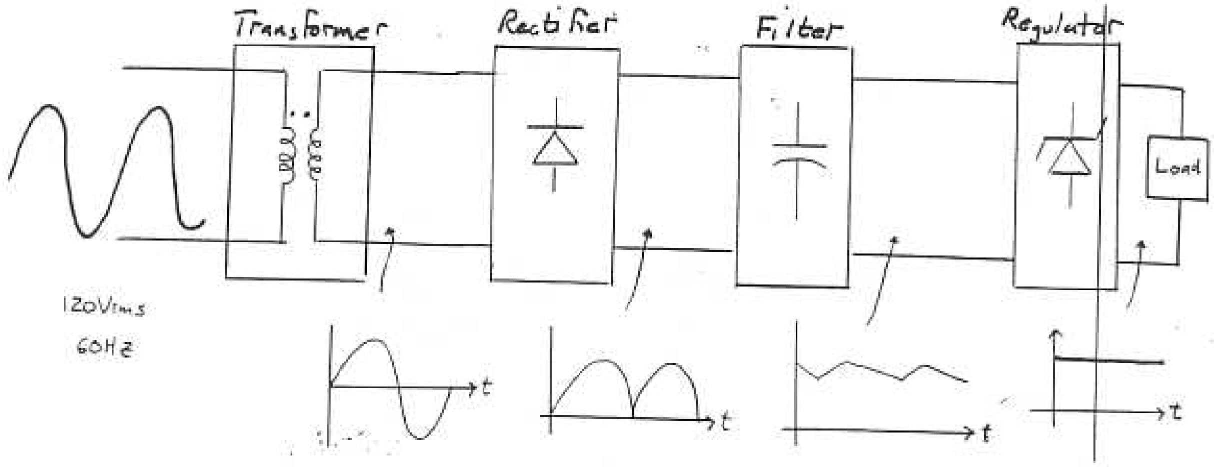
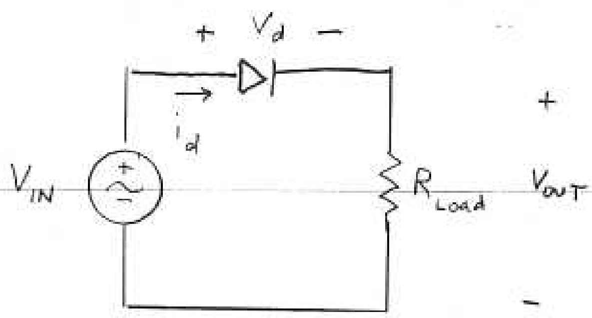
Recap from last time

(*) Most electronic equipment require a conversion from AC to DC



(*) The simplest structure of rectifier is the half-wave rectifier

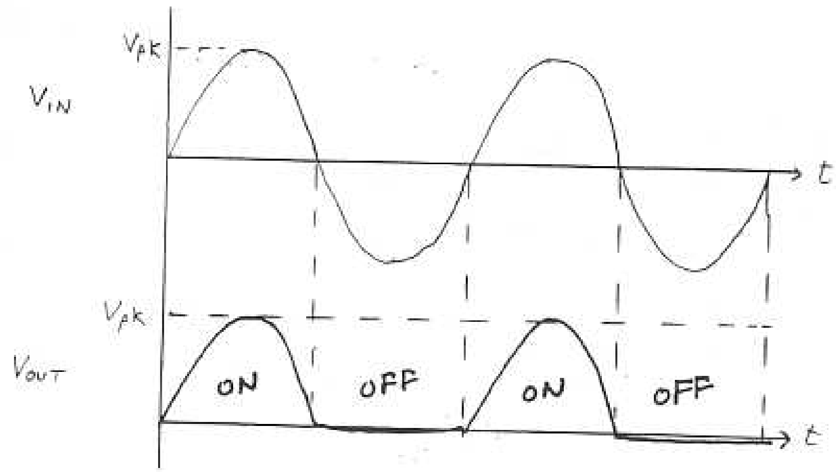


which contains a diode, which for its simplest model is governed by

Forward Biased : "ON" $V_d \approx 0$ $i_d > 0$

Reverse Biased : "OFF" $i_d = 0$ $V_d < 0$

(*) Upon assuming a "state" and performing circuit analysis then testing our assumption, we found



which we then used to find the average and RMS values

$$V_{OUT,ave} = \frac{V_{pk}}{\pi} \qquad V_{OUT,rms} = \frac{V_{pk}}{2}$$

(*) Finally, we asserted that those values would be influenced by two real-world phenomenon : diode voltage drop and the output resistance of the transformer so we would use

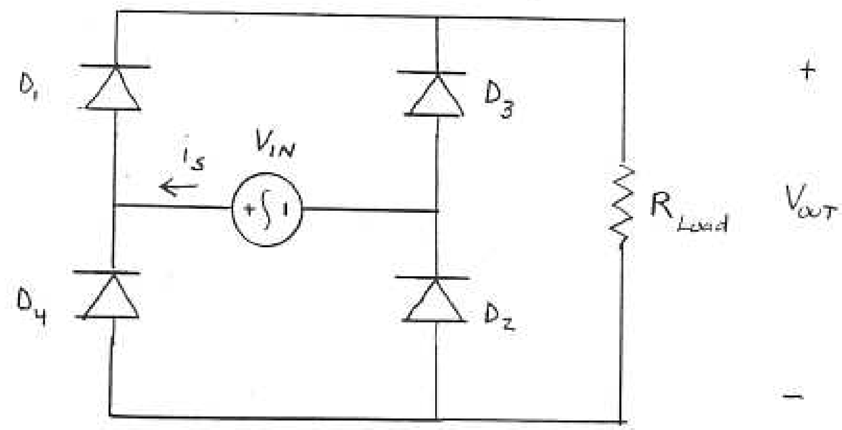
$$V_{pk} = \left(V_{sec,pk,NL} - 0.7 \right) \frac{R_{Load}}{R_{Load} + R_{TRAN}}$$

where 0.7V : assumed diode ON-state voltage drop

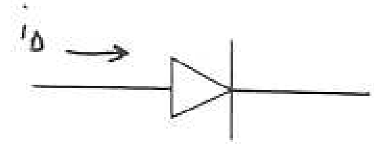
R_{TRAN} : assumed output resistance of the transformer

Full-Wave Rectifier

(*) We next want to consider a way to reduce the output voltage ripple and increase the average value. Consider the following topology



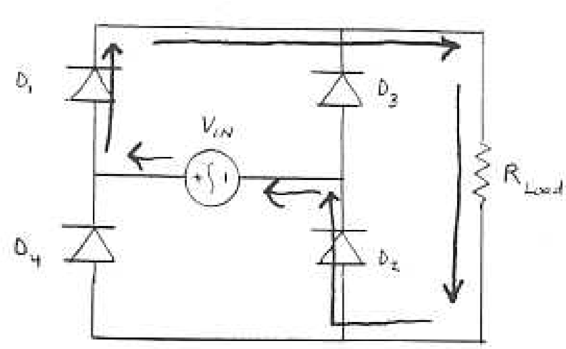
Since the diode only conducts current from anode to cathode



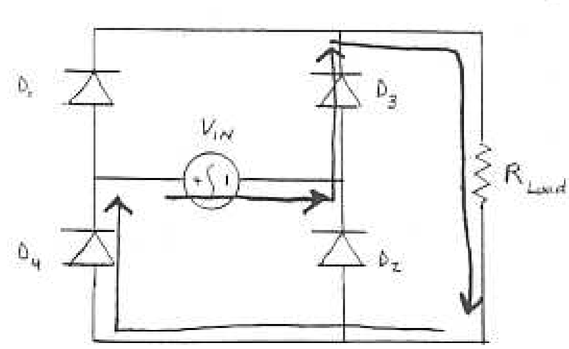
anode cathode

we have 2 possible paths from the source (and have a complete path)

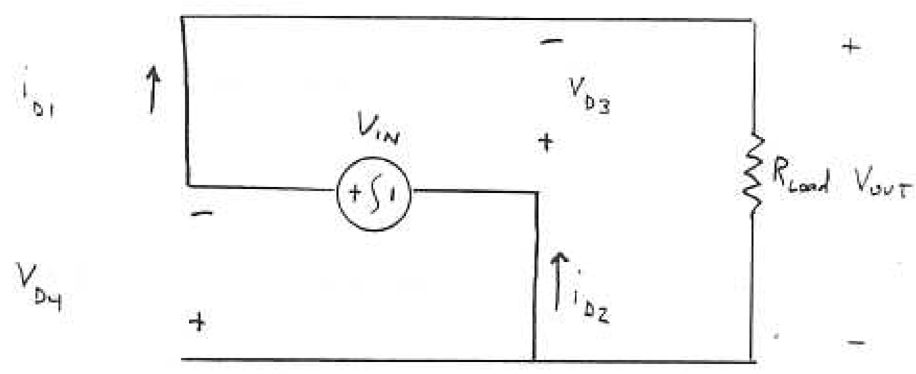
i_S positive



i_S negative



Step 1: Assume D_1 and D_2 are "ON" and D_3 and D_4 are "OFF"

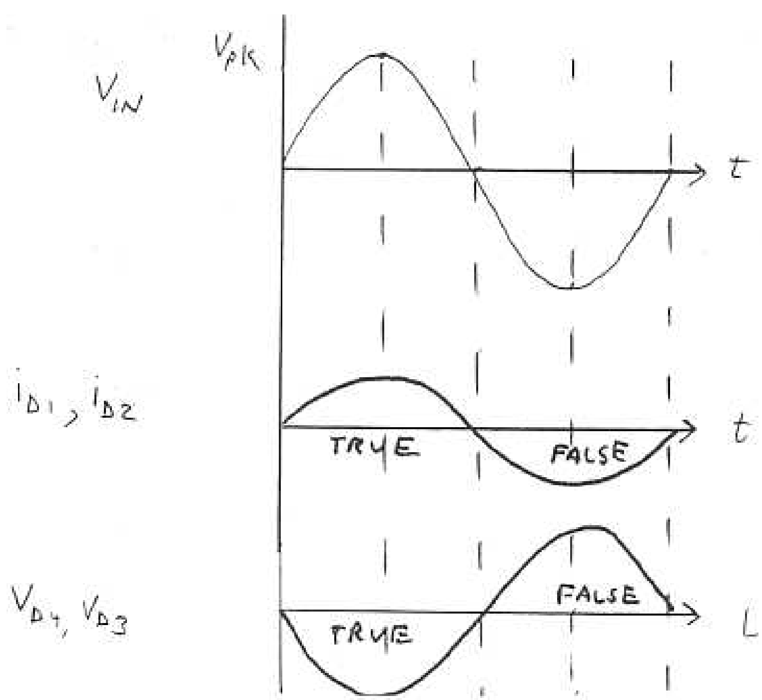


- KVL Demands : $V_{OUT} = V_{IN}$

  Further : $V_{D4} = V_{D3} = -V_{IN}$

- Ohm's Law Asserts : $i_{D1} = i_{D2} = \dfrac{V_{IN}}{R_{Load}}$

- IF we assume $V_{IN} = V_{pk} \sin \omega t$

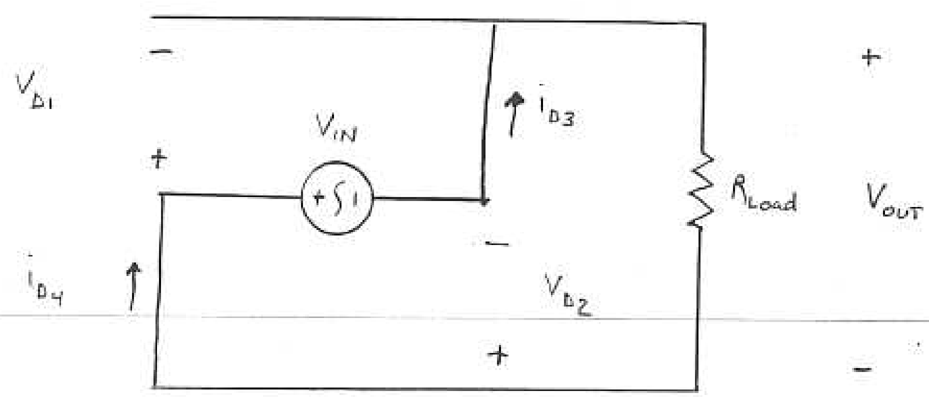


(*) The assumption on the status (D_1, D_2 ON D_3, D_4 OFF) is TRUE if $i_{D1} = i_{D2} > 0$ AND $V_{D3} = V_{D4} < 0$

and FALSE otherwise

→ Thus our assumption is TRUE for the positive half cycle

Step 2: Assume D_3 and D_4 are "ON" and D_1 and D_2 are "OFF"

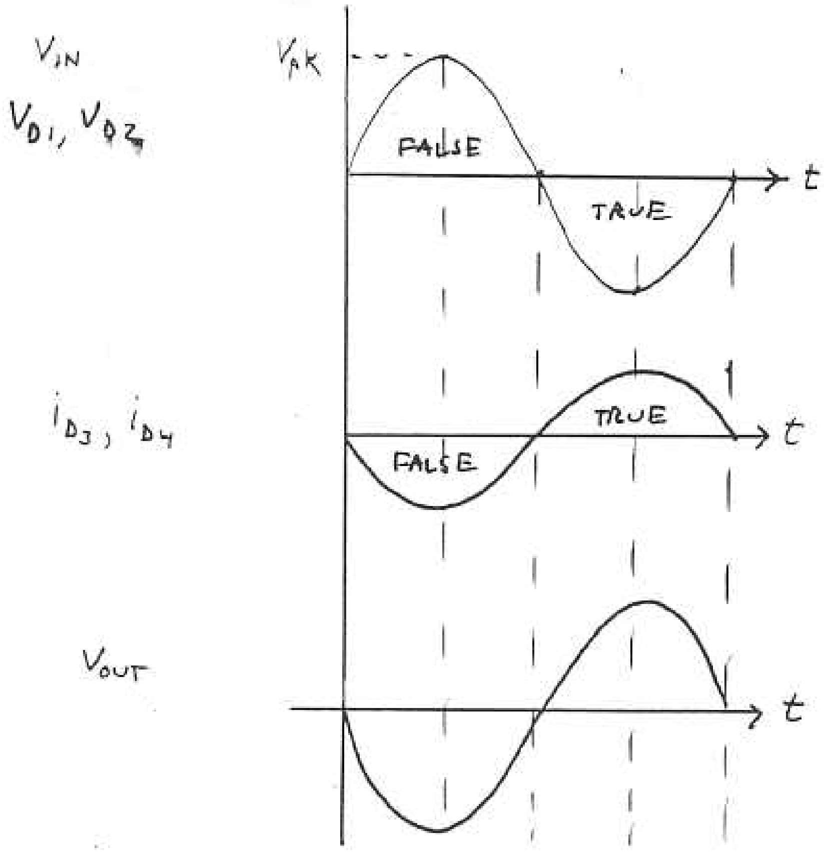


- KVL Demands: $V_{OUT} = -V_{IN}$

  And: $V_{D1} = V_{D2} = V_{IN}$

- Ohm's Law Gives: $i_{D3} = i_{D4} = \dfrac{-V_{IN}}{R_{Load}}$

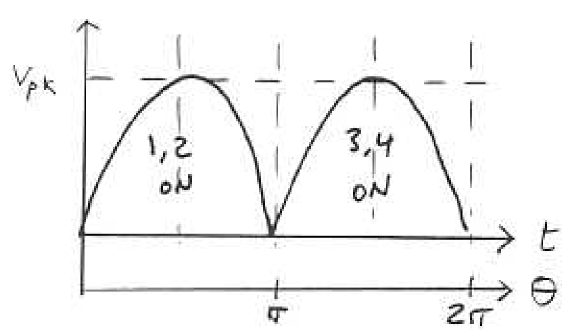
- With $V_{IN} = V_{pk} \sin\omega t$, sketching



(*) The assumption of the status (D_3, D_4 ON D_1, D_2 OFF) is TRUE if $i_{D3} = i_{D4} > 0$ AND $V_{D1} = V_{D2} < 0$ and FALSE otherwise

→ Thus our assumption is TRUE for the negative half cycle

Step 3 Recover V_{OUT}



(*) We can now determine the AVERAGE and RMS values as we did for the half-wave rectifier:

$$V_{OUT,ave} = \frac{1}{\pi}\left[\int_0^{\pi} V_{pk}\sin\theta\, d\theta\right] = \frac{2V_{pk}}{\pi}$$

$$V_{OUT,RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_{OUT})^2\, d\theta} = \frac{V_{pk}}{\sqrt{2}}$$

since squaring V_{OUT} gives the same waveform as squaring V_{IN} and thus it must have the same RMS value!

(*) The average power delivered to the load resistance is found as

$$P_{Load,ave} = \frac{V_{OUT,RMS}^2}{R_{Load}}$$

(*) To account for non-ideal effects, we must modify V_{pk} but this time we have 2 diode drops so

$$V_{pk} = (V_{sec,pk,NL} - 1.4V)\frac{R_{Load}}{R_{Load} + R_{TRAN}}$$

ex. Suppose you wish to deliver 2W to a 20Ω load through a transformer with a 2Ω output resistance, determine the No-Load RMS value of the transformer secondary assuming a Full-wave rectifier is used

$$\rightarrow \quad V_{OUT,RMS} = \sqrt{P_{Load,ave} R_{Load}} = 6.325V$$

$$\rightarrow \quad V_{pk} = \sqrt{2}\, V_{OUT,RMS} = 8.944V$$

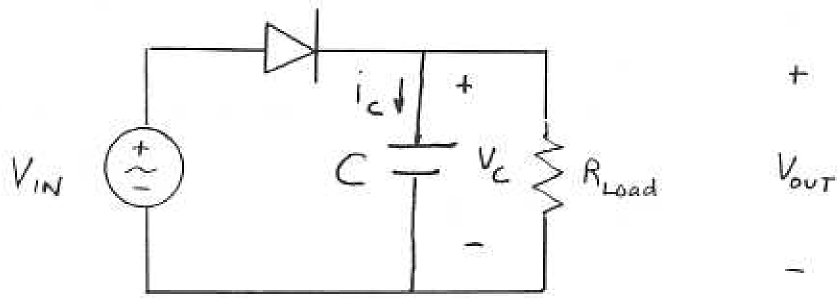
$$\rightarrow \quad V_{sec,pk,NL} = \frac{20+2}{20}(V_{pk}) + 1.4V = 11.23V$$

$$\rightarrow \quad V_{sec,RMS,NL} = \frac{V_{sec,pk,NL}}{\sqrt{2}} = 7.95V$$

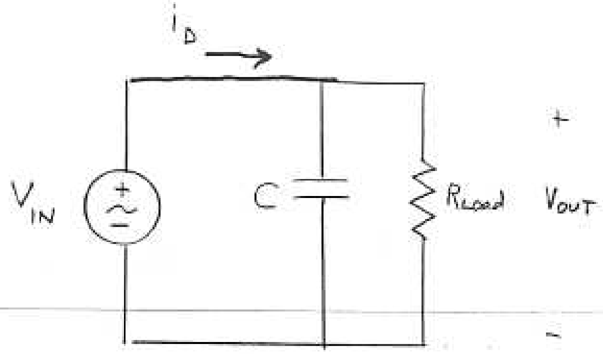
(*) Many loads require "smoother" output voltage and will not operate properly without it. Thus, we need to Filter the output -- with the simplest example being a capacitor

Filtering

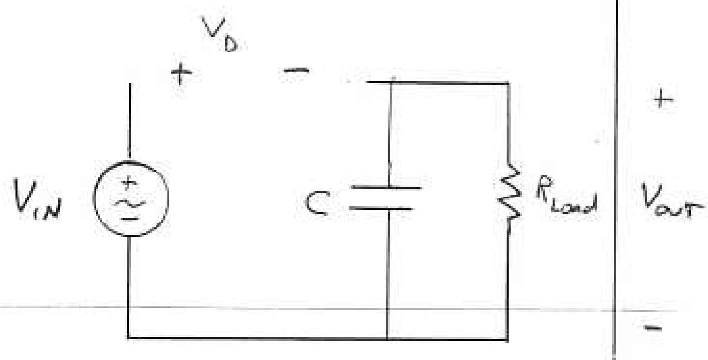
(*) Half-Wave Rectifier



Assuming an "ideal" diode, the circuit has two possible states



DIODE ON



DIODE OFF

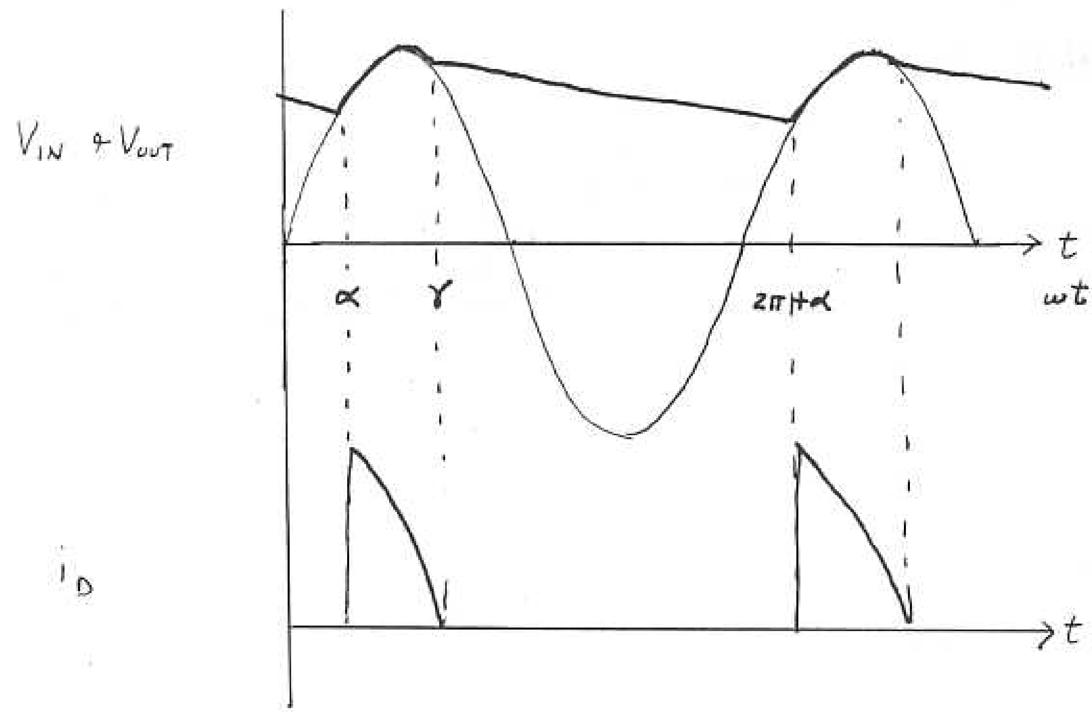
→ Capacitor charging

→ $V_{OUT} = V_{IN}$

→ Capacitor discharging

→ $V_{OUT} = V_C$

"Basic Idea"

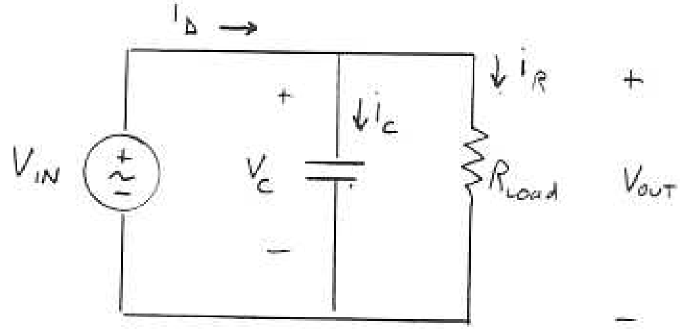


"The Analysis"

<u>Define</u>: α as angle at which diode starts to conduct

<u>Define</u>: γ as angle at which diode stops conducting

<u>Step 1</u> Analyze the circuit when the diode is assumed "ON" and Find when $i_D > 0$



(*) The components are in parallel and if we let

$V_{IN} = V_{pk} \sin \omega t = V_{pk} \sin \theta,$ then

$$i_R = \frac{V_{IN}}{R_{Load}} = \frac{V_{pk}}{R_{Load}} \sin \omega t$$

$$i_C = C \frac{dV_c}{dt} = C \frac{d}{dt}\left[V_{pk} \sin \omega t\right] = V_{pk}\, \omega C \cos \omega t$$

Then KCL demands that

$$i_D = i_C + i_R$$

which we can add using phasors to get

$$i_D = V_{pk} \sqrt{(\omega C)^2 + (R_{Load})^{-2}}\; \sin\left[\omega t + \tan^{-1}(\omega R_{Load} C)\right]$$

The diode conducts until this current goes to zero.

Setting

$$i_D = 0 \longrightarrow \sin\left[\omega t + \tan^{-1}(\omega R_{Load} C)\right] = 0$$

which occurs when

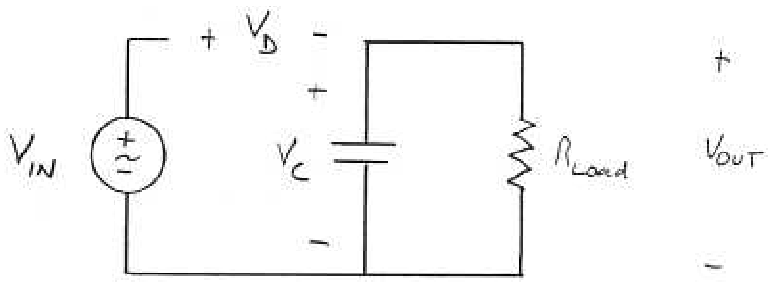
$$\omega t + \tan^{-1}(\omega R_{Load}\, C) = 0, \pi, 2\pi$$

But we defined $\omega t = \gamma$
at this point

So we find based on the figure on page 32-10

$$\gamma = \pi - \tan^{-1}(\omega R_{Load} C)$$

<u>Step 2</u> Analyze the circuit when the diode is assumed "OFF" and Find when $V_D < 0$



(*) Since the diode turns off at $\omega t = \gamma$, at that point $V_{IN} = V_C = V_{pk} \sin(\gamma)$ and that voltage cannot change instantaneously and must be the starting point for this circuit.

$$V_I = V_{pk} \sin \gamma$$

(*) The capacitor voltage will exponentially decay. governed by

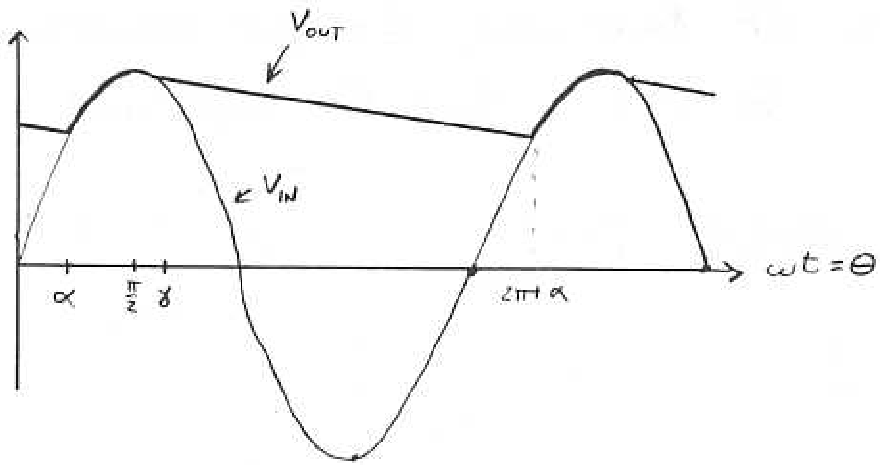
$$V_C = V_I \, e^{-\left(\frac{t - t_\gamma}{RC}\right)}$$

or in terms of angles

$$V_C = V_I \, e^{-\left(\frac{\theta - \gamma}{\omega RC}\right)}$$

(*) The diode voltage is given by

$$V_D = V_{IN} - V_C$$



(*) The diode conducts until $V_D = 0$ when $\theta = 2\pi + \alpha$

$$V_D = V_{pk} \sin(2\pi + \alpha) - V_{pk} \sin(\gamma)\, e^{-\left(\frac{2\pi + \alpha - \gamma}{\omega R_{Load} C}\right)} = 0$$

even with γ known, this is a non-linear equation that is transcendental (multiple solutions) (Fortunately our calculator solves it!)

<u>Step 3</u> Once we know γ and α, we can find the capacitor ripple by

$$\Delta V_{OUT} = V_{pk} - V_{pk} \sin \alpha$$

and the average value is very close to

$$V_{OUT,AVE} = V_{pk} - \frac{\Delta V_{OUT}}{2}$$

or

$$V_{OUT,AVE} = (V_{pk} + V_{pk} \sin \alpha)/2$$

ex. IF $R_{Load} = 500\Omega$, $C = 100\mu F$, $\omega = 377 \text{ rad/s}$ and $V_{pk} = 15V$, Find the ripple and average voltage

Step 1 Evaluate $\omega R_{Load} C = 18.85 \text{ rad}$

Step 2 Find γ

$$\gamma = \pi - \tan^{-1}(\omega R_{Load} C) = 1.62 \text{ rad}$$

1.51779

Step 3 Find α

$$\sin(\alpha) - \sin(\gamma)\, e^{-\left(\frac{2\pi + \alpha - \gamma}{\omega R_{Load} C}\right)} = 0$$

Make sure to have MODE → ANGLE → RADIAN

$$\alpha = 0.843 \text{ rad}$$

Step 4 Find ΔV_{OUT} and $V_{OUT,AVE}$

$$\Delta V_{OUT} = V_{pk} - V_{pk} \sin\alpha = 3.85V$$

$$V_{OUT,AVE} = 15 - \frac{3.85}{2} = 13.07V$$