Name: Key
Section: ___

EE331 Homework PS8 – fall 2012

Problems from Alexander & Sadiku:

CH 10

- 10.55 *(Just find the **Thevenin** for parts a and b)*

CH 11

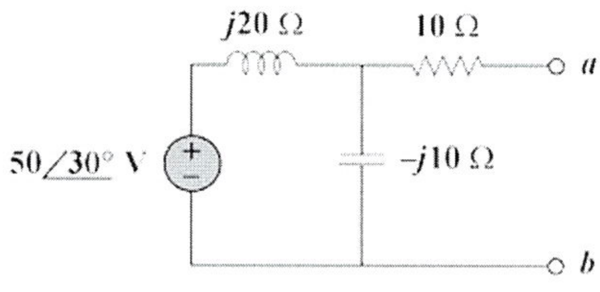
- 11.2 (Ans: P_{avg} 5 Ohm = 2.647 W, 0 W for inductor and capacitor, why?)
- 11.3
- 11.6 (Ans: 276.8 W)
- 11.9

Additional Problems (Instructor Option):

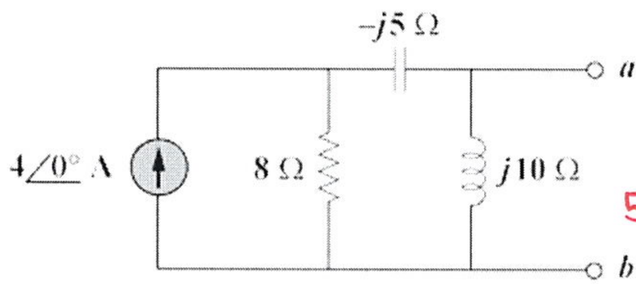
- Any as assigned by instructor

Chapter 10, Problem 55.

Find the Thevenin and ~~Norton~~ equivalent circuits at terminals *a-b* for each of the circuits in Fig. 10.98.



(a)



(b)

STEP 6

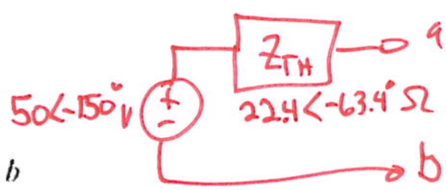
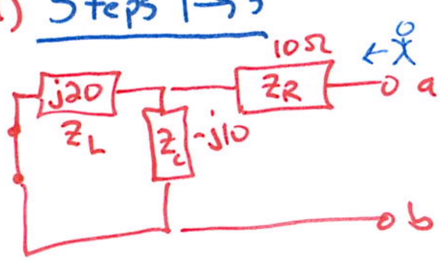


Figure 10.98

a) Steps 1→3



STEP 4

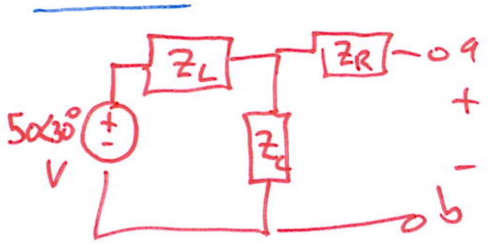
$$Z_{TH} = Z_R + Z_L \| Z_c$$

$$= (10\Omega + j0) + \frac{(0+j20)(0-j10)}{(0+j20)+(0-j10)}$$

$$= 10\Omega + (-j20)$$

$$\boxed{Z_{TH} = 22.4\angle -63.4^\circ \Omega}$$

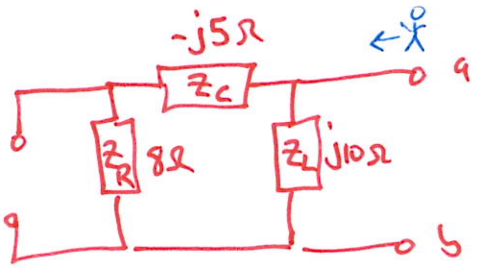
STEP5



$$V_{TH} = \frac{(50\angle 30^\circ V)\cdot Z_c}{Z_L + Z_c} = \boxed{50\angle -150^\circ V}$$

10-55 b

STEPS 1-3



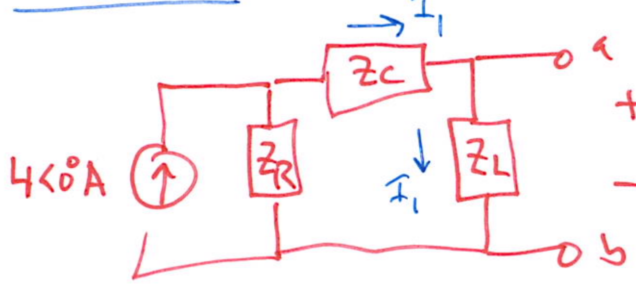
STEP 4

$$Z_{TH} = (Z_C + Z_R) \| Z_L$$

$$= \frac{(8 - j5)\,\Omega \cdot (0 + j10)\,\Omega}{8\Omega - j5 + j10}$$

$$\boxed{Z_{TH} = 10\angle 26^\circ\ \Omega}$$

STEP 5



$$V_{TH} = I_1 \cdot Z_L$$

$$= 3.4\angle{-32^\circ}\,A \cdot (j10)$$

$$\boxed{V_{TH} = 33.9\angle 58^\circ\ V}$$

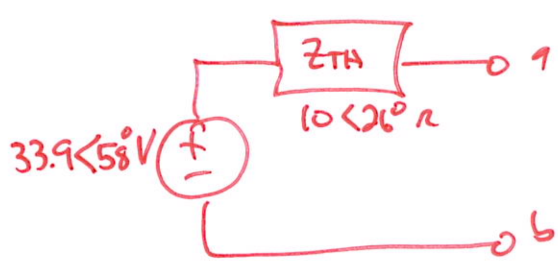
By Current Division

$$I_1 = \frac{4\angle 0^\circ A \cdot Z_R}{Z_R + Z_C + Z_L}$$

$$= \frac{4\angle 0^\circ A \cdot (8 + 0j)\,\Omega}{8\Omega - j5 + j10}$$

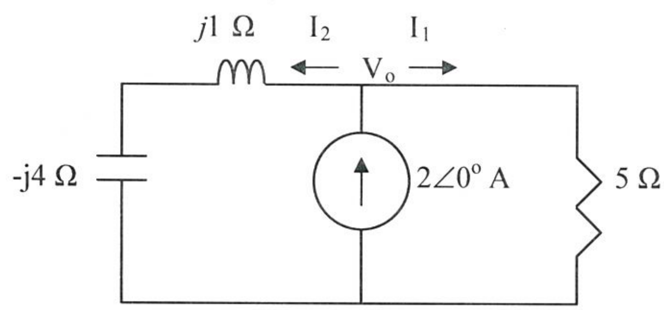
$$= 3.4\angle{-32^\circ}\,A$$

STEP 6



Chapter 11, Solution 2.

Using current division,



$$I_1 = \frac{j1 - j4}{5 + j1 - j4}(2) = \frac{-j6}{5 - j3}$$

$$I_2 = \frac{5}{5 + j1 - j4}(2) = \frac{10}{5 - j3}$$

For the inductor and capacitor, the average power is zero. For the resistor,

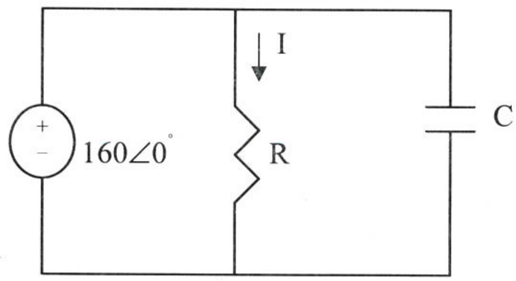
$$P = \frac{1}{2}|I_1|^2 R = \frac{1}{2}(1.029)^2(5) = 2.647 \text{ W}$$

$$V_o = 5I_1 = -2.6471 - j4.4118$$

$$S = \frac{1}{2}V_o I^* = \frac{1}{2}(-2.6471 - j4.4118)x2 = -2.6471 - j4.4118$$

Hence the average power supplied by the current source is **2.647 W**.

Chapter 11, Solution 3.



$$90\ \mu F \quad \longrightarrow \quad \frac{1}{j\omega C} = \frac{1}{j90x10^{-6}x2x10^{3}} = -j5.5556$$

$$I = 160/60 = 2.667A$$

The average power delivered to the load is the same as the average power absorbed by the resistor which is

$$P_{avg} = 0.5|I|^2 60 = \mathbf{213.4\ W}.$$

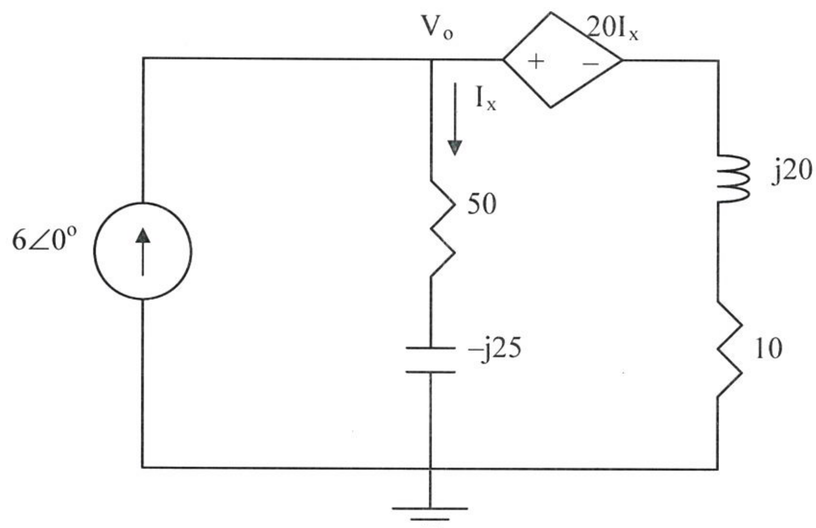
Chapter 11, Solution 6.

$$20\text{ mH} \longrightarrow j\omega L = j10^3 x 20 x 10^{-3} = j20$$

$$40\mu\text{F} \rightarrow \frac{1}{j\omega C} = \frac{1}{j10^3 x 40 x 10^{-6}} = -j25$$

We apply nodal analysis to the circuit below.

$6\cos 10^3 t \Rightarrow$



$$-6 + \frac{V_o - 20I_x}{10 + j20} + \frac{V_o - 0}{50 - j25} = 0$$

But $I_x = \dfrac{V_o}{50 - j25}$. Substituting this and solving for V_o leads

$$\left(\frac{1}{10 + j20} - \frac{20}{(10 + j20)}\frac{1}{(50 - j25)} + \frac{1}{50 - j25}\right)V_o = 6$$

$$\left(\frac{1}{22.36\angle 63.43^\circ} - \frac{20}{(22.36\angle 63.43^\circ)(55.9\angle -26.57^\circ)} + \frac{1}{55.9\angle -26.57^\circ}\right)V_o = 6$$

$$(0.02 - j0.04 - 0.012802 + j0.009598 + 0.016 + j0.008)V_o = 6$$

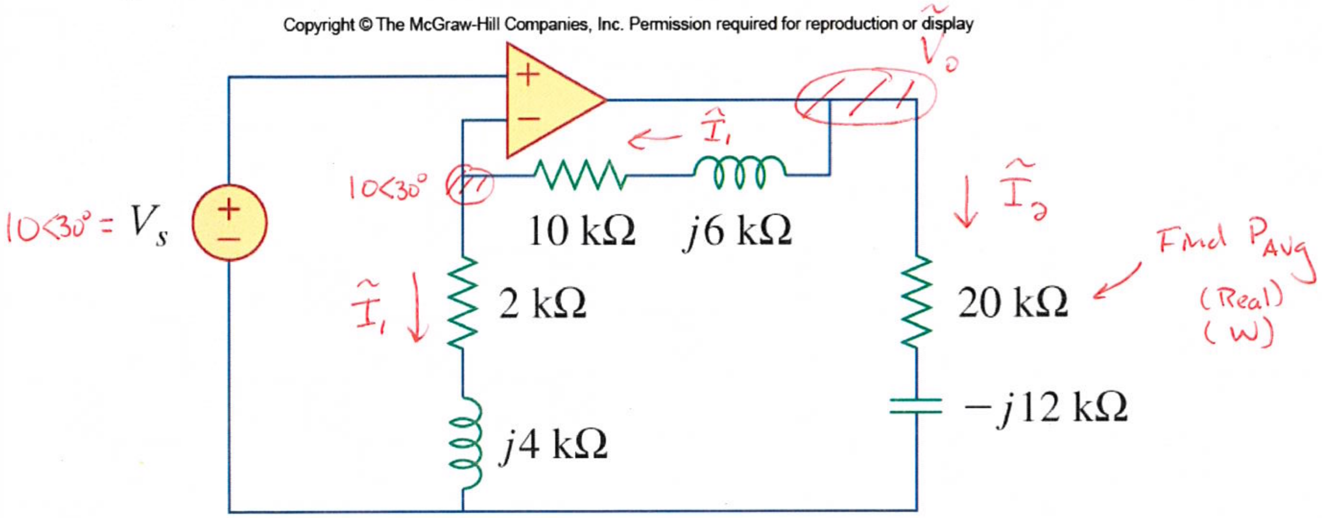
$(0.0232 - j0.0224)V_o = 6$ or $V_o = 6/(0.03225\angle -43.99^\circ) = 186.05\angle 43.99^\circ$ volts.

$|I_x| = 186.05/55.9 = 3.328$

We can now calculate the average power absorbed by the 50-Ω resistor.

$P_{avg} = [(3.328)^2/2]x50 =$ **276.8 W**.

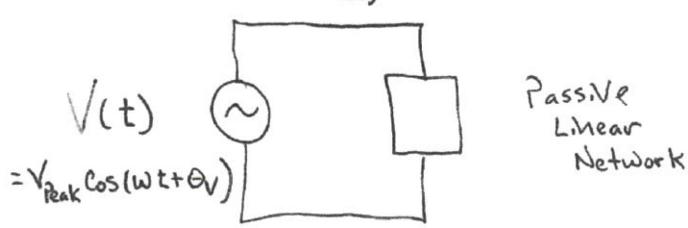
Figure 11.41



- $\hat{I}_1 = \dfrac{10\angle 30^\circ\ V}{(2k\Omega + j4k\Omega)} = 2.24\angle -33.4\ mA$
- $\hat{V}_o = 10\angle 30^\circ + \hat{I}_1 \cdot (10k\Omega + j6k\Omega)$
  $= 34.93\angle 6.37^\circ\ V$
- $\hat{I}_2 = 1.5\angle 37.3^\circ\ mA$
- (Assumes Peak Value)
  $P_{Avg_{20k\Omega}} = \frac{1}{2} \cdot (1.5_{mA})^2 \cdot 20k\Omega = \boxed{22.5\ mW}$

Instantaneous & Average Power

$i(t) = i_{peak} \cos(\omega t + \theta_i)$



- $P(t) = V(t) \cdot i(t)$

$$= V_{peak} \cos(\omega t + \theta_V) \cdot i_{peak} \cos(\omega t + \theta_i)$$

$$= \tfrac{1}{2} V_{peak} \cdot i_{peak} \left[\cos(\cancel{\omega t} + \theta_V - \cancel{\omega t} - \theta_i) + \ldots \right.$$

$$\left. \cos(2\omega t + \theta_V + \theta_i) \right]$$

$\cos A \cdot \cos B = \tfrac{1}{2}\cos(A-B) + \tfrac{1}{2}\cos(A+B)$

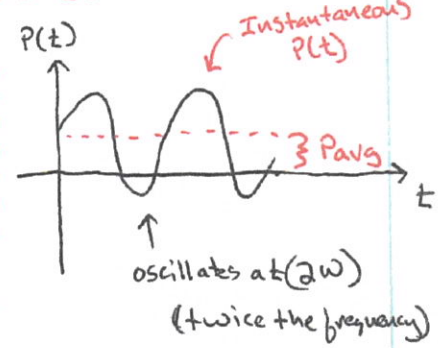
Instantaneous Power →

- $$P(t) = \tfrac{1}{2} V_{peak} \cdot i_{peak} \left[\underbrace{\cos(\theta_V - \theta_i)}_{\text{Constant}} + \underbrace{\cos(2\omega t + \theta_V + \theta_i)}_{\text{Averages to Zero}} \right]$$

- $$P_{avg} = \tfrac{1}{2} V_{peak} i_{peak} \cdot \cos(\underbrace{\theta_V - \theta_i}_{\theta_Z \leftarrow \text{Impedance Angle}})$$

$$= \tfrac{1}{2} (\sqrt{2} \cdot V_{rms})(\sqrt{2}\, i_{rms}) \cos(\theta_Z)$$

$$= V_{rms}\, i_{rms} \cos \theta_Z \qquad \text{(Watts)}$$



Notes

- Purely Resistive Load → $R + 0j = Z_R$
  - $\theta_V = \theta_i$, $\therefore \theta_V - \theta_i = 0$

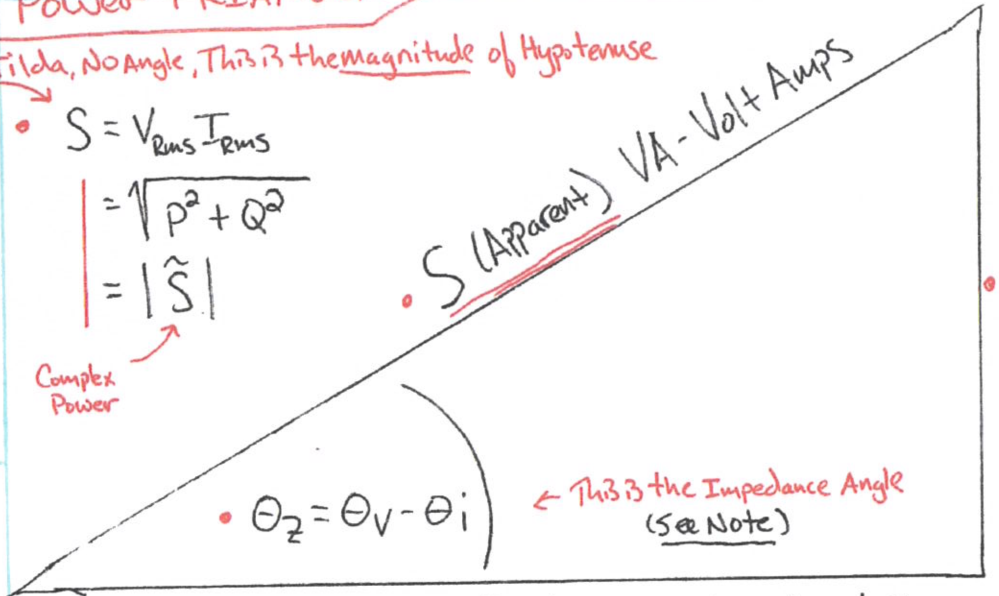
  $\Rightarrow \boxed{P_{avg} = V_{Rms} I_{Rms} \cos(0^\circ) = V_{Rms} I_{RMS} = \tfrac{1}{2} V_{peak} I_{peak}}$

- Purely Reactive Load (Inductive or Capacitive) → $0 + Xj = Z$
  - $\theta_V - \theta_i = \pm 90^\circ$

  $\Rightarrow \boxed{P_{avg} = V_{Rms} I_{Rms} \cos(\pm 90^\circ) = 0}$ ← No Avg. Power! (Sloshes Back and forth)

Power TRIANGLE

* No Tilda, No Angle, This is the magnitude of Hypotenuse



- $S = V_{Rms} I_{Rms}$
  $= \sqrt{P^2 + Q^2}$
  $= |\hat{S}|$ ← Complex Power

- S (Apparent) VA - Volt Amps

- $\Theta_z = \Theta_V - \Theta_i$ ← This is the Impedance Angle (see Note)

- P (Real or Average) Watts

- $P = S \cos\Theta_z$
  $= I_{Rms} V_{Rms} \cos\Theta_z$
  $= \frac{1}{2} I_{peak} V_{peak} \cos\Theta_z$
  $= \text{Real}[\hat{S}]$

*Note

- $Z = \frac{\tilde{V}}{\tilde{I}} = \frac{V\angle\Theta_V}{I\angle\Theta_i} = V/I \angle \Theta_V - \Theta_i = V/I \angle\Theta_z$

Impedance Angle

- Q (Reactive or Imaginary) VAR - Volt Amps Reactive

$Q = I_{Rms} V_{Rms} \sin\Theta_z$
$= \frac{1}{2} I_{peak} V_{peak} \sin\Theta_z$
$= S \sin\Theta_z$
$= \text{Imaginary}(\hat{S})$

Power FACTOR

- $PF = \cos\Theta_z = P/S$

- for Purely Resistive Load, $R + 0j = Z$
  $\rightarrow \Theta_z = 0°$
  $\rightarrow \hat{S} = V_{Rms} I_{Rms} \cos(0°) + V_{Rms} I_{Rms} \sin(0°) j$ (VA)
  $= V_{Rms} I_{Rms} + 0 j$ (VA or W)
  $= S\angle 0°$ VA or Watts ← in this case as No Reactive Portion

Putting it ALL Together with Complex Power

- $\hat{S} = \hat{V}_{Rms} \tilde{I}^*_{Rms}$ ← Complex Conjugate
  $= |\tilde{S}| \angle\Theta_z = S\angle\Theta_z$ ← Apparent Power

Polar Form →

$= V_{Rms}\angle\Theta_V \cdot I_{Rms}\angle -\Theta_i$ ← This is the Impedance Angle, Θ_z
$= V_{Rms} I_{Rms} \angle \Theta_V - \Theta_i$

Rectangular form →

$= V_{Rms} I_{Rms} \cos\Theta_z + V_{Rms} I_{Rms} \sin\Theta_z j$
$= P + Q j$
$= S\cos\Theta_z + S\sin\Theta_z j$
$= \hat{I}\cdot\hat{I}^* Z = |\hat{I}|^2 Z$
$= \hat{V}\cdot\hat{V}^*/Z^* = |\hat{V}|^2/Z^*$ ← Complex Conjugate => $+\Theta_z$ for $S\angle\Theta_z$