

1. Math/Algebra: Simplify $1 + e^{j\pi} = \underline{\quad 0 \quad}$.

$$1 + e^{j\pi} = 1 + \cos\pi + j\sin\pi = 1 - 1 + j0 \\ = 0$$

Note:

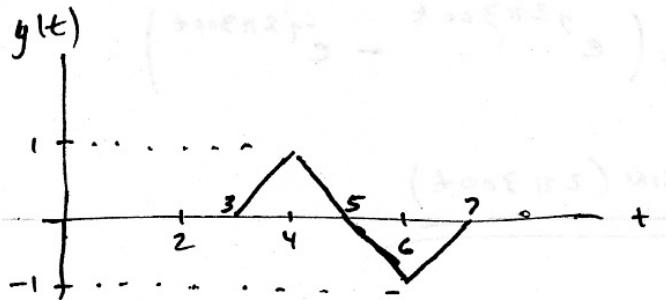
$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}, \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

2. Sketch the convolution: $y(t) = \text{tri}(t) * [\delta(t-4) - \delta(t-6)]$.

$$x(t) * A\delta(t-t_0) = Ax(t-t_0)$$

$$\text{So } \text{tri}(t) * (\delta(t-4) - \delta(t-6)) = \text{tri}(t-4) - \text{tri}(t-6)$$



3. A discrete-time system is described by $4y[n] + 3y[n-1] = x[n]$. Solve for the impulse response $h[n]$. Determine if the system described by $h[n]$ is BIBO stable.

$$y[n] = \frac{x[n]}{4} - \frac{3}{4}y[n-1] \quad \text{To find } h[n], \text{ let } x[n] = \delta[n], \text{ then } y[n] = h[n]$$

n	$x[n]$	$y[n]$	$y[n-1]$
0	1	$\frac{1}{4}$	0
1	0	$-\frac{3}{4}(\frac{1}{4})$	$\frac{1}{4}$
2	0	$(-\frac{3}{4})^2(\frac{1}{4})$	$-\frac{3}{4}(\frac{1}{4})$
3	0	$(-\frac{3}{4})^3(\frac{1}{4})$	$(-\frac{3}{4})^2(\frac{1}{4})$
⋮	⋮	⋮	⋮

$$h[n] = y[n] = \left(\cancel{-\frac{3}{4}}\right)^n \cdot \frac{1}{4} u[n]$$

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} \frac{1}{4} \left(\frac{4}{3}\right)^n$$

$$= \frac{1}{4} \left(\frac{1}{1-3/4} \right) = 1 < \infty$$

BIBO STABLE

Do either problem 4 or problem 5 – indicate clearly which problem should be graded:

4. A continuous-time Fourier series harmonic function is given by:

$$X[k] = \frac{j}{2}\delta[k+3] - \frac{j}{2}\delta[k-3],$$

with fundamental period $T_F = 0.01$ sec. Write the corresponding time signal in terms of sinusoid(s).

$$f_F = \frac{1}{T_F} = 100 \text{ Hz}$$

$$\begin{aligned} x(t) &= \frac{j}{2} \left(e^{j2\pi(100)t} - e^{-j2\pi(100)t} \right) \quad \text{THEN } j = -\frac{1}{j}, \text{ so...} \\ &= \frac{1}{2j} \left(e^{j2\pi 300t} - e^{-j2\pi 300t} \right) \\ &= \underline{\underline{\sin(2\pi 300t)}} \end{aligned}$$

5. Find the harmonic function $X[k]$ for the signal $x(t) = 4 \cos(2\pi 250t)$ if $f_F = 50\text{Hz}$.

$$250 \text{ Hz} = 5 f_F$$

$$\begin{aligned} x(t) &= 4 \left(\frac{e^{j2\pi 5(50)t} + e^{-j2\pi 5(50)t}}{2} \right) \\ &= 2 e^{j2\pi(5)50t} + 2 e^{-j2\pi(-5)50t} \end{aligned}$$

$$\therefore \underline{\underline{X[k] = 2\delta[k-5] + 2\delta[k+5]}}$$

Bonus: On October 25, 1917, the Bolsheviks seized control over the provisional government at the Winter Palace, located in what Russian city?

St PETERSBURG