Name: Key
Section: ______

EE331 Homework PS9 – fall 2012

Problems from Alexander & Sadiku:

CH 11 –

- 11.51
- 11.61
- 11.69
- 11.71
- 11.74 (ans: pf = 0.9 lagging, C = 5.74 mF)

Additional Problems (Instructor Option):

- 11.85
- Any as assigned by instructor

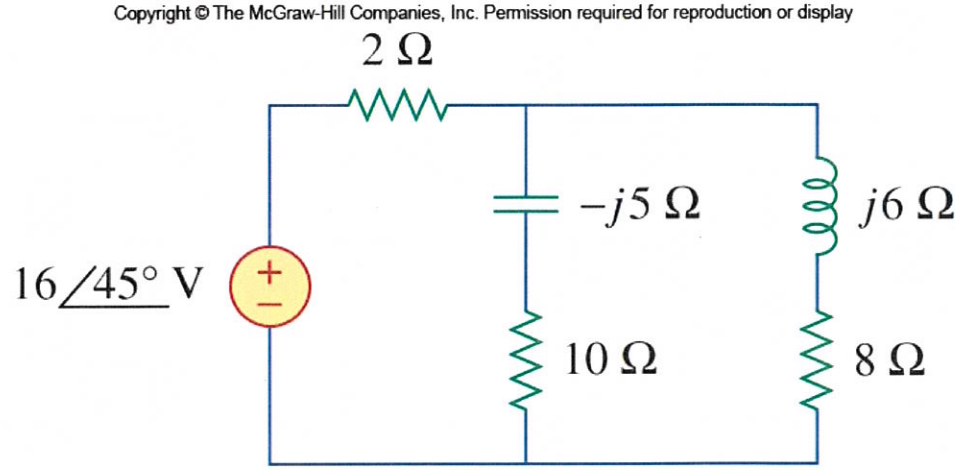
Chapter 11, Solution 51.

(a) $\mathbf{Z}_T = 2 + (10 - j5) \parallel (8 + j6)$

$$\mathbf{Z}_T = 2 + \frac{(10 - j5)(8 + j6)}{18 + j} = 2 + \frac{110 + j20}{18 + j}$$

$$\mathbf{Z}_T = 8.152 + j0.768 = 8.188\angle 5.382°$$

$\text{pf} = \cos(5.382°) = \mathbf{0.9956} \quad \textbf{(lagging)}$



(b) $$S = \mathbf{V}\mathbf{I}^* = \frac{|\mathbf{V}|^2}{\mathbf{Z}^*} = \frac{(16)^2}{(8.188\angle -5.382°)}$$

$S = 31.26\angle 5.382°$

$P = S\cos\theta = \mathbf{31.12\ W}$

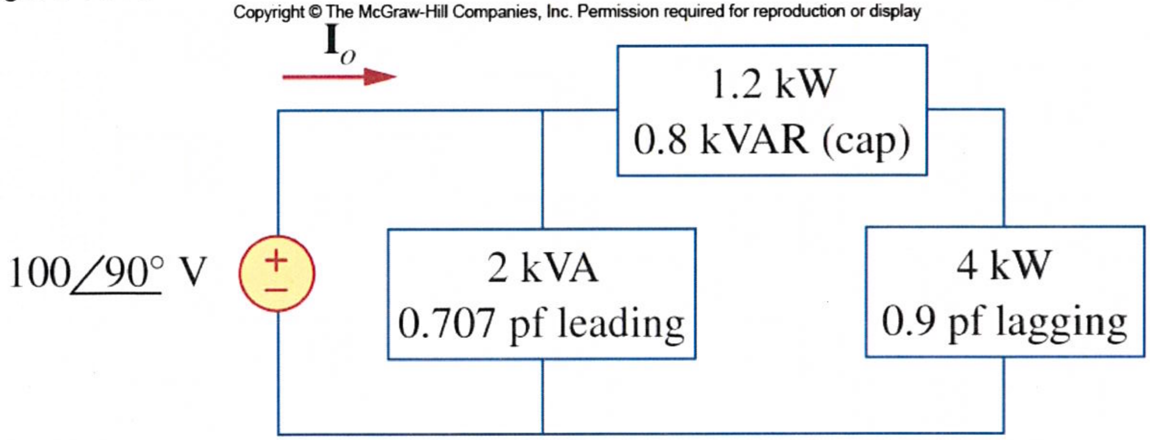
(c) $Q = S\sin\theta = \mathbf{2.932\ VAR}$

(d) $S = |S| = \mathbf{31.26\ VA}$

(e) $S = 31.26\angle 5.382° = \mathbf{(31.12 + j2.932)\ VA}$

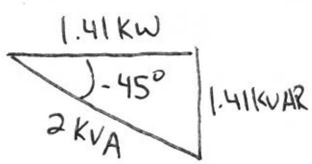
(a) 0.9956 (lagging, (b) 31.12 W, (c) 2.932 VAR, (d) 31.26 VA, (e) [31.12+j2.932] VA

Figure 11.80



$\hat{S}_{TOT} = \hat{S}_1 + \hat{S}_2 + \hat{S}_3$

$=$



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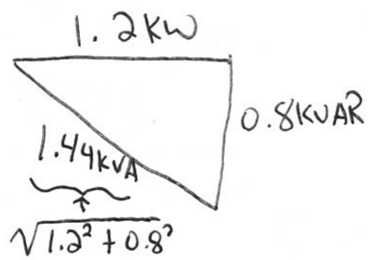

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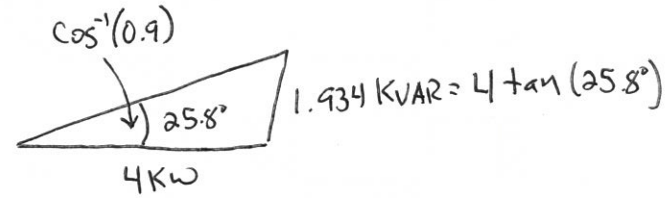

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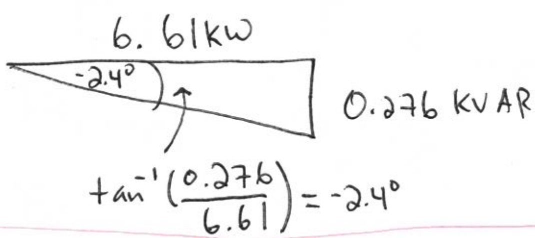


$$\boxed{\hat{S}_{TOT} = 6.61 + j0.276 = 6.62\angle -2.4° \text{ KVA}}$$

$$\hat{I}^* = \frac{\hat{S}_{tot}}{\hat{V}_{Source}} = \frac{6.62\angle -2.4° \text{ kVA}}{100\angle 90° \text{ V}} = 66.2\angle -92.4° \text{ A}$$

Complex Conjugate

$$\Rightarrow \boxed{\hat{I} = 66.2\angle 92.4° \text{A}}$$

Chapter 11, Problem 69.

Refer to the circuit shown in Fig. 11.88.

(a) What is the power factor?
(b) What is the average power dissipated?
(c) What is the value of the capacitance that will give a unity power factor when connected to the load?

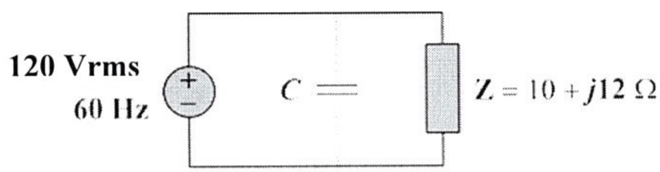


Figure 11.88

a) $Z = 10 + 12j = 15.6\angle 50.2^\circ\ \Omega$

$\theta_Z = \theta_V - \theta_i$

$\rightarrow pf = \cos(\theta_Z) = 0.64$ Lagging (ELI)

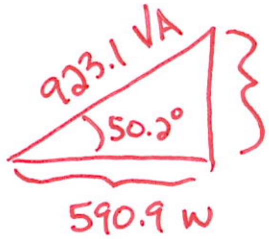
b) 1ST $\rightarrow$ $\hat{S} = \dfrac{V_{rms}^2}{Z^*} = \dfrac{(120V)^2}{15.6\angle -50.2^\circ} = 923.1\angle 50.2^\circ$ VA

or $\underbrace{590.9}_{P_{AVG}} + 709j$ VA

Complex Conjugate

$\rightarrow \boxed{P_{AVG} = 590.9\ W}$

c)



709 VAR

Want to Eliminate VARS

$\Rightarrow$

New Power Triangle w/ $\theta_Z = 0^\circ$

$\Downarrow$

590.9W

(pf = 1)

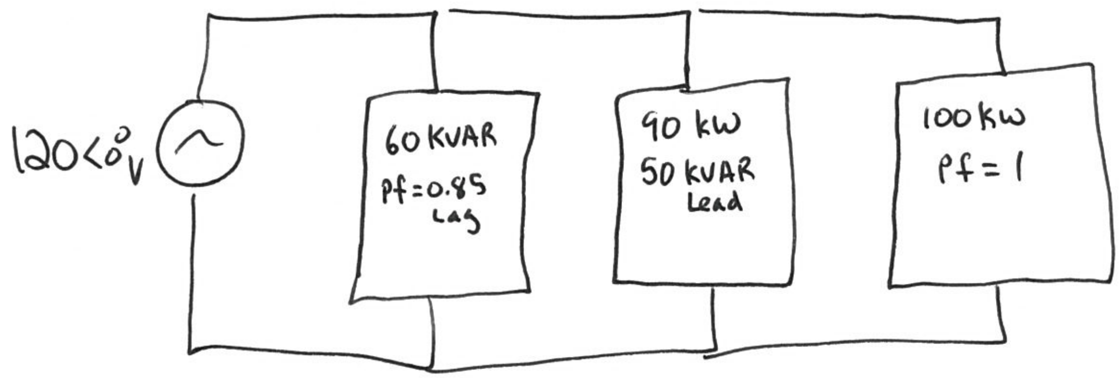
$\therefore$ NEED -709 VAR to Counter $+709$ VAR

(11.60) in Book $\rightarrow$ $Q = 709\ VAR = \dfrac{(120V)^2}{1/\omega C} = \dfrac{(120V)^2}{\dfrac{1}{(2\pi \cdot 60 \cdot C)}}$

$\Rightarrow \boxed{C = 130.6\ \mu F}$

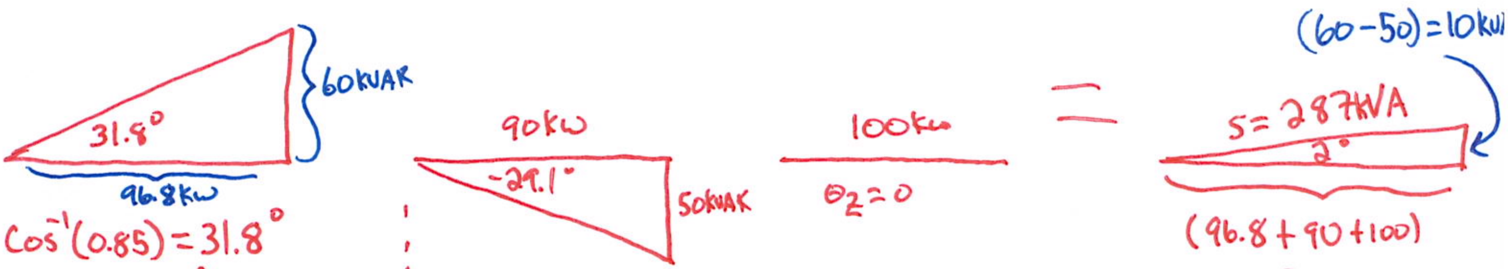
Chapter 11, Problem 71.

Three loads are connected in parallel to a $120\angle 0^\circ$ V rms source. Load 1 absorbs 60 kVAR at pf =0.85 lagging; load 2 absorbs 90 kW and 50 kVAR leading; and load 3 absorbs 100 kW at pf = 1. (a) Find the equivalent impedance. (b) Calculate the power factor of the parallel combination. (c) Determine the current supplied by the source.



1st USE Summation of Power Triangles

Load: #1 #2 #3 Total



- $\cos^{-1}(0.85) = 31.8^\circ$
  + as Inductive (ELI)
- $P = \dfrac{60\text{ kVAR}}{\tan 31.8^\circ} = 96.8\text{ kW}$
- $\theta_Z = \tan^{-1}\left(\frac{-50}{90}\right) = -29.1$
- $\theta_Z = \tan^{-1}\left(\frac{10}{286.8}\right) = 2^\circ$
- $S = \sqrt{286.8^2 + 10^2} = 286.97\text{ kVA}$

- $\tilde{S} = 287\angle 2^\circ$ kVA

a) $\tilde{S} = \dfrac{|\tilde{V}_{rms}|^2}{Z^*} \Rightarrow Z^* = \dfrac{(120V)^2}{287\angle 2^\circ \text{ kVA}} = 50.2\angle -2^\circ \text{ m}\Omega$

$\therefore \boxed{Z = 50.2\angle 2^\circ \text{ m}\Omega}$

b) $\boxed{pf = \cos\theta_Z = \cos(2^\circ) = 0.9994 \text{ LAgging}}$

c) $\tilde{I}_{Source} = \dfrac{\tilde{V}_{Source}}{Z} = \dfrac{120V\angle 0^\circ}{50.2\angle 2^\circ \text{ m}\Omega} = \boxed{2390.4\angle -2 \text{ A}}$

Chapter 11, Problem 74.

A 120-V rms 60-Hz source supplies two loads connected in parallel, as shown in Fig. 11.89.

(a) Find the power factor of the parallel combination.

(b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.

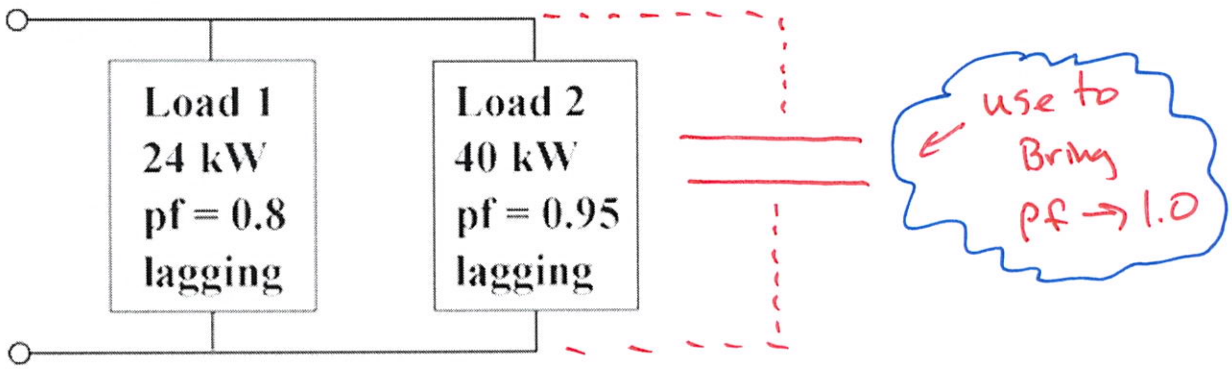
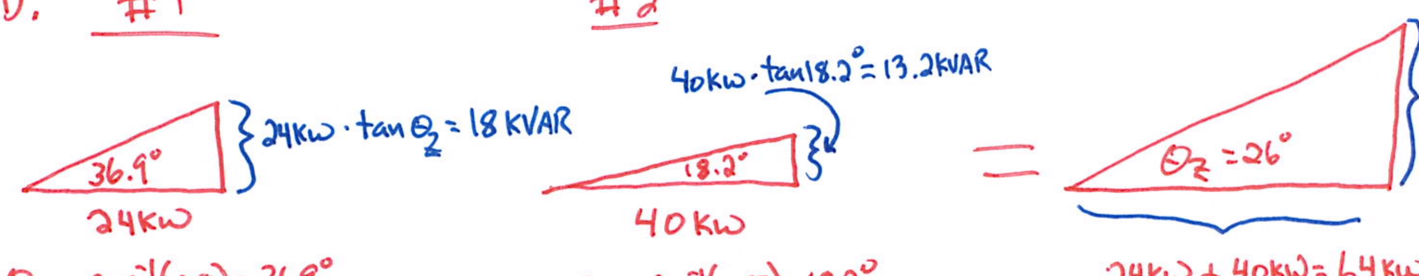


Figure 11.89

USE SUMMATION of Power Triangles

LOAD: #1 #2 Total



• $\theta_Z = \cos^{-1}(0.8) = 36.9°$

(+) ELI → Inductive

• $\theta_Z = \cos^{-1}(0.95) = 18.2°$

• $\theta_Z = \tan^{-1}\left(\frac{31.2}{64}\right) = 26°$

a) $\boxed{pf = \cos\theta_{Z_{Total}} = 0.9 \text{ LAGGING}}$

b) want Final Power Triangle → $\xrightarrow[64\text{KW}]{\theta_Z = 0°}$ } 0 VARS

(11.60) in Book

Need to cancel with ↗ $31.2\text{ KVAR} = \frac{V_{rms}^2}{|Z_c|} = \frac{(120V)^2}{\frac{1}{(2\pi\cdot 60)\cdot C}} \Rightarrow \boxed{C = 5.74\text{mF}}$

Chapter 11, Problem 85.

A regular household system of a single-phase three-wire allows the operation of both 120-V and 240-V, 60-Hz appliances. The household circuit is modeled as shown in Fig. 11.96. Calculate: (a) the currents $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_n$, (b) the total complex power supplied, (c) the overall power factor of the circuit.

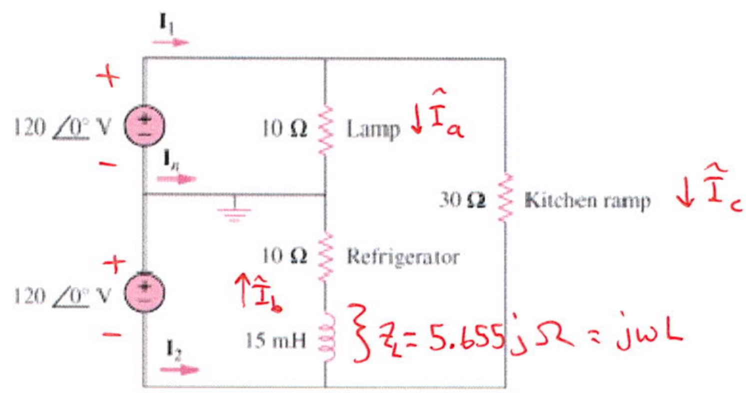


Figure 11.96

$$\hat{I}_a = \frac{120\,V\angle 0° - 0\,V}{10\,\Omega} = 12\angle 0°\ A$$

$$\hat{I}_b = \frac{-120\,V\angle 0° - 0}{10 + 5.655j\ \Omega} = -10.45\angle -29.5°\ A$$

$$\hat{I}_c = \frac{120\angle 0° - -120\angle 0}{30} = 8\angle 0°\ A$$

a)

$$\boxed{\begin{aligned} \hat{I}_1 &= \hat{I}_a + \hat{I}_c = 20\angle 0°\ A \\ \hat{I}_2 &= \hat{I}_b - \hat{I}_c = 17.85\angle 163.2°\ A \\ \hat{I}_n &= -\hat{I}_a - \hat{I}_b = 5.91\angle -119.4°\ A \end{aligned}}$$

b)

$$\hat{S}_{TOT} = \hat{S}_{Lamp} + \hat{S}_{Refrigerator} + \hat{S}_{KR} = \hat{I}_a^* \cdot 120\angle 0° + \hat{I}_b^*(-120\angle 0°) + \hat{I}_c^* \cdot 240\angle 0°$$

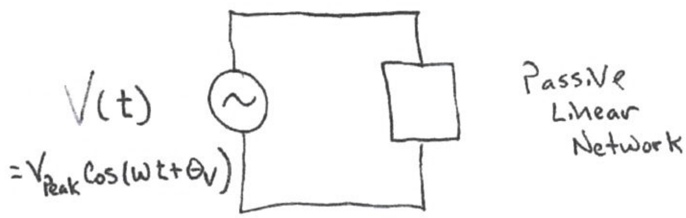
$$= 1440\angle 0° + 1254\angle 29.5° + 1920\angle 0°$$

$$= 4494.1\angle 7.9°\ \ VA$$

c) $\boxed{P.f. = \cos(7.9°) = 0.9905\ \text{Lagging}}$

Instantaneous & Average Power

$\rightarrow i(t) = i_{peak} \cos(\omega t + \theta_i)$



- $P(t) = V(t) \cdot i(t)$

$$= V_{peak} \cos(\omega t + \theta_V) \cdot i_{peak} \cos(\omega t + \theta_i)$$

$$= \tfrac{1}{2} V_{peak} \cdot i_{peak} \left[\cos(\cancel{\omega t} + \theta_V - \cancel{\omega t} - \theta_i) + \ldots \right.$$
$$\left. \cos(2\omega t + \theta_V + \theta_i) \right]$$

$\cos A \cdot \cos B = \tfrac{1}{2}\cos(A-B) + \tfrac{1}{2}\cos(A+B)$

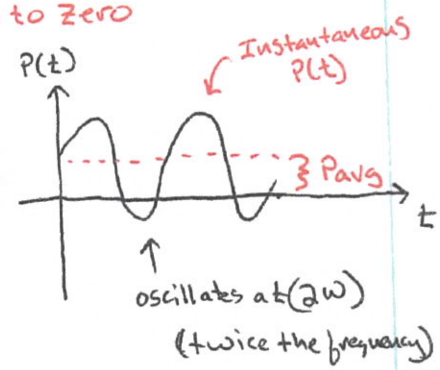
Instantaneous Power $\rightarrow$

- $$P(t) = \tfrac{1}{2} V_{peak} \cdot i_{peak} \left[\underbrace{\cos(\theta_V - \theta_i)}_{\text{Constant}} + \underbrace{\cos(2\omega t + \theta_V + \theta_i)}_{\text{Averages to Zero}} \right]$$

- $$\underline{P_{avg}} = \tfrac{1}{2} V_{peak} i_{peak} \cdot \cos(\underbrace{\theta_V - \theta_i}_{\theta_Z \leftarrow \text{Impedance Angle}})$$

$$= \tfrac{1}{2} (\sqrt{2} \cdot V_{rms})(\sqrt{2}\, i_{rms}) \cos(\theta_Z)$$

$$= V_{rms}\, i_{rms} \cos\theta_Z \qquad \text{(Watts)}$$



Notes

- Purely Resistive Load $\rightarrow R + 0j = Z_R$
  - $\theta_V = \theta_i$, $\therefore \theta_V - \theta_i = 0$

  $\Rightarrow \boxed{P_{avg} = V_{Rms} I_{Rms} \cos(0°) = V_{Rms} I_{Rms} = \tfrac{1}{2} V_{peak} I_{peak}}$

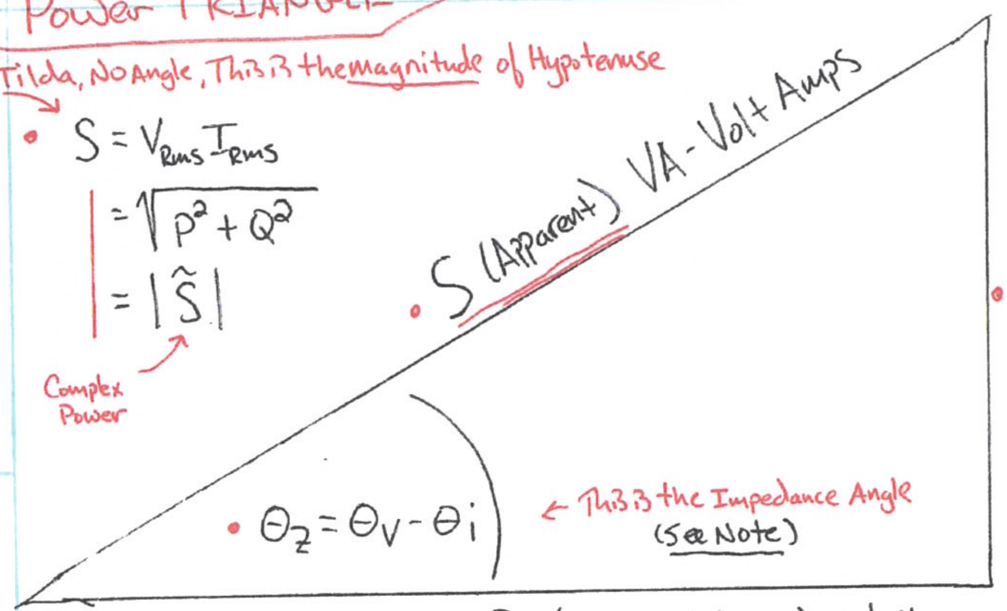
- Purely Reactive Load (Inductive or Capacitive) $\rightarrow 0 + Xj = Z$
  - $\theta_V - \theta_i = \pm 90°$

  $\Rightarrow \boxed{P_{avg} = V_{Rms} I_{Rms} \cos(\pm 90°) = 0}$ $\leftarrow$ No Avg. Power! (Sloshes Back and forth)

Power TRIANGLE

* No Tilda, No Angle, This is the magnitude of Hypotenuse

- $S = V_{Rms} I_{Rms}$
  $= \sqrt{P^2 + Q^2}$
  $= |\hat{S}|$ ← Complex Power



- S (Apparent) VA - Volt Amps
- $\theta_Z = \theta_V - \theta_i$ ← This is the Impedance Angle (See Note)
- P (Real or Average) Watts

* Note
- $Z = \dfrac{\tilde{V}}{\tilde{I}} = \dfrac{V\angle\theta_V}{I\angle\theta_i} = \dfrac{V}{I}\angle\theta_V - \theta_i = \dfrac{V}{I}\angle\theta_Z$ ← Impedance Angle

- Q (Reactive or Imaginary) VAR - Volt Amps Reactive

$Q = I_{Rms} V_{Rms} \sin\theta_Z$
$= \frac{1}{2} I_{peak} V_{peak} \sin\theta_Z$
$= S \sin\theta_Z$
$= \text{Imaginary}(\hat{S})$

- $P = S\cos\theta_Z$
  $= I_{Rms} V_{Rms} \cos\theta_Z$
  $= \frac{1}{2} I_{peak} V_{peak} \cos\theta_Z$
  $= \text{Real}[\hat{S}]$

Putting it ALL Together with Complex Power

- $\hat{S} = \tilde{V}_{Rms} \tilde{I}^*_{Rms}$ ← Complex Conjugate $= |\tilde{S}|\angle\theta_Z = S\angle\theta_Z$ ← Apparent Power

Polar Form
$= V_{Rms}\angle\theta_V \cdot I_{Rms}\angle{-\theta_i}$ ← This is the Impedance Angle, θ_Z
$= V_{Rms} I_{Rms} \angle\theta_V - \theta_i$

Rectangular form
$= V_{Rms} I_{Rms} \cos\theta_Z + V_{Rms} I_{Rms} \sin\theta_Z\, j$
$= P + Q\, j$
$= S\cos\theta_Z + S\sin\theta_Z\, j$
$= \tilde{I}\cdot\tilde{I}^* Z = |\hat{I}|^2 Z$
$= \tilde{V}\cdot\tilde{V}^*/Z^* = |\hat{V}|^2/Z^*$ ← Complex Conjugate $\Rightarrow +\theta_Z$ for $S\angle\theta_Z$

Power FACTOR

- $PF = \cos\theta_Z = P/S$

- for Purely Resistive Load, $R + 0j = Z$
  $\rightarrow \theta_Z = 0^\circ$
  $\rightarrow \hat{S} = V_{Rms} I_{Rms} \cos(0^\circ) + V_{Rms} I_{Rms} \sin(0^\circ)\, j$ (VA)
  $= V_{Rms} I_{Rms} + 0\, j$ (VA or W)
  $= S\angle 0^\circ$ VA or Watts ← in this case as No Reactive portion