEL the axes and give the plot a meaningful title!!!

Note: This function has a vector in the denominator (since *f* runs from -2 to 2 in small steps). In order for MATLAB to provide a vector output, for each value of *f*, the denominator must be divided into 1. This means you should use the " **. /** " operator instead of just "/" to get a magnitude or phase vector output.

2. Using MATLAB, record the magnitude <u>and</u> phase of *H*(*f*) at a frequency of 0.5 Hz and at a frequency of −0.5Hz.

Frequency	**Magnitude**	**Phase**
+0.5 Hz		
-0.5 Hz		

3. Consider the following function:

$$H(f) = \frac{-2\pi f}{1 + j0.5\pi f}$$

Plot the magnitude and phase using MATLAB for $-3 \le f \le 3$. Use a small frequency step such as 0.001 to get an accurate plot. Use the subplot command to plot the magnitude on the top plot, and phase (in DEGREES) right below it on the same figure. Turn on the grid (type ***help grid*** if needed). Be sure to LABEL the axes and give the plot a meaningful title!

4. Plot the magnitude and phase of the following function using MATLAB for $-20 \le f \le 20$.

$$H(f) = \frac{j2\pi f + 2}{\left(j2\pi f + 2\right)^2 + \left(2\pi 10\right)^2}$$

Use a small frequency step such as 0.001 to get an accurate plot. Use the subplot command to plot the magnitude on the top plot, and phase (in DEGREES) right below it on the same figure. Turn on the grid (type ***help grid*** if needed). Be sure to LABEL the axes and give the plot a meaningful title!

By looking at the magnitude and phase plot, are there any frequencies for which this function is <u>real-valued</u>? Are there any frequencies for which the function is <u>purely imaginary</u> (that is, has no real component)?

For a lab write-up, be sure to address all questions asked on a separate sheet, and provide code and plots.