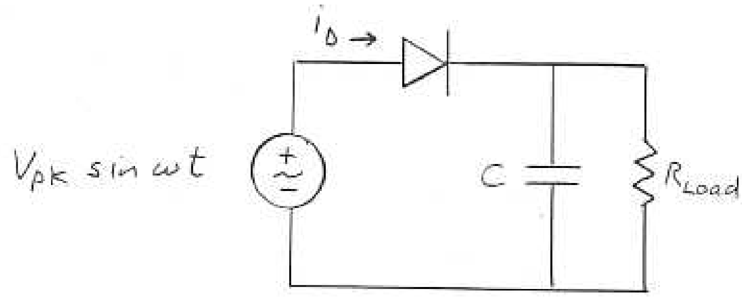
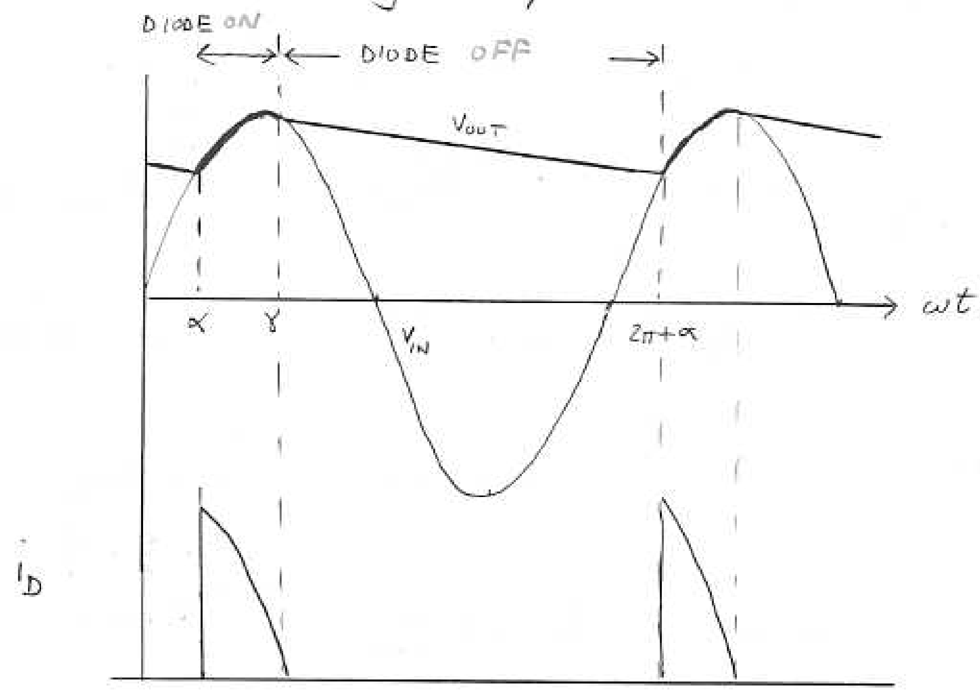
FILTERING SIMPLIFICATION

(*) For the half-wave rectifier with capacitor filter



we found the following steady-state waveforms



we determined

- $\gamma = \pi - \tan^{-1}(\omega R_{Load} C)$
- $\sin\alpha - \sin(\gamma)\, e^{-\left(\frac{2\pi+\alpha-\gamma}{\omega R_{Load} C}\right)} = 0$
- $\Delta V_{OUT} = V_{pk} - V_{pk} \sin\alpha$
- $V_{OUT,AVE} = V_{pk} - \frac{\Delta V_{OUT}}{2}$

(*) These are cumbersome equations to design a filter!

Assumptions:

1. $\omega R_{Load} C$ is typically big

2. Such that $\gamma \approx \pi - \frac{\pi}{2} \approx \frac{\pi}{2}$

3. If the diode does not conduct much, then $\alpha \approx \frac{\pi}{2}$

When the diode is OFF (pg 33-1 Figure), the output voltage is given by

$$V_{OUT} = (V_{pk} \sin\gamma) e^{-\left(\frac{\theta - \gamma}{\omega R_{Load} C}\right)} \qquad \gamma < \theta < 2\pi + \alpha$$

which hits its minimum value at $\theta = 2\pi + \alpha$

(*) Using the above assumptions, we find

$$V_{OUT,MIN} = V_{pk} \sin\left(\frac{\pi}{2}\right) e^{-\left(\frac{2\pi + \frac{\pi}{2} - \frac{\pi}{2}}{\omega R_{Load} C}\right)} = V_{pk} e^{-\frac{2\pi}{\omega R_{Load} C}}$$

$(\omega R_{Load} C >> 2\pi)$

This can be approximated by its Taylor expansion as

$$V_{OUT,MIN} \approx V_{pk} \left[1 - \frac{2\pi}{\omega R_{Load} C}\right]$$

(*) The ripple is then approximated as

$$\Delta V_{OUT} = V_{pK} - V_{OUT,MIN} = V_{pK} \frac{2\pi}{\omega R_{Load} C}$$

But with $\omega = 2\pi F$, we substitute and simplify

$$\Delta V_{OUT} = \frac{V_{pK}}{F R_{Load} C}$$

Finally the average value is found to be

$$V_{OUT,ave} = V_{pK} - \frac{\Delta V_{OUT}}{2}$$

(*) Which gives us a couple of good "design" equations

Note 1 : The ripple is largest when R_{Load} is smallest

Note 2 : The average voltage decreases with R_{Load} small

Note 3 : The ripple can be made small by making C big ex. 1 Suppose R_{Load} varies between 50Ω and 250Ω. The input voltage is 60Hz and we desire the minimum average output voltage to be 12V and the maximum ripple is 4V. Find the required C and V_{pk}

Recall

$$\Delta V_{OUT} \approx \frac{V_{pk}}{F R_{Load} C}$$

$$V_{OUT,ave} \approx V_{pk}\left[1 - \frac{1}{2F R_{Load} C}\right]$$

(*) Note $\Delta V_{OUT,max}$ occurs when $R_{Load} = 50\Omega$

$V_{OUT,ave,\,min}$ occurs when $R_{Load} = 50\Omega$

(*) To eliminate having to know V_{pk} form

$$\frac{\Delta V_{OUT,max}}{V_{OUT,ave,min}} = \frac{\dfrac{1}{F R_{Load,min} C}}{\dfrac{2F R_{Load,min} C - 1}{2F R_{Load,min} C}}$$

or

$$\frac{\Delta V_{OUT,max}}{V_{OUT,ave,min}} = \frac{2}{2F R_{Load,min} C - 1}$$

substituting

$$\frac{4V}{12V} = \frac{2}{2(60)(50)C - 1}$$

and solving for C gives $C = 1167\mu F$

which is a Fair amount of capacitance!

(*) We can then solve for V_{pk} using

$$\Delta V_{OUT,max} = \frac{V_{pk}}{f R_{Load,min} C}$$

$$4V = \frac{V_{pk}}{60(50)(1167\mu F)}$$

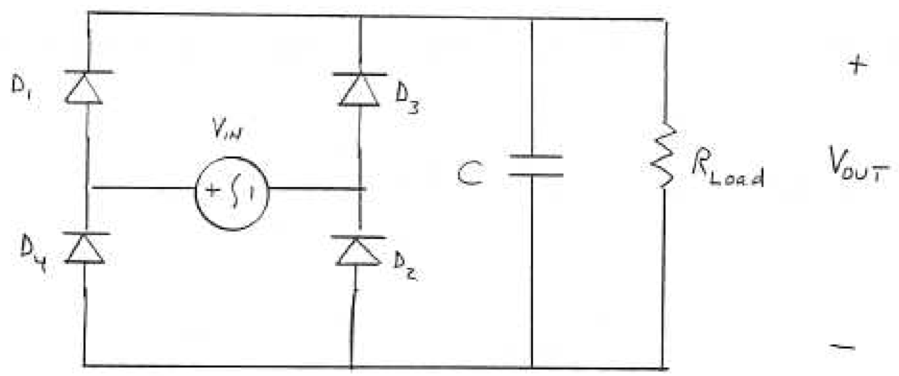
giving

$$V_{pk} = 14V$$

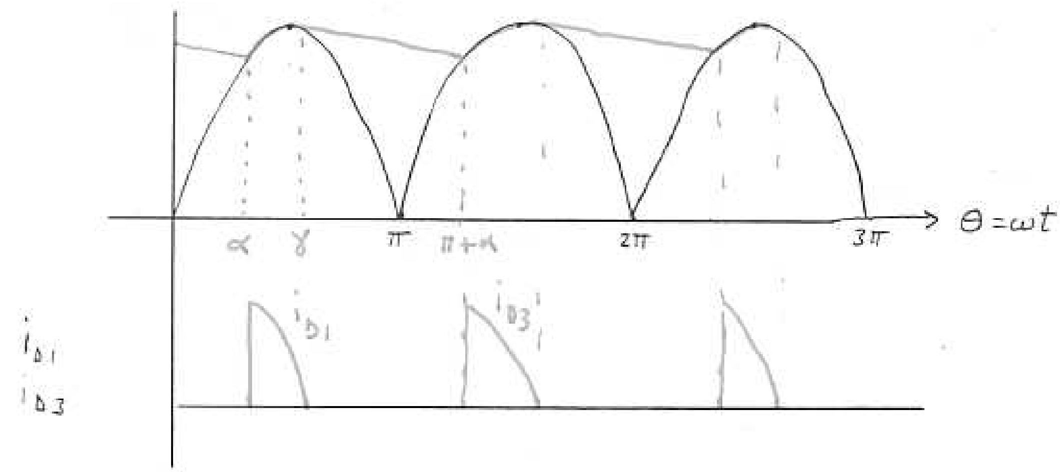
Q. For a varying load, how can we minimize the ripple across the load and maintain the average voltage constant?

→ we need to know more!

FULL-WAVE RECTIFIER WITH CAPACITOR FILTER



(*) The resultant steady-state waveforms appear as follows



(*) The analysis follows as before (shown in the Appendix) with the following approximate results

$$\Delta V_{OUT} \approx \frac{V_{pk}}{2 F R_{Load} C}$$

$$V_{OUT,ave} \approx V_{pk} - \frac{\Delta V_{OUT}}{2}$$

$$V_{OUT,ave} \approx V_{pk} \left[\frac{4 F R_{Load} C - 1}{4 F R_{Load} C} \right]$$

and Following some algebra

$$\frac{\Delta V_{OUT}}{V_{OUT,ave}} \approx \frac{2}{4 F R_{Load} C - 1}$$

(*) We notice that the ripple is reduced by a Factor of 2 From the half-wave case, meaning less capacitance is required.

ex. 2 R_{Load} varies between 50Ω and 250Ω. The input voltage is 60Hz. We want the minimum average output voltage to be 12V and the maximum ripple to be 4V. Find the required C and V_{pk}

→ $\Delta V_{OUT,max}$ occurs when $R_{Load} = 50\Omega$

→ $V_{OUT,ave,min}$ occurs when $R_{Load} = 50\Omega$

$$\frac{\Delta V_{OUT}}{V_{OUT,ave}} = \frac{4V}{12V} = \frac{2}{4(60)(50)C - 1}$$

solving For C gives

$$C = 583\mu F$$

Substituting back into the ripple formula yields

$$\Delta V_{OUT,max} = \frac{V_{pk}}{2 F R_{Load,min} C}$$

$$4V = \frac{V_{pk}}{2(60)(50)(583 \times 10^{-6})}$$

which solves as $V_{pk} = 14V$

Note: to size the required secondary voltage of the transformer, you would need to account for diode drops and transformer output resistance!