EE322 Lab 9: Bode Diagrams

Introduction

The first step in determining the asymptotic gain (Bode) plot is to put the frequency response in standard form as a function of $j\omega$. As an example, given the $H(j\omega)$ below, determine the form to plot the asymptotic gain by hand. The equation you use is shown in equation (1).

$$
\begin{aligned}
H(j\omega) &= \frac{10}{(j\omega+1)(j\omega+2)(j\omega+10)} \\
&= \frac{10}{\left(1+\frac{j\omega}{1}\right)2\left(1+\frac{j\omega}{2}\right)10\left(1+\frac{j\omega}{10}\right)} \\
&= \frac{\left(\frac{10}{20}\right)}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)} \\
&= \frac{0.5}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)} \qquad \textbf{(1)}
\end{aligned}
$$

MATLAB has many functions available to determine the frequency response of systems. Many of these functions first require you to input the system's frequency response ($H(j\omega)$), in the standard form as you see above, but then determine the numerator and denominator polynomials. In order for MATLAB to read in a frequency response, the coefficients of the numerator polynomial and the denominator polynomial are treated as two different vectors, as shown in equation (2) below.

$$
\begin{aligned}
H(j\omega) &= \frac{0.5}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)} \\
&= \frac{0.5}{.05(j\omega)^3+.65(j\omega)^2+1.6j\omega+1} \qquad \textbf{(2)}
\end{aligned}
$$

In (2), we see that the numerator vector is [0.5] and the denominator vector is [0.05 0.65 1.6 1]. The vectors must be written from left to right using the coefficient of the highest power of s that appears (on the left end of the vector) down to the constant term in the polynomial. If any powers of s are skipped in the polynomial, then that means their coefficient is 0 and that needs to be incorporated into the vector. The coefficient of the constant term must appear in the vector. For example, the vector representing the polynomial

$$(j\omega)^2 + j\omega$$

is [1 1 0]. In this way, MATLAB automatically knows the highest power of $j\omega$ in the vector. If you tried to represent this polynomial as [1 1], MATLAB would interpret the polynomial as $j\omega$+1, and your results would be wrong.

To generate the Bode diagram in MATLAB for the above frequency response in equation (2), run this code:

```
>> N=[0.5];                          % numerator polynomial
>> D=[0.05 0.65 1.6 1];              % denominator polynomial
>>bode(N,D)
```

This generates both the magnitude and phase response (we haven't emphasized the phase response in this class, so disregard this part of the Bode diagram). It is difficult to pick out values on the curves without a grid, so type:

>> **grid on**

Also, note that the frequency axis runs from $\omega = 0.01$ rad/sec out to 1000 rad/sec. If I wanted to adjust the axis to a different x-axis range or a different y-axis range, you can edit the plot. To adjust the frequency axis to run from 0.1 rad/sec out to 10000 rad/sec, do the following. In the figure window, select **Tools→Edit Plot**, then double-click on the x-axis on the plot itself, and adjust the X Axis limits to be 0.1 to 5000. After changing the numbers as desired, click elsewhere on the figure to update the axis.

Note that in this problem, the gain starts at -6dB at the left end, since 20 $\log_{10}(0.5)$ is -6dB.

Side note: Discrete convolution can be used to multiply polynomials, when the coefficients of each polynomial are used as the inputs to convolution. For example, using the MATLAB *conv* function, you could use the following code to determine the denominator coefficient of equation (2):

$$H(j\omega) = \frac{0.5}{\left(1+\frac{j\omega}{1}\right)\left(1+\frac{j\omega}{2}\right)\left(1+\frac{j\omega}{10}\right)}$$
$$= \frac{0.5}{.05(j\omega)^3 + .65(j\omega)^2 + 1.6j\omega + 1}$$

```
>> d1=[1 1];, d2=[0.5 1];     % remember, the coefficient of the highest
                              %     power of jω is on the left
>> D = conv(d1, d2);          % convolve the first two polynomials
>> D = conv(D, [0.1 1])       % now convolve that result with the 3rd term

       D =

          0.0500    0.6500    1.6000    1.0000
```

I. Problems

A. Given the frequency response:

$$H(j\omega) = \frac{10000(j\omega + 0.1)}{(j\omega + 1)\left((j\omega)^2 + 4j\omega + 100\right)}$$

1. Put this frequency response in the proper form as a function of $j\omega$.

   $H(j\omega) =$

2. What are the corner frequencies? Do they correspond to poles or zeros? Any complex conjugate pole pairs?

3. Plot (by hand on semi-log paper) the asymptotic gain curve for this system as a function of ω. Your plot should include frequencies from **0.01** Hz out to **100** Hz. Indicate corner frequencies and slopes.

4 Now use MATLAB's *bode* function to obtain the Bode diagram. **Be sure the grid is turned on. Print this out and on top of the MATLAB magnitude plot, plot (by hand) in a different color, <u>your</u> asymptotic gain curve.**

5. Is your asymptotic gain curve a good approximation to the system gain? Why or why not?

6. What type of filter would you call this?

B. Given the frequency response:

$$H(j\omega) = \frac{200j\omega + 200}{(j\omega)^5 + 32(j\omega)^4 + 361(j\omega)^3 + 2630(j\omega)^2 + 4300j\omega + 2000}$$

1. Put this frequency response in the proper form as a function of $j\omega$. Your calculator can factor the denominator, and you can simplify the notation in your calculator by replacing $j\omega$ with s in the above equation.

$H(s) =$

2. What are the corner frequencies? Do they correspond to poles or zeros? Any complex conjugate pole pairs?

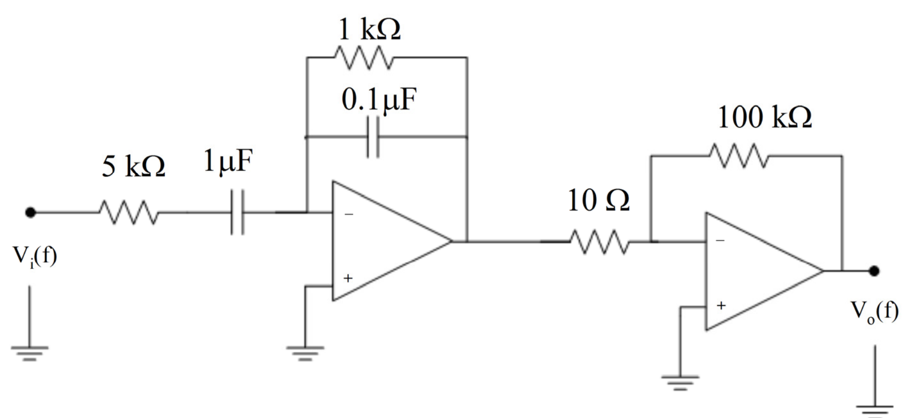
3. Plot (by hand on semi-log paper) the asymptotic gain curve for this system as a function of ω. Your plot should include frequencies from **0.01** Hz out to **1000** Hz. Indicate corner frequencies and slopes.

4 Now use MATLAB's *bode* function to obtain the Bode diagram. **Be sure the grid is turned on. Print this out and on top of the MATLAB magnitude plot, plot (by hand) in a different color your asymptotic gain curve.**

5. Is your asymptotic gain curve a good approximation to the system gain? Why or why not?

6. What type of filter would you call this?

C. Use the circuit below to work the problems that follow:.



1. Find the frequency response of this system in the proper form as a function of *s*.

$H(s) =$

2. What are the corner frequencies? Do they correspond to poles or zeros? Any complex conjugate pole pairs?

3. Plot (by hand on semi-log paper) the asymptotic gain curve for this system as a function of ω. Your plot should include frequencies from **1** Hz out to **100,000** Hz. Indicate corner frequencies and slopes.

4 Now use MATLAB's *bode* function to obtain the Bode diagram. **Be sure the grid is turned on. Print this out and on top of the MATLAB magnitude plot, plot (by hand) in a different color your asymptotic gain curve.**

5. Is your asymptotic gain curve a good approximation to the system gain? Why or why not?

6. What type of filter would you call this?

For a lab write-up, be sure to turn in your derivations of the frequency responses to the correct forms and address all questions asked. Turn in your plots and label your asymptotic gain curves with corner frequencies and slopes. No code is required.