EE334 Homework – Gain and Decibels

Work the following problems:

1. Express the following gains in decibels.

   a. V_{in} = 0.005 V, V_{out} = 1.5 V

   $$G = 20\log\left(\frac{V_{out}}{V_{in}}\right) = 20\log\left(\frac{1.5\ V}{0.005\ V}\right) = \boxed{49.5\ dB}$$

   b. P_{in} = 10 W, P_{out} = 5 W

   $$\mathrm{G}{=}10\log\left(\frac{\mathrm{P_{out}}}{\mathrm{P_{in}}}\right){=}10\log\left(\frac{5\ \mathrm{W}}{10\ \mathrm{W}}\right) = \boxed{-3\ \mathrm{dB}}$$

   c. P_{in} = 5 W, P_{out} = 10 W

   $$G = 10\log\left(\frac{P_{out}}{P_{in}}\right) = 10\log\left(\frac{10\ W}{5\ W}\right) = \boxed{3\ dB}$$

2. Express the following gains as ratios.

   a. 12.5 dB in reference to power.

   $$A_P = 10^{\left(\frac{G_{dB}}{10}\right)} = 10^{\left(\frac{12.5\ dB}{10}\right)} = \boxed{17.8}$$

   b. -30 dB in reference to power

   $$A_P = 10^{\left(\frac{G_{dB}}{10}\right)} = 10^{\left(\frac{-30\ dB}{10}\right)} = \boxed{0.001 = \frac{1}{1000}}$$

   c. 6 dB in reference to voltage.

   $$A_V = 10^{\left(\frac{G_{dB}}{20}\right)} = 10^{\left(\frac{6\ dB}{20}\right)} = \boxed{2}$$

   d. -6 dB in reference to voltage.

   $$A_V = 10^{\left(\frac{G_{dB}}{20}\right)} = 10^{\left(\frac{-6\ dB}{20}\right)} = \boxed{0.5 = \frac{1}{2}}$$

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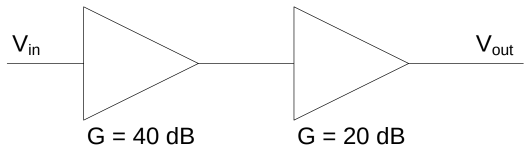
3. Find the requested values for the systems described below.

a. Two voltage amplifiers with gains of 40 dB and 20 dB, respectively, are cascaded and have an input voltage of 8.2 μV. What is the overall gain in dB and what is the output voltage?

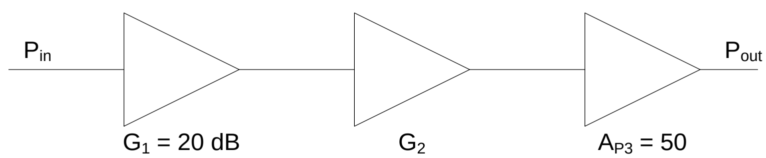


$$G_{Total} = G_1 + G_2 = 40\ dB + 20dB = \boxed{60\ dB}$$

$$G_{Total} = 20\log A_V = 20\log\left(\frac{V_{out}}{V_{in}}\right)$$

$$V_{out} = V_{in}\left(10^{\frac{G_{Total}}{20}}\right) = 8.2\ \mu V\left(10^{\frac{60\ dB}{20}}\right) = 8.2\ \mu V(10^3) = \boxed{8.2\ mV}$$

b. A three-stage power amplifier used a series of three amplifiers. The overall gain of the system is 60 dB. The first amplifier in the chain has a gain of 20 dB and the last one in the chain has a gain of 50 (ratio). What is the gain of the middle amplifier as a ratio and in decibels?



$$G_{Total} = 60dB = G_1 + G_2 + G_3 = 20dB + G_2 + 10\log\left(\frac{50}{1}\right)$$

$$G_2 = 60dB - G_1 - G_3 = 60dB - 20dB - 10\log(50) = 40dB - 17dB = \boxed{23dB}$$

$$A_{P2} = 10^{\left(\frac{G_2}{10}\right)} = 10^{\left(\frac{23\ dB}{10}\right)} \cong \boxed{200}$$

4. You are driving around town in your car while tuned to "FM 93.7 The All Hit Wonder Station." The station's transmitter is emitting an average power, P_t, of 8,000 Watts at a frequency, f, of 93.7 MHz. You are 15 kilometers from the station. Both you (the receiver) and the station (the transmitter) are using dipole antennas with gains of 3dB each ($G_t = G_r =$ 3dB)

   a. Using the equations below, how much power is being received, P_r, by your car's antenna?

$$c = \lambda f$$

$$P_r = \frac{P_t G_t G_r \lambda^2}{16\pi^2 d^2}$$

   where c is the speed of light (3x10^8 m/s), λ is the signal's wavelength, and d is the distance in meters between the transmitter and receiver.

$$\lambda = \frac{c}{f} = \frac{3x10^8\ {}^{m}/_{s}}{93.7x10^6\ {}^{cycles}/_{s}} = 3.2\ {}^{m}/_{cycle} = \boxed{3.2\ m}$$

$$G_{dB} = 10\log(G) \rightarrow G = 10^{3/10} = 2$$

$$P_r = \left(\frac{(8000W)(2)(2)(3.2m)^2}{16(\pi)^2(15{,}000m)^2}\right) = 9.2x10^{-6}W = \boxed{9.2\ \mu W}$$

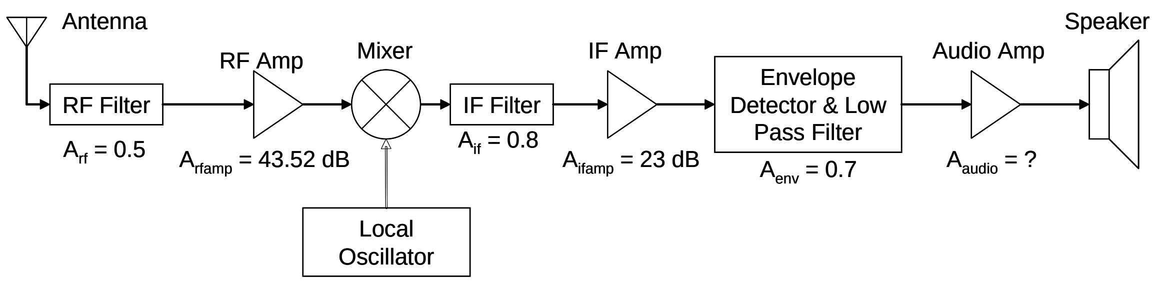
   b. What fraction of the transmitted power actually reaches your radio receiver?

$$A_{P,total} = \left(\frac{P_{received}}{P_{transmitted}}\right) = \left(\frac{9.2x10^{-6}W}{8000\ W}\right) = \boxed{1.15x10^{-9}}$$

   c. Express the number in part (b) in decibels.

$$G_{total} = 10\log 1.15x10^{-9} = \boxed{-89.4\ dB}$$

5. As the last problem showed, the power received at a car radio's antenna is extremely small even if the car is relatively close to the transmitter. The received energy is not enough to power a set of loudspeakers. Below is a typical radio system consisting of several devices that boost (amplify) and reduce the power of the received broadcast as it makes its way from the antenna to your ear.



a. For comfortable listening the loudspeaker needs to receive 5 Watts. The power into the antenna is a measly 56.69 nW. Based on the diagram above, how much gain does the audio amplifier need to provide? Express this number as a power ratio and in decibels.

$$A_{P.total} = \left(\frac{P_{out}}{P_{in}}\right) = \left(\frac{5\ W}{56.69\ nW}\right) = 88.2x10^6 = (0.5)\left(10^{43.52/10}\right)(0.8)\left(10^{23/10}\right)(0.7)(A_{audio})$$

$$A_{audio} = 88.2x10^6 \Big/ (0.5)\left(10^{43.52/10}\right)(0.8)\left(10^{23/10}\right)(0.7) \cong \boxed{70}$$

b. What is the loudspeaker's input power in dBm? Now work the problem by first converting the input power into dBm and then working the problem entirely in decibels.

$$P_{in} = 56.69\ nW = 10\log\left(\frac{56.69\ nW}{1\ mW}\right) dBm = -42.45dBm$$

$$P_{out} = 5\ W = 10\log\left(\frac{5\ W}{1\ mW}\right) dBm = 37\ dBm$$

$$G_{total} = 37dBm - (-42.45\ dBm) = 79.45dB = 10\log(0.5) + 43.52dB + 10\log(0.8) + 23dB + 10\log(0.7) + G_{audio}$$

$$G_{audio} = 79.45dB + 3dB - 43.52dB + 0.97dB - 23dB + 1.55dB = \boxed{18.45dB}$$

$$A_{audio} = 10^{\left(\frac{18.45dB}{10}\right)} \cong \boxed{70}$$

6. Say that an amplifier's maximum output is rated at -0.15 dBW. What is the amp's maximum output in dBm? What is the difference between these two values, and can you draw a larger conclusion about engineering units and decibels?

$$P_{max} = -0.15dBW = 10^{\left(-0.15/10\right)}W = .966W = 10\log\left(\frac{.966W}{1mW}\right) dBm = \boxed{29.85dBm}$$