EE322 Fall 2012 Quiz 5 **Name:** KEY

1. Find the continuous time Fourier Transform (CTFT) for:

a. $x(t) = 2\sin(40\pi t)$. $X(f) = \frac{1}{j}[\delta(f-20) - \delta(f+20)]$

$2\sin(40\pi t) = 2\left(\frac{e^{j40\pi t} - e^{-j40\pi t}}{2j}\right) \Leftrightarrow \frac{1}{j}\left(\delta(f-20) - \delta(f+20)\right)$

$40\pi = 2\pi(20)$

b. $x(t) = 5\text{sinc}(t/5)$. $X(f) =$ 25 rect(5f)

$x(at) \Leftrightarrow \frac{1}{|a|} X\left(\frac{f}{a}\right)$

$5\,\text{sinc}\,t \Leftrightarrow 5\,\text{rect}(f)$

$a = \frac{1}{5}$

$5\,\text{sinc}\left(\frac{t}{5}\right) = |5| \cdot 5\,\text{rect}(5f)$

2. Find the inverse CTFT for:

a. $X(f) = \delta(f+5)$. $x(t) =$ $e^{-j2\pi 5t}$

$\delta(f-f_0) \xleftrightarrow{\mathcal{F}^{-1}} e^{j2\pi f_0 t}$

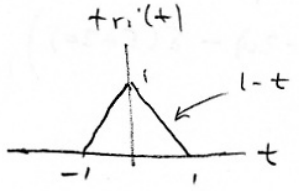
$f_0 = -5\text{Hz}$ here

b. $X(f) = \frac{1}{(j\pi f + 3)}$. $x(t) =$ $2e^{-6t}u(t)$

$= \frac{2}{j2\pi f + 6} \Leftrightarrow 2e^{-6t}u(t)$

3. Use Parseval's theorem to solve:

$$\int_{-\infty}^{\infty} |\mathrm{sinc}(f)|^4 \, df = \underline{\quad 2/3 \quad}$$



$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

$X(f) = \mathrm{sinc}^2(f) \overset{\mathcal{F}^{-1}}{\Longleftrightarrow} \mathrm{tri}(t) = x(t)$

So solve $\int_{-\infty}^{\infty} |\mathrm{tri}(t)|^2 \, dt$

$$= 2\int_0^1 (1-t)^2 \, dt = 2\int_0^1 (1-2t+t^2)\, dt$$

$$= 2\left(t - \frac{2t^2}{2} + \frac{t^3}{3}\right)\Big|_0^1 = 2\left(1-1+\frac{1}{3}\right)$$

4. A certain system impulse response and output response are given by:

$$h(t) = \delta(t-4)$$
$$y(t) = e^{-3(t-10)}u(t-10)$$

Determine the system input $x(t)$ ~~and the CTFT of the input $X(f)$~~.

EASIEST WAY: $y(t) = x(t) * A\delta(t-t_0) = Ax(t-t_0)$ AND NOTE THAT $A=1$, $t_0 = 4$

So BY INSPECTION $\boxed{x(t) = e^{-3(t-6)}u(t-6)}$

OR $H(f) = e^{-j2\pi f(4)}$, $Y(f) = \dfrac{1}{j2\pi f + 3} \cdot e^{-j2\pi f(10)}$

THEN $X(f) = \dfrac{Y(f)}{H(f)} = \dfrac{1}{j2\pi f+3} \cdot \dfrac{e^{-j2\pi f(10)}}{e^{-j2\pi f(4)}} = \dfrac{e^{-j2\pi f(6)}}{j2\pi f+3}$

$\overset{\mathcal{F}^{-1}}{\Longleftrightarrow} x(t) = e^{-3(t-6)}u(t-6)$

Bonus: In the book, *The Day of the Jackal*, by Frederick Forsyth, what is the name of the President that the Jackal is hired to assassinate?

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