Name: Key
Section:

EE331 Homework PS5 – fall 2012

Problem 1: You are given a aircraft launch system that has a catapult length of 80m, a maximum flux density of B = 1T, an effective length across of L = 100m, a maximum power supply current of 30kA, a maximum average power available from the ship power system of 7.5MW, and we desire that the pilot be subject to no more than 4g of acceleration ($1g = 9.8\frac{m}{s^2}$).

a. Determine the minimum launch time and the terminal velocity (Hint: velocity is the time integral of acceleration and distance is the time integral of velocity). *Ans:* (2.02s, 79.2 m/s)
b. Determine the maximum mass (largest plane and payload) that can be launched at this acceleration. *Ans:* (76,531 kg)
c. Determine the energy required to launch the plane (find the induced voltage, calculate the power, then integrate to find the energy). *Ans:* ($2.41*10^8$ J)
d. Determine the total required energy if only 75% of the total energy goes to launching the plane (the rest is lost in heat and overcoming drag). *Ans:* ($3.21*10^8$ J)
e. Given the restriction on available average power from the ship power system, find the minimum time required between catapult shots. *Ans:* (42.8s)

Problem 2: You are given a PM DC motor intended for use in an electric car that uses a 72V battery. You wish to determine the three parameters of the model: r_a, K_v, and T_{Loss}. You run the following two tests:

Test 1: Apply $72V_{DC}$ and measure $I_a = 4.196A$ and $\omega = 198.6 rad/s$ with no load

Test 2: Apply $72V_{DC}$ and measure $I_a = 61A$ and $\omega = 180.0 rad/s$ with a load

a. Determine r_a, K_v, and T_{Loss}.
[$r_a = 0.118\Omega$, $K_v = 0.36Vs$, $T_{Loss} = 1.51Nm$]

b. Determine the efficiency $\left(\frac{P_{Load}}{P_{In}}\right)$ of the motor for the conditions of Test 2.
[83.8%]

c. If we set $V_a = 50V_{DC}$ and $T_{Load} = 7.5Nm$, determine the resultant armature current and rotor speed (Assume that T_{Loss} is constant with speed).
[$I_a = 25A$, $\omega = 130.7 rad/s$]

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Problem 3: Given a $24V_{DC}$ PM motor with $r_a = 0.2279\Omega$ and $K_v = 0.1253V\cdot s$,

a. Sketch the electromagnetic torque (T_e) versus speed (ω on the x-axis) characteristic. Calculate the y-intercept, x-intercept, and slope. (y-int = 13.1953 Nm, x-int = 191.5 rad/s, slope = -0.0689)

b. If $T_{Load} = \left(250\times10^{-6}\,\mathrm{N\cdot m\cdot s^2}\right)\omega^2$ and T_{Loss} is negligible, what is the steady-state operating speed and the armature current?
[ω = 130.1 rad/s, I_a = 33.78A]

c. What is the efficiency at this operating point?
[η = 67.9%]

Problem 4: We wish to reduce the speed of the motor above to half its current value by adjusting the armature voltage.

a. Find the load torque at this speed. (1.056 Nm)

b. Find the new armature current. (8.43 A)

c. Find the required armature voltage. (10.1 V)

d. What is the efficiency at this operating point? (80.6%)

Problem 5: A 10HP PM DC motor is controlled by a power converter that can adjust its terminal voltage between 0 and $120V_{DC}$. The input voltage to the power converter is $144V_{DC}$. The parameters of the machine are $r_a = 0.143\Omega$, $K_v = 1.104V\cdot s$ and T_{Loss} is negligible. If T_{Load} is fixed at 70N·m (regardless of speed):

a. To achieve a rotor speed of 60rad/s,
   i. Find the required armature current and armature voltage.
      [I_a = 63.4A, V_a = 75.3V]
   ii. Find the converter duty cycle.
      [D = 52.3%]
   iii. If the converter is 93% efficient, determine the average current out of the 144V source.
      [I_s = 35.6A]

Additional Problems (Instructor Option):

- Any as assigned by instructor

#1

a) $a \leq 4g = 39.2\, m/s^2$

- $V(t) = \int a\,dt = a \cdot t + \cancel{V_0}^{\,0} = 39.2 \cdot t$
- $S(t) = \int V(t)\,dt = \frac{1}{2} \cdot 39.2\, t^2 + \cancel{S_0}^{\,0}$

$80m = \frac{1}{2} \cdot 39.2 \cdot t^2$

Catapult Length

$\boxed{t = 2.02s}$

← min. as @ max. acceleration

- $V_{Terminal}(2.02s) = 39.2 \cdot 2.02 = \boxed{79.2\, m/s}$

$(m/s^2) \quad (s)$

b) $F = ma = ILB = (30{,}000\,A)(100m)(1T)$

↳ $m = \frac{ILB}{a} = \frac{(30{,}000A)(100m)(1T)}{39.2\, m/s^2} = \boxed{76{,}531\, Kg}$

c) • $E_{ind}(t) = BLu = (1T)(100m)(39.2 \cdot t) = 3920\, V/s \cdot t$

- $P(t) = E(t) \cdot I = 3920\, V/s \cdot t \cdot 30{,}000A = 1.18 \cdot 10^8\, W/s \cdot t$
- $W_{Launch} = \int_0^{t=2.02s} P(t)\,dt = \int_0^{2.02s} 1.18 \cdot 10^8\, W/s \cdot t\,dt$

$= \frac{1}{2}\left[1.18 \cdot 10^8\, W/s \cdot t^2\right]\Big|_0^{2.02s}$

$= \boxed{2.41 \cdot 10^8\, J}$

Next →

#1 cont.

d) $W_{Launch} = 0.75 \cdot W_{TOT}$

$$W_{TOT} = \frac{2.41 \cdot 10^8 \text{ J}}{0.75} = \boxed{3.21 \cdot 10^8 \text{ J}}$$

e) $P_{Ave} = \frac{W_{TOT}}{\Delta t}$

$$7.5 \text{ MW} = \frac{3.21 \cdot 10^8 \text{ J}}{\Delta t}$$

$$\boxed{\Delta t = 42.8 \text{ s}}$$

2

$V_A = I_A R_A + K_v \omega$

a)

Test1: $72V = 4.196A \cdot R_A + K_v \cdot 198.6\ \text{rad/s}$

Test2: $72V = 61A \cdot R_A + K_v \cdot 180.0\ \text{rad/s}$

Solve ⟹

$r_a = 0.118\,\Omega$

$K_v = 0.36\ V \cdot s$

- $\tau_e = \tau_{Loss} + \tau_{Load}$

- $\tau_{Loss} = \tau_e(\text{NoLoad}) = K_v I_a\ (\text{No Load})$

  $\tau_{Loss} = \left(0.36\ \frac{N \cdot m}{A}\right)(4.196A) = 1.51\ N \cdot m$

b)

- $P_{Load} = \tau_{Load}\,\omega$

- $\tau_{Load} = \tau_e(\text{Loaded}) - \tau_{Loss} = K_v I_a(\text{Loaded}) - \tau_{Loss} = 0.36\frac{N \cdot m}{A} \cdot 61A - 1.51 N \cdot m$

  $\tau_{Load} = 20.45\ N \cdot m$

- $P_{Load} = (20.45\ N \cdot m) \cdot (180\ \text{rad/s}) = 3681\ W$

- $P_{in} = V_a I_a = (72V)(61A) = 4392\ W$

$$\eta = \frac{3681W}{4392W} \cdot 100\% = 83.8\%$$

#a cont

c) $\tau_e = \tau_{Load} + \tau_{Loss} = 7.5\ N\cdot m + 1.51\ N\cdot m = 9.01\ N\cdot m$

$\tau_e = K_v I_a = 9.01\ N\cdot m$

- $I_a = \dfrac{9.01\ N\cdot m}{0.36\ \frac{N\cdot m}{A}} = \boxed{25.03A}$

- $V_a = I_a r_a + K_v \omega$

$50V = (25.03A)(0.118\Omega) + (0.36\ V\cdot s)\cdot\omega$

$\boxed{\omega = 130.7\ rad/s}$

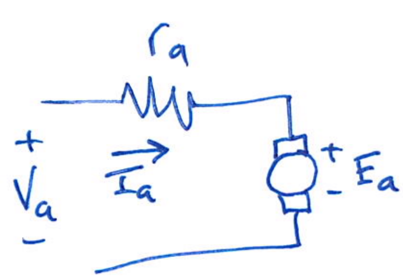
#3

a)

$V_a = 24\,V$

$r_a = 0.2279\,\Omega$

$k_v = 0.1253\,V\cdot s$

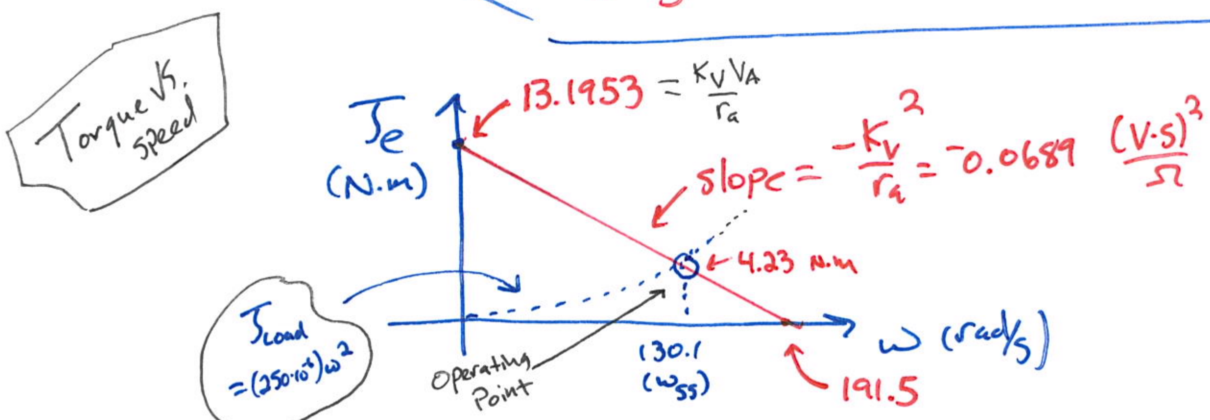


- $V_a = I_a r_a + \underbrace{K_v \omega}_{\text{Find}} \Rightarrow I_a = \dfrac{V_a - K_v\omega}{r_a}$

- $J_e = K_v I_a \Rightarrow J_e(\omega) = K_v\left(\dfrac{V_a - k_v\omega}{r_a}\right) = \underbrace{\dfrac{K_v V_a}{r_a}}_{\text{Y-intercept}} - \underbrace{\dfrac{K_v^2}{r_a}}_{\text{Slope}} \cdot \omega$

$$J_e(\omega) = \frac{(0.1253\,V\cdot s)\cdot(24\,V)}{0.2279\,\Omega} - \frac{(0.1253\,V\cdot s)^2}{0.2279\,\Omega}\cdot\omega$$

$$\boxed{J_e(\omega) = 13.1953 - 0.0689\cdot\omega \quad [N\cdot m]}$$



b)

$$J_e = \cancelto{0}{J_{Loss}} + J_{Load} = (250\cdot 10^{-6}\,N\cdot m\cdot s^2)\cdot\omega^2$$

- $J_e(\omega) = 13.1953 - 0.0689\cdot\omega = (250\cdot10^{-6}\,N\cdot m\cdot s^2)\cdot\omega^2$

Solve → $\boxed{\omega_{ss} = 130.1\ rad/s}$

(See Above plot)

- $$\boxed{I_a = \frac{V_a - K_v\omega}{R_a} = \frac{(24V - 0.1253\,V\cdot s\cdot 130.1\,rad/s)}{0.2279\,\Omega} = 33.78\,A}$$

#3 cont.

c) $\eta = \frac{P_{out}}{P_{in}} \cdot 100 = \frac{K_v \cancel{I_a} \omega}{V_a \cancel{I_a}} \cdot 100 = \frac{(0.1253\ V \cdot s) \cdot (130.1\ \frac{rad}{s})}{24V} \cdot 100$

- $P_{out} = K_v I_a \omega$
- $P_{in} = V_a I_a$

$\eta = 67.9\%$

(*) $P_{out} = \tau_{Load} \cdot \omega = \tau_e \cdot \omega$ (as $\tau_{Loss} = 0$)

#4

New $\omega_{ss} = 65\ \text{rad/s}$ ← (1/2 speed)

a)

$$T_e = T_{Load} \text{ as } (T_{loss} = 0)$$

$$= (250\cdot10^{-6}\ \text{N·m·s}^2)(65\ \text{rad/s})^2$$

$$= \boxed{1.056\ \text{N·m}}$$

b) $T_e = K_v I_a$

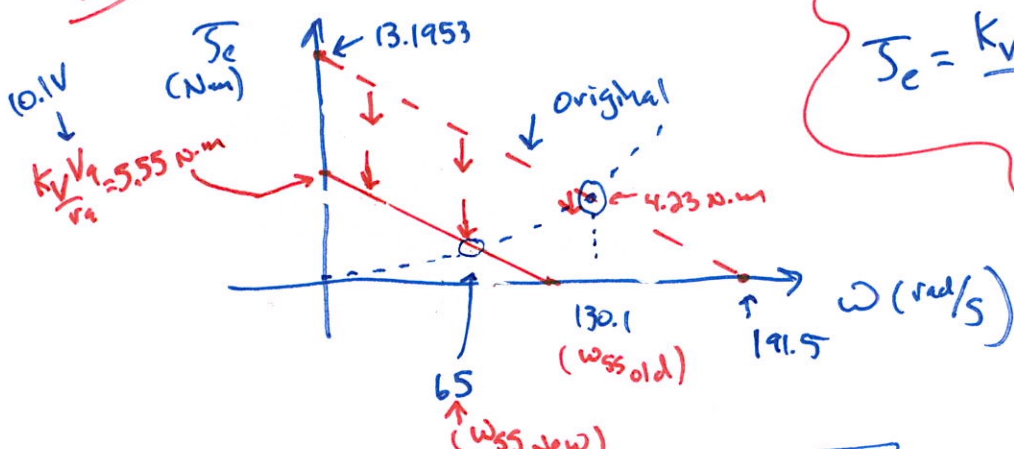
$$\Rightarrow I_a = \frac{T_e}{K_v} = \frac{1.056\ \text{N·m}}{0.1253\ \text{V·s}} = \boxed{8.43\ \text{A}}$$

c) $V_a = I_a r_a + K_v \omega = (8.43\text{A})\cdot(0.2279\,\Omega) + (0.1253\ \text{V·s})\cdot 65\ \text{rad/s}$

$$= \boxed{10.1\ \text{V}}$$

(We shifted curve Downward)

Notes



(*) Lower V_a (Affects Intercept but Same slope)

$$T_e = \frac{K_v V_a}{r_a}\downarrow - \frac{K_v^2}{r_a}\cdot\omega$$

d) $\boxed{\eta = \dfrac{K_v \omega}{V_a}\cdot 100\% = 80.6\%}$

#5

0–120V Input = 144V

$r_a = 0.143\,\Omega$ $K_v = 1.104\ V\cdot s$

$\tau_{Load} = 70\ N\cdot m$ $\tau_{Loss} \approx 0$

a) for $\omega = 60\ rad/s$

i) • $\tau_e = \tau_{Load} + \tau_{Loss}^{\nearrow 0} = 70\,N\cdot m = K_v I_a$

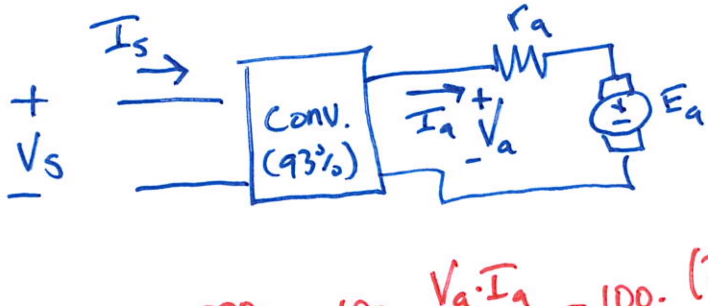
$$\Rightarrow I_a = \frac{70\ N\cdot m}{1.104\ V\cdot s} = 63.4\,A$$

• $V_a = I_a r_a + K_v \omega = (63.4A)\cdot(0.143\,\Omega) + (1.104\,V\cdot s)\cdot(60\,rad/s)$

$$V_a = 75.3V$$

ii) $V_a = D V_s \Rightarrow D = \frac{V_a}{V_s} = \frac{75.3V}{144V} = 52.3\%$

iii) $\eta_{Converter} = \frac{P_{out}\text{ to Motor Circuit}}{P_{in}\text{ from 144V Source}} \cdot 100 = 93\%$



$$93\% = 100\cdot\frac{V_a \cdot I_a}{V_s \cdot I_s} = 100\cdot\frac{(75.3V)(63.4A)}{(144V)\cdot I_s}$$

$$\Rightarrow I_s = 35.6\,A$$